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A DETERMINISTIC PARAMETRIC WATER-BALANCE MODEL

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A simple deterministic model, that needs little input-data, is derived. It is based on an analysis of the recession curve, where new considerations are introduced. Input data consist of time-series of precipitation, initial values of storage in upper and lower soil-storages and in interception storage, and initial runoff. Output data are time-series of evapotranspiration, interception, upper and lower soil-storages and runoff, all given as 2-hour mean values.

Introduction

In 1968 in the north-west of Västmanland, central Sweden, in a moraine-covered granitic zone close to the village of Kloten, a project was started to study the effects of forest-fertilization and clear cutting on the chemical and biological status of lakes as well as on the hydrological cycle.

In order to prove hydrological changes in the area, deterministic water-balance models for five experimental basins will be built up. They are all small plots ranging from 0.27 to 3.17 km².

The result published here is of the attempt at constructing such a model for the area. Although the model is not yet tested against independent data, it is felt it is of sufficient interest for publication.

The experimental basin

This study deals with a catchment of 0.63 km² (Fig. 1). It has a total range of relief of 60 m. The stream coming from the lake Vitalampa (0.03 km²) in the

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upper part of the catchment has no branches. The stream is surrounded by swamps (0.02 km^2), which are concentrated at the top of the stream. The efflux area around the stream is well-defined by its characteristic flora, consisting mainly of sphagnum. The basin, excluding the lake and the swamp, is covered by mature pine-forest (11 %), by young pine-forest (64 %), and the rest is clear-cut.

The data

The data used are 2-hour values of precipitation and runoff. The precipitation was recorded by a Hellman-recorder and the runoff by an Ott water-stage recorder at a 120°-triangular overflow weir.

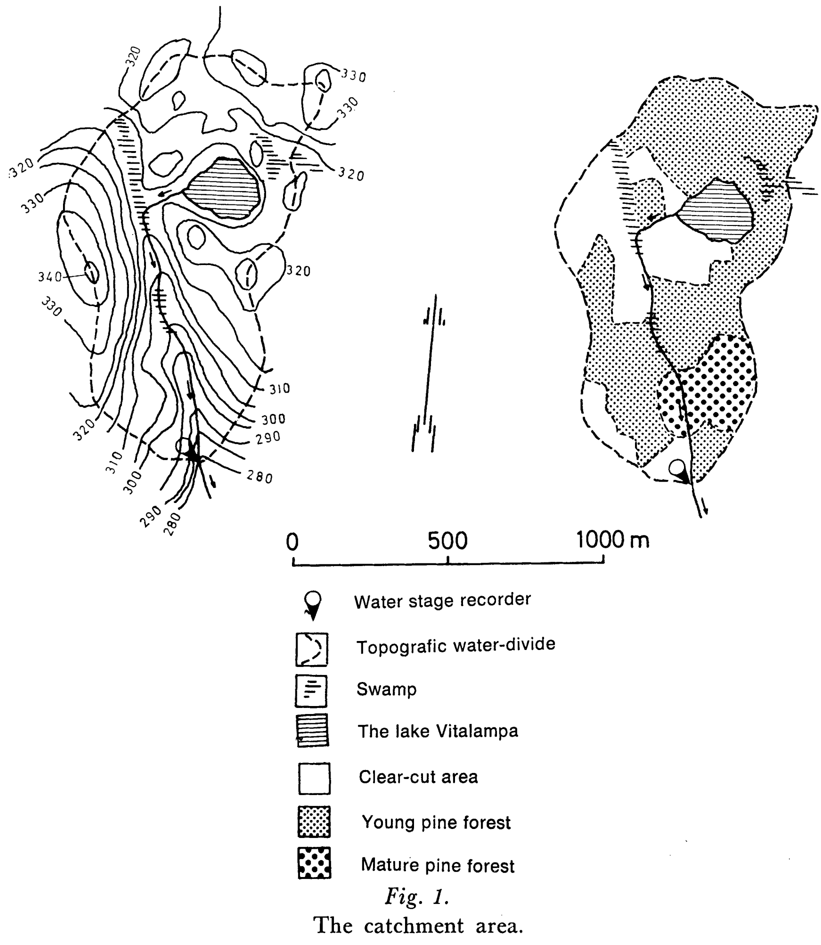

Fig. 1.
The catchment area.

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Output

The output from the model is 2-hour values of runoff. As a by-product, a time-series of storage in upper and lower soil storages is obtained, as well as a series of computed evapotranspiration.

The model

To some extent the model is built up on the same principles as the Stanford Watershed model (Crawford & Linsley 1966). The drawback with the latter is that it is too complicated and needs too many parameters to be estimated. Not least from an economic point of view, it is advantageous if the hydrological processes can be described in a simpler way.

A deterministic model of the same type as the one described here has been built up in the Kericho Experimental Basin, Tanzania (Gathuru Mburu 1971).

The elements of the model

It is assumed that the drainage area can be described as a series of boxes put on top of each other. Each of them defines a part of the hydrological cycle. The model is built up of four boxes representing, from the top, interception storage, upper soil storage, lower soil storage and storage in the saturated zone (Figs. 5, 6).

The model neglects overland flow for two reasons: (i) It has never been observed in moraine catchments with coniferous forest in Sweden, and (ii) 2-hour values of precipitation do not say enough about the intensity of precipitation for the overland flow to be calculated.

Fig. 7 shows an idealized view of the catchment as consisting of an infiltration area and an efflux area. In the model the efflux area, the swamps, the lake and the stream are regarded as a part of the saturated zone. This makes it possible to allow precipitation to enter and evaporation to leave this box.

1. *Interception storage.* The interception storage can be looked upon as a bucket. When its water content exceeds a certain value, the interception storage capacity, the excess will immediately run out.

$$\begin{aligned} AP &= AA - ISC \\ AB &\equiv ISC \end{aligned} \quad \text{if } AA > ISC$$

$$\begin{aligned} AP &= 0 \\ AB &\equiv AA \end{aligned} \quad \text{if } AA \leqq ISC$$

where AP is precipitation that infiltrates, AA and AB are actual interception storages at time t and t + 1, respectively. ISC is interception storage capacity.

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The evaporation acts upon the interception storage as upon a free water surface, *i.e.* evaporation is equal to potential evaporation. When the bucket is empty there will, of course, be no evaporation from it. The value of potential evaporation used in the model is taken as an optimized coefficient (= 1.122) times the mean evapotranspiration, because of lack of data on meteorological factors such as temperature and wind.

The daily mean evapotranspiration has been calculated from the water balance equation applied to the 73-day period, 24.5–3.8, 1970.

$$\Sigma P = \Sigma Q + \Sigma E + \Delta S + \Sigma Gl,$$

where

P = precipitation
Q = runoff
E = evapotranspiration
ΔS = change in storage
Gl = ground water leakage.

Gl is assumed to be small, which is not unreasonable for the area. It is therefore neglected. As an assumption ΔS can also be neglected since the stream discharge was equal at the beginning and at the end of the period. On the other hand, the gradient of the recession curve was steeper at the end than at the beginning of the period. This indicates a decrease in total storage during the period.

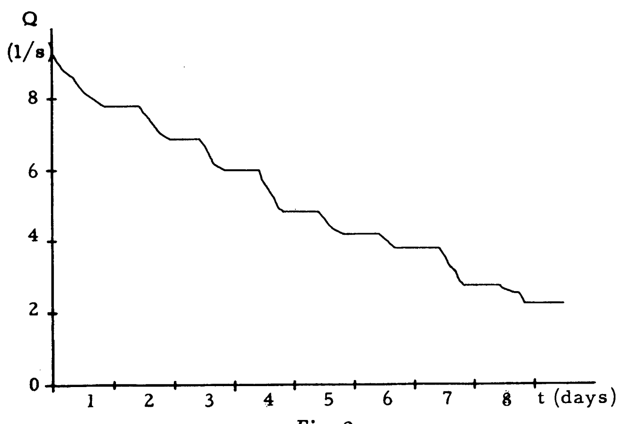

Fig. 2.
A part of the observed recession curve showing the well-defined daily variation used in the derivation of the daily evaporation distribution.

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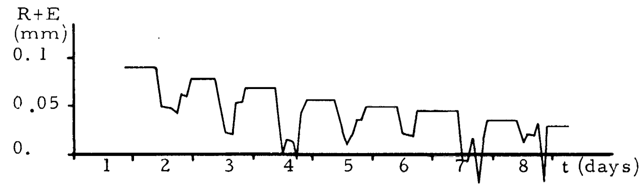

Fig. 3.
Computed recharge, RE, as the sum of recharge, R, and evaporation, E.

The daily variations of the recession curve (Fig. 2) have been used to calculate a daily mean evaporation distribution.

The observed runoff was routed backwards through the box of the saturated zone (see Fig. 5) during periods without precipitation. That gives the difference of the recharge and the evaporation from the saturated zone.

$$Q_t = A \cdot Q_{t-1} + (1-A) \cdot RE_t$$
$$RE_t = (B-C) \cdot R_t - C \cdot E_t$$

Where Q_t is runoff, RE_t is net recharge, R_t is recharge and E_t is evaporation from the saturated zone (= potential evaporation), all at time t. A is a constant, B and C are scaling factors.

RE_t was plotted in a diagram (Fig. 3) and, assuming a smooth depletion for the recharge, R, the daily evaporation distribution was found as the difference between RE_t and R_t. The daily mean evaporation distribution was defined as the mean of all calculated daily evaporation distributions. It was afterwards divided by the daily mean evaporation to get the normalized daily evaporation distribution (Fig. 4). A time-lag of 4 hours was introduced due to the time of concentration (Svenberg 1970).

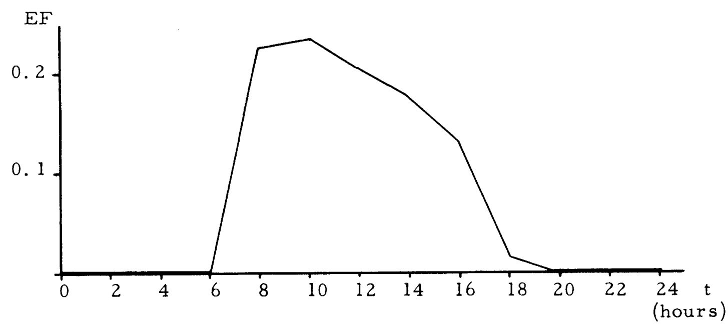

Fig. 4.
Normalized evaporation curve, as derived from Fig. 3, with a time-lag of 4 hours.

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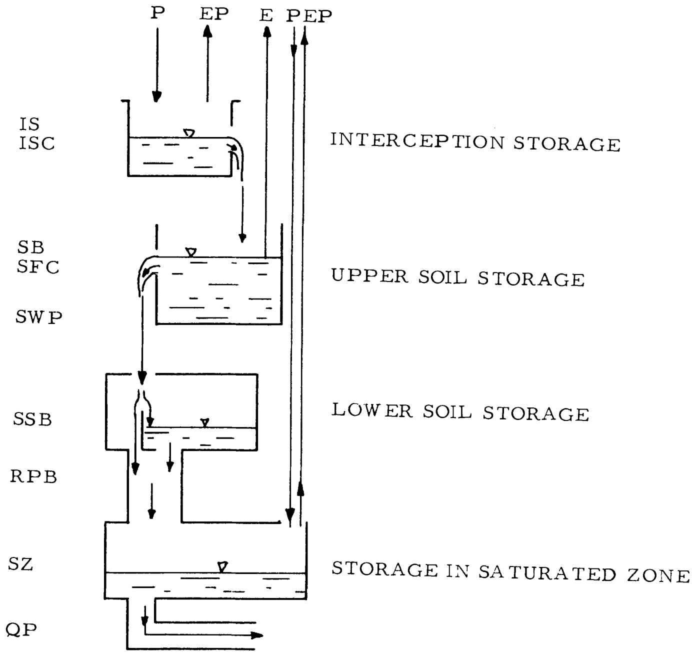

Fig. 5.
Structure of the model.

2. *Upper soil storage.* It is built up in exactly the same way as the interception storage, with the only difference that the evapotranspiration is proportional to the square root of ((SB – SWP)/(SFC – SWP)), where SB is actual upper soil storage, SFC is soil field capacity, and SWP is soil wilting point capacity. Fig. 8 shows this proportionality. If SB is greater than SFC, the evapotranspiration is regarded as potential, i.e. E/EP in Fig. 8 is equal to 1.

3. *Lower soil storage.* Except for the evapotranspiration, the output from the upper soil storage flows in two directions. One fraction goes straight down to the saturated zone, while the other is diverted to the lower soil storage, which acts as a long-time buffer.

$$\mathrm{RPB} = \mathrm{A} \cdot \mathrm{SSB} + \mathrm{B} \cdot \mathrm{SSA}$$
$$\mathrm{SSA} = (1 - \mathrm{A}) \cdot \mathrm{SSB} + (1 - \mathrm{B}) \cdot \mathrm{SSA}$$

where RPB is recharge, SSB is seepage from the upper soil storage, SSA is lower soil storage, A and B are coefficients.

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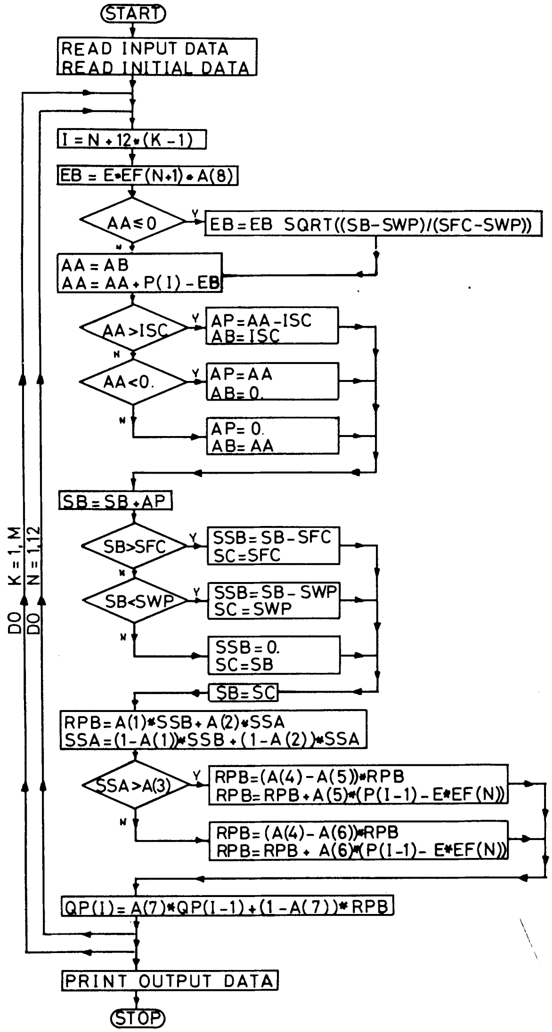

Fig. 6.
Flow-chart of the model for runoff simulation.

4. *The saturated zone*. This is the main controlling box of the model. The theory used has been outlined by Prof. E. Eriksson, Uppsala, Sweden (personal communication) for ground water storages. It is based on an analysis of the recession curve.

In the model, the box of the saturated zone contains the active ground water

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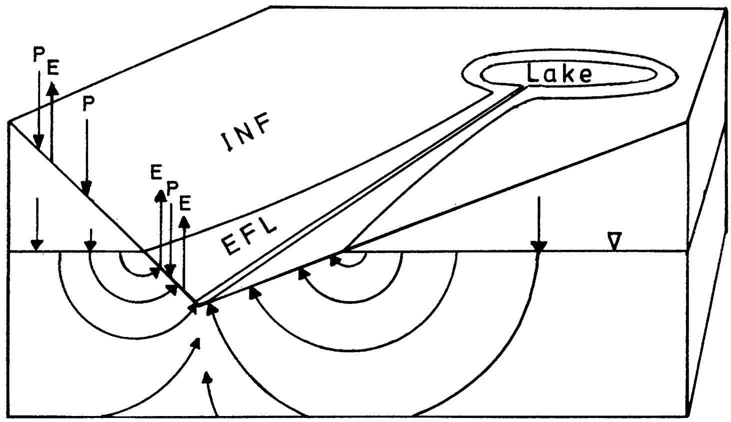

Fig. 7.
Idealized view of the catchment area showing the zones of infiltration and efflux.

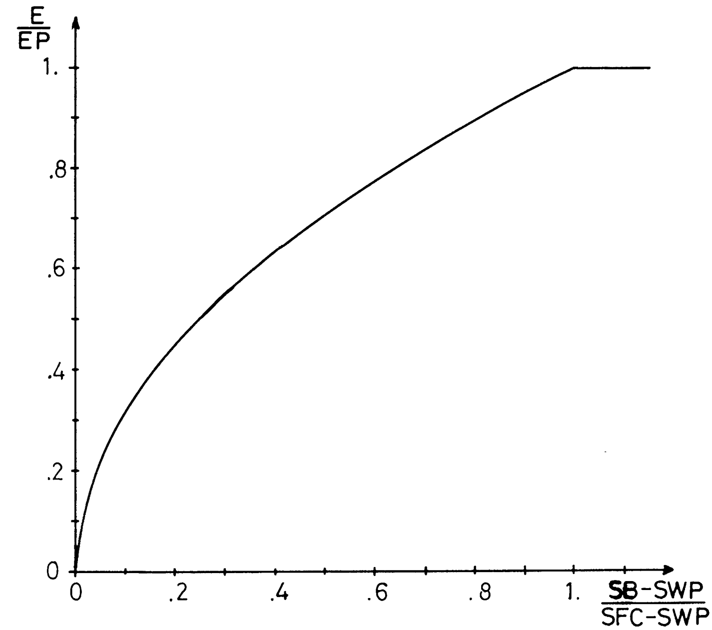

Fig. 8.
The suggested dependence of evapotranspiration on soil moisture.

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Table 1.
Values of coefficients.

A(1)	0.085
A(2)	0.0086
A(3)	7.76
A(4)	88.06
A(5)	8.81
A(6)	4.77
A(7)	0.915
A(8)	1.122

storage, water in the stream, the lake, the swamps and also water in the efflux area. This means that there is an extra hole in this box through which it can communicate with the atmosphere, e.g. rain coming in, evaporation going out. The basic considerations of this box are:

Here, as for the others, the equation of continuity holds:

$$\frac{dV}{dt} \equiv R - Q, \tag{1}$$

where V is the water volume in it, R is input and Q is output. As the aquifer is unconfined, it can be regarded as a bucket, with a hole in the side. Input is recharge and precipitation, minus evaporation. Output is runoff. As the area of the water-table is large, compared to the area-changes due to water-level fluctuations, the area is considered independent of water-level and constant. Probably the mean porosity is a linear function of depth, i.e. the water volume is a linear function of the water level.

Now the dependence of the output, Q, formulated through the water level, h, to the volume, V.

$$V(t) = V(h(Q(t))) \tag{2}$$

The change in storage with respect to t:

$$\frac{dV}{dt} = \frac{\delta V}{\delta h} \cdot \frac{\delta h}{\delta Q} \cdot \frac{dQ}{dt}, \tag{3}$$

where $\frac{\delta V}{\delta h} \equiv$ const.

In (3) let
$$L \equiv \frac{1}{\frac{\delta V}{\delta h} \cdot \frac{\delta h}{\delta Q}} \tag{4}$$

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For finite differences (1), (3) and (4) give

$$Q_t - Q_{t-1} = L \cdot \frac{R_t + R_{t-1}}{2} - L \cdot \frac{Q_t + Q_{t-1}}{2} \tag{5}$$

With the time interval studied $\frac{R_t + R_{t-1}}{2} \approx R_t$ (6)

Q_t evaluated from (5) with the approximation (6) gives

$$Q_t \equiv \frac{2 - L}{2 + L} \cdot Q_{t-1} + \frac{2 \cdot L}{2 + L} \cdot R_t \tag{7}$$

Let $\lambda = \frac{2 - L}{2 + L}$. Then (7) can be written as

$$Q_t = \lambda \cdot Q_{t-1} + (1 - \lambda) \cdot R_t, \tag{8}$$

which is a first order Markov process. To obtain a value of λ, Q_{t-1} is plotted against Q_t on log-log paper (Fig. 9) for all recession periods of the runoff record. All points will lie below a 45° line and as Q_t is the sum of runoff àt time $t - 1$ and recharge at time t, the line which gives λ must lie below all points. How and where to draw this control line is a matter of judgment. If it can be drawn parallel to the 45°-line, however, this means that λ is not a function of Q and then that $\frac{\delta h}{\delta Q}$ in eq. (3) is constant. From Fig. 9 we can see that this is a reasonable assumption. The resulting $\lambda = 0.91$ from the optimization indicates that there was always recharge during the period studied.

The parameters, coefficients and starting values of the variables

The parameters are

ISC = interception storage capacity
SFC = soil field capacity
SWP = soil wilting-point capacity

Values of ISC have been suggested to lie between 2.5 and 5.0 mm (Crawford et al. 1966). The value of SFC has been derived from values given by Tamm & Wiklander (1967), together with observed ground-water levels in the area. Actually the model is not appropriate for alterations in the value of SFC, because once it is chosen, the actual storage, SB, will adjust itself around SFC. Furthermore, only the value (SFC-SWP) is essential since the value of SWP does not affect the computations. Therefore a transformation was done, which reduced SWP to zero (SFC = SFC - SWP, SWP = SWP - SWP).

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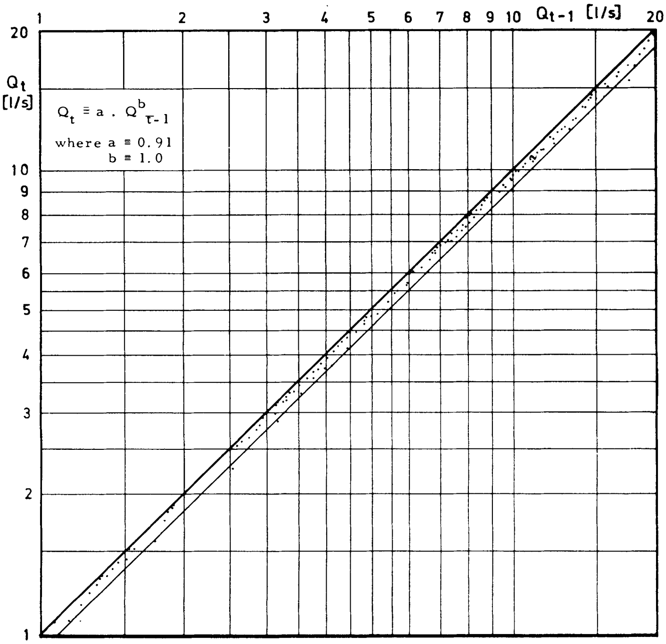

Fig. 9.

Plot of runoff at time t, Q_t, against runoff at time t–1, Q_{t-1}, for the recession part of the runoff record, showing the control line (the lower line) which defines the runoff from the saturated zone.

FITTING METHOD

The optimization of parameters, coefficients and initial values of the variables (all called parameters below) has been done successively, with six at the same time as a maximum.

The method has been a random search method based on a narrow range, moving scan, $p(x, x^*)$, where the range of possible trial values of each parameter is constrained as follows:

$$x_i^* - \frac{r}{2} \leqq x_i \leqq x_i^* + \frac{r}{2}$$

where x_i^* is the former value of parameter no. i, x_i is the new value of the same parameter and r is the scan range (Karnopp 1963). The scan range has continuously been adjusted to give the optimal adjustment of x_i at each step. This random search parameter fitting method is a powerful tool, and compared

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with what probably is the best deterministic search method (Rosenbrock 1960), it has the advantage of simplicity and of being less likely to just find relative optima.

On the other hand, Ibbitt & O'Donnell (1971) have shown that with good starting values of the parameters, Rosenbrock's deterministic search method gives the optimum more rapidly. As a criterion of goodness of fit between measured and computed runoff, the sums of squared deviations were used. To be able to compare the results obtained with those given by others, the normalized objective function value, F_0, was calculated.

$$F_0 = \frac{\left(\frac{F}{m}\right)^{1/2}}{\frac{\Sigma Q_{obs}}{m}} = \frac{(m \cdot F)^{1/2}}{\Sigma Q_{obs}},$$

where $F = \sum_{1}^{m} (Q_{obs} - Q_{comp})^2$, Q is the observed, respectively, the computed runoff, and m is the number of observations (Ibbitt et al. 1971).

Final discussion and results

Fig. 10 shows the observed and the computed runoff, precipitation, as well as the derived time series of the upper soil storage and lower soil storage. The aim to get a good over-all fit is achieved for the peaks and the recessions, as well as for the time of maxima. Compared to the few input data and the simplicity of the model, the computed runoff values agree fairly well with those observed (Table 2). The worst fit is located at the first big peak. This error contains about 40 % of the total error. Probably it is due to errors in the precipitation record, or to wrong initial values of the variables. As far as the water-balance is concerned, the model seems to be good. The evapotranspiration stands for more than 2/3 of the total water loss during the period and the runoff only for about 1/3. It is amazing what good results can be obtained without any initial information about the time distribution of this major term. By the water-balance equation the evapotranspiration was calculated to be 152.8 mm and the model gives 161.3 mm. The change in storage was assumed to be zero, but the model gives – 10.8 mm, or about as much as the extra loss by evapotranspiration. The normalized objective function value is $F_0 = 0.295$,

Fig. 10.
Observed and generated time-series for the period 24.5–3.8, 1970.

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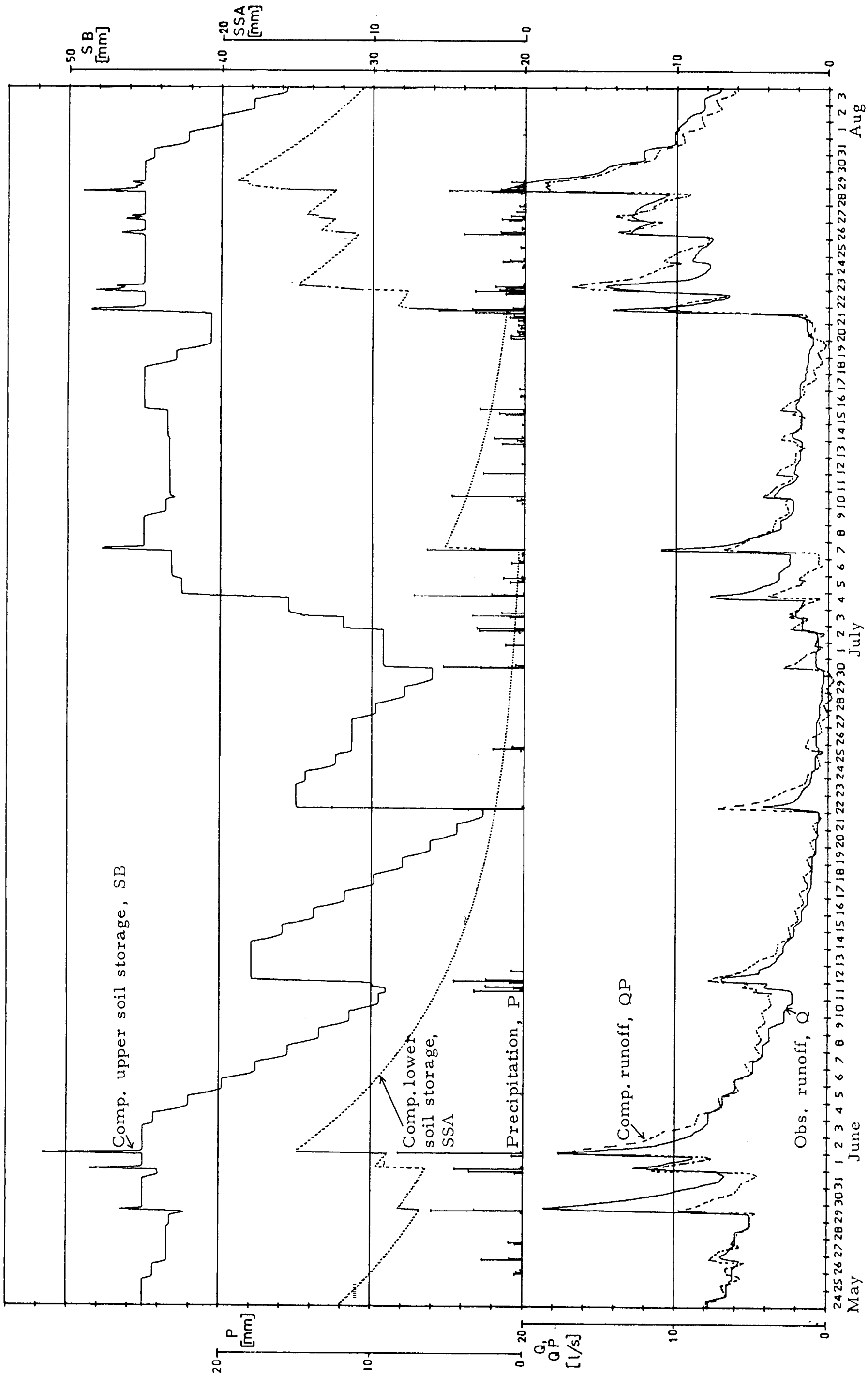

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Table 2.
Some numerical results.

	Observed	Calculated	
Sum of runoff	4230.5 l/s	4192.3 l/s	
Mean runoff	4.90 l/s	4.85 l/s	
Variance	18.8	17.1	
Sums of deviations			– 38.6 l/s
Sums of squared deviations			1.643×10^3
Mean error			1.38 l/s
Residual variance			1.9
Normalized obj. fcn. value, F_0			0.282

Table 3.
List of symbols.

Parameters	Symbol	Value
Interception storage capacity	ISC	4.0 mm
Soil field capacity	SFC	45.0 mm*
Soil wilting point capacity	SWP	0. mm*
* after transformation		
Constants		
Mean evapotranspiration	E	2.09 mm
Normalized daily evaporation distribution	EF(I), I = 1, 12	
Variables		
Precipitation	P	
Precipitation, infiltrating	AP	
Evapotranspiration, calculated	EB	
Interception storage at time t resp. t + 1	AA, AB	
Upper soil storage at time t resp. t + 1	SB, SC	
Percolation	SSB	
Lower soil storage	SSA	
Recharge	RPB	
Runoff, observed	Q	
Runoff, calculated	QP	

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which seems high compared to experiments done in laboratory by Ibbitt et al. (1971).

The model was built with the intention of being able to detect hydrological changes within the catchment in the future. Changes in the variability of the runoff will be detected by a change in the variance. This change must exceed the residual variance (Table 2).

The work to make the model more sophisticated will continue. More field data and better understanding of the laws governing the hydrological processes will certainly improve the model.

ACKNOWLEDGMENTS

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