A Constitutive Equation for High-Temperature Flow

TERENCE G. LANGDON.

(1) According to the constitutive equation, \( \sigma_0 = 0.72 \Gamma / d \), so that \( \sigma_0 \) decreases in magnitude with increasing grain size. However, the experimental results show the opposite trend, with a large value of \( \sigma_0 \) for the material having the larger grain size. Why is this?

(2) What is the rationale for taking \( \delta / d = 0.1 \) and \( D_B / D_v = 2 \) in the calculation of \( K_j \)?

The effective width of the grain boundary, \( \delta \), is generally taken as \( 2b \), so that, for \( d = 12 \mu m \), \( \delta / d \approx 10^{-4} \). The relative magnitudes of \( D_B \) and \( D_v \) depend very critically on the testing temperature, such that \( D_B > D_v \) at low temperatures and \( D_v > D_B \) at high temperatures.

Author’s Closure

The comments on the magnitude of \( \sigma_0 \) and its dependence on grain size are valid. However, it was noted in the original paper[1] that the calculated value of \( \sigma_0 \) is much smaller than the measured value. Quoting from reference 1, “If a reasonable magnitude for the grain boundary energy \( \Gamma \) is taken to be 2000 ergs/cm², the equation yields a value of 0.041 ksi for \( \sigma_0 \)—considerably different from the computed values of 0.353 and 2.07 ksi in Figs. 1(a) and 1(b), respectively. This is probably because there are contributions to the background friction stress (\( \sigma_0 \)) other than simply grain boundary energy, in an alloy as complex as the one examined here. In addition, the elongated grain shape in Fig. 3(b) would be expected to severely limit the ease with which grain switching might occur, a factor which is not taken into account in the derivation of Ashby and Verrall.” It is therefore realized that there are certainly other contributions to \( \sigma_0 \) which have not been quantified, but which may well have a different dependence on grain size from the 0.72 \( \Gamma / d \) term. Ashby and Verrall[2] make the same comment in their paper.

As for the second point, the value of \( \delta / d = 0.1 \) was taken based on the fact that at the high temperatures involved here, the value of \( \delta = 2b \) as suggested by the discusser, is a minimum for low angle boundaries at low temperatures. If \( \delta \) is taken as 10 to 100\( \mu m \) which is more realistic for grain boundaries at high temperature, then for \( d = 12 \mu m \), \( \delta / d = 2.5 \times 10^{-4} \) to \( 2.5 \times 10^{-5} \). The value of \( \delta / d = 0.1 \) is probably an overestimate, as pointed out by the discusser, but if the values of \( D_B \) and \( D_v \) are not several orders of magnitude different, then the precise value taken for \( \delta / d \) is not critical. As for the comment on the magnitudes of \( D_B \) and \( D_v \), \( D_v \) cannot be greater than \( D_B \), even at high temperatures. They may approach one another at high temperatures, but published data on several metals show that they do not cross. \( D_B / D_v = 2 \) was taken as a good estimate of high temperature behavior in the absence of any actual data on the magnitude of \( D_B \) for titanium. If data exist showing \( D_v > D_B \) at high temperature (and this author is not aware of any) then the data are probably in error.

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2Department of Materials Science, University of Southern California, Los Angeles, Calif.

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