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A new Expression for the Uncertainty of a Current Meter Discharge Measurement

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A method for the conduction of routine current meter discharge measurements practised in Denmark and Norway is described. Contrary to traditional methods (ISO-recommendations) it incorporates information about the geometry of the stream profile in question into the choice of measurement layout. An associated method for calculation of the measurement uncertainty is developed. By tentative application to a testset of mesaurements it is rendered probable that the developed method will be able to provide satisfactory estimates of the uncertainty of current meter measurements while at the same time having a wider applicability and being easier to use than existing methods.

Introduction

The paper is in four parts. The first part reviews contemporary methodologies for the conduction of routine current meter discharge measurements. In doing so, a more rigorous outline of principles employed in Denmark and Norway than has previously been given is attempted. This is done by formulation of *the principle of uniform resolution.* The described Scandinavian measuring practice is termed the Flex method. In the second part, a method for calculation of the uncertainty of a Flex measurement is developed. This consists of two uncertainty equations termed "coarse" and "fine", respectively, as the former only takes into account overall measurement characterisics while the latter considers individual vertical characteristics. The two newly developed equations parallel the two traditional uncertainty equations, and these are included to facilitate direct comparison. The third part is a comparative application of the two times two uncertainty equations to a Danish testset of 154 measurements and the fourth part provides a concluding discussion.

Traditional notation is used as widely as possible, but in a few cases it has been necessary to supplement that. Nevertheless, it is believed that the notation is self-explanatory and thus definitions are not necessarily included in the text. They are, however, given in a complete list of notations p. 200.

As an exception it is specifically emphasized at this point that when i and j are used as summation indexes, i always refers to one of the m verticals in a traditional current meter discharge measurement while j refers to one of the N single point measurements.

Present Methodologies

k-point Methods

The most authoritative directions for the conduction of current meter discharge measurements are found in (Herschy 1978, 1985) and (ISO 1979).

For routine measurements the so-called Reduced point methods are advanced. At least 20 verticals per measurement is recommended, – generally with an equal distance spacing between each, provided that no single vertical accounts for more than 10 % of the total discharge. A fixed number k of measuring points per vertical, to be located at specific relative depths is prescribed. k may equal 1, 2 or alternatively 3, 5 or 6.

The virtues of the k-point methods are their simplicity and the fact that they are associated with a well-documented method for calculation of the measurement uncertainty, (Herschy 1978, 1985) and (ISO 1978, 1979). This method, however, incorporates no information about profile geometry in the structure of its equations.

The k-point methods also have serious drawbacks. The discharge uncertainty calculation requires the evaluation of a number of individual contributory uncertainties. Suggested values for these are tabulated but to quote Herschy (1985): "It is always advisable ... for users to either confirm these values *for a particular gauging site or* to establish their own values", (emphasis imposed). This subject is further discussed in (ISO 1985).

From the above, it is evident that rigid application of k-point methods necessitates substantial effort beyond the mere performing of measurements. Alternatively the tabulated values may be used with no confirmation but then, of course, one has to spend a certain amount of effort in a specific manner deemed "optimal" by the fixed content of the tables irrespective of the stream profile in question. It is readily demonstrated that the latter approach is suboptimal in the sense that an improved reproducibility may often be achieved with a differing layout of measurements, possibly comprising a reduction of the total number of measuring points (Frost, Lintrup and Krogdahl 1988). The inherent loss in accuracy may be considerable.

An Alternative Method

For the reasons given, the *k*-point methods have never been widely used in Denmark. Instead, an empirically based practice as yet not named and with no rigorous definition has evolved from sound but heuristic arguments (Hedeselskabets Hydrometriske Undersøgelser 1986). Because of its inherent flexibility in incorporating information about the profile geometry into the choice of measurement layout this practice will be referred to as the *Flex method.* Below an exposition of the main principles employed is attempted.

For any stream profile, subareas with high velocities are more important in the determination of the total discharge and thus deserve special attention. Likewise, the areas where the velocity is most variable make large relative contributions to the uncertainty of the total discharge and thus deserve special attention too. These two principles will tend to cancel each other as generally velocity is most variable where shear stress is most significant and causes velocities to be low. It is thus deemed reasonable to make no explicit a priori assumption about the optimal distribution of measuring points from velocity distribution considerations. As a guiding principle a measurement consequently ought to have a uniform resolution of the profile in measuring points, possibly differing in horisontal and vertical direction.

The guiding principle explained above may be formally stated as *the principle of uniform resolution* in a measurement. It may be written

$$\frac{m}{p} \equiv P_0 \frac{B}{\tilde{D}} \qquad (1)$$

That is: The number of verticals m and the average number of points in these p should be chosen so that their ratio approximately equals the ratio of profile breadth B to the average profile depth D multiplied by the aspect ratio P_0. As $p \equiv N/m$, Eq. (1) provides a sufficient basis for discriminating between different measurement layouts.

Eq. (1) only takes into account overall characteristics of a measurement. The expression may be generalised to yield an equation governing the characteristics of individual verticals

$$\frac{1}{p_i} = P_0 \frac{b_i}{d_i} \; , \quad i = 1 \; ... \; m \qquad (2)$$

When considering individual vertical characteristics another relation should also be satisfied for the measurement layout to comply with the guiding principle

$$\frac{p_i}{b_i d_i} \equiv \frac{N}{A} \; , \quad i = 1 \; ... \; m \qquad (3)$$

That is: The distribution of measuring points among verticals should correspond to the vertical areas relative to the profile area. It is observed that the $2m$ equations Eq. (2) and Eq. (3) provide a sufficient basis for discriminating between different choices of b_i and p_i, $i = 1 \ldots m$.

When the geometry of a profile is not known in advance, Eq. (1) or Eq. (2) and Eq. (3) provide only an implicit clue to the arrangement of a measurement. Nevertheless, it is found that experienced technicians can perform measurements that largely comply with the principle of uniform resolution, though of course, for a limited number of points the above equations can never be completely satisfied.

Hydrometrical experience and recommendations in Norway are similar to the situation described for Denmark (Tilrem and Viken 1987).

The Flex Calculation of Uncertainty

Thus far, the main drawback of Flex measurements has been the lack of an expression for calculating the uncertainty associated with them. The very purpose of this paper is to ameliorate this situation. It is emphasised that only the calculation of the *random* uncertainty is considered in the following.

Firstly, the various sources of random uncertainty in a measurement are described. Next, an expression is found for the random uncertainty considering only overall characteristics of a measurement. Lastly, the corresponding expression which takes into account individual vertical characteristics is given.

The Sources of Random Uncertainty

In an ordinary current meter measurement, assuming textbook procedures for integrating over the profile area from computed point velocities to obtain the total discharge, sources of random uncertainty come in three groups: 1) Uncertainty in the various values read off for the relevant parameters, 2) Uncertainty in the determination of the velocity profile because of imprecise placement of points – only a formal difference from the first group, 3) Computational uncertainty in the integration of the velocity profile because of interpolation between points and extrapolation to the surface.

Developing each group by member and corresponding contributory uncertainty one finds:

1) Breadth of stream (B): X_B
   Depths of soundings (d_i, $i = 1 \ldots m$): X_{d_i}
   Recorded revolutions (n_j, $j = 1 \ldots N$): X_{c_j}
   Exposure time (t_j, $j = 1 \ldots N$): X_{e_j}

2) Horisontal placing of verticals (m times): X_b, $(=X_B)$
   Vertical placing of points (N times): X_d

3) Interpolation of velocity profile: X_N

The Coarse Flex Uncertainty Equation

The sources of uncertainties just identified are assumed independent except for the m horisontal placings of verticals. Errors in these correspond to the uncertainties of vertical widths. A strong negative correlation in adjacent verticals is allowed for them (in accordance with computation of the total discharge under the restriction $\Sigma b_i = B$) by substituting the error of the full stream width, X_B. A correction factor f is introduced to take into account the "degree of improper distribution" of the points over the profile and the uncertainty contributions are combined using ordinary root-mean-square technique yielding

$$X_Q \equiv \pm (f X_N^{\,2} + \frac{m+1}{m} X_B^{\,2} + \frac{m+N}{mN} X_d^{\,2} + \frac{1}{N} (X_c^{\,2} + X_e^{\,2}))^{\frac{1}{2}} \tag{4}$$

To find an expression for f it is natural to consider Eq. (1). Upon restructuring it reads

$$1 = P_0 \frac{Bp}{\bar{D}m} = P_0 \frac{B^2 N}{Am^2} \tag{5}$$

We can thus very naturally write

$$f = f_c \equiv f_c(P_1) \quad , \quad P_1 = P_0 \frac{B^2 N}{Am^2} \tag{6}$$

It is scarcely possible to derive a functional relationship between f_o and P_1 along strictly analytical lines. It is known that

$$1 < f < \frac{X^2_{N=1}}{X^2_N}$$

– the upper bound being of no relevance to real measurements. Additionally heuristic arguments must be employed to arrive at a simple expression for the calculation of the correction factor. Assuming a linear relationship between f_c and m, and making no discrimination whether realised values of m are too small or too large, the relation is

$$f_c(P_1) = \begin{cases} P_1^{-\frac{1}{2}} & , \quad P_1 < 1 \\ P_1^{\frac{1}{2}} & , \quad P_1 > 1 \end{cases} \tag{7}$$

The Fine Flex Uncertainty Equation

Taking into account individual vertical characteristics and weighting vertical uncertainties according to vertical discharges, the expression corresponding to Eq. (4) is found to be

$$X_Q = \pm\{f\ X_N^2 + X_B^2 + \frac{\sum_{i=1}^{m} q_i^2 (X_b^2 + X_{d_i}^2 + \frac{1}{p_i^2} \sum_{k=1}^{p_i} (X_{c_k} + X_{e_k} + X_d))}{(\sum_{i=1}^{m} q_i)^2}\}^{\frac{1}{2}} \quad (8)$$

To find an expression for f it is now natural to consider Eqs. (2) and (3). Restructuring and defining

$$1 \cong P_0 \frac{b_i p_i}{d_i} \equiv r_i \quad , \quad i = 1 \ldots m \quad (9a)$$

$$1 \cong \frac{N b_i d_i}{A p_i} \equiv g_i \quad , \quad i = 1 \ldots m \quad (9b)$$

We can thus very naturally write

$$f = f_f = f_f(r_i, g_i) \quad , \quad i = 1 \ldots m \quad (10)$$

Assuming Eqs. (9a) and (9b) to be equally important, weighting vertical discharges and making no discrimination whether the ratios r_i and g_i are too small or too large, the relation is

$$f_f(r_i, g_i) \equiv (\sum_{i=1}^{m} (\frac{r_i^* + q_i^*}{2}) \frac{g_i}{Q})^{\frac{1}{2}} \quad (11)$$

where

$$r_i^* \equiv \begin{cases} r_i^{-1} & , \quad r_i < 1 \\ r_i & , \quad r_i > 1 \end{cases} \qquad i = 1 \ldots m \quad (12)$$

and a similar relation exists betwen g_i^* and g_i. The square root is still extracted to balance the quadratic influence of one or the other ratio in each vertical.

The assumptions stated might be applied in a different order to yield a slightly different expression.

The Traditional Uncertainty Equations

The Flex uncertainty Eqs. (4) and (8) closely parallel the two traditional expressions for the uncertainty of a Reduced point measurement (Herschy 1985)

$$X_Q = \pm(X_m^2 + \frac{X_b^2 + X_d^2 + X_e^2 + X_p^2 + X_c^2}{m})^{\frac{1}{2}} \quad (13)$$

$$X_Q = \pm\{X_m^2 + \frac{\sum_{i=1}^{m} (b_i d_i \bar{v}_i)^2 (X_{b_i}^2 + X_{d_i}^2 + X_{e_i}^2 + X_{p_i}^2 + X_{c_i}^2)}{(\sum_{i=1}^{m} b_i d_i \bar{v}_i)^2}\}^{\frac{1}{2}} \quad (14)$$

In the following results, Eqs. (4), (8), (13) and (14) are compared.

Table 1 – Main statistics of the distribution of variables in Danish testset (154 measurements).

Variable	Mean	Standard deviation	Minimum value	Maximum value
Q, l/s	1618.	3050.	7.	15878.
X_Q (13), %	9.7	3.3	5.0	22.4
X_Q (14), %	10.5	3.6	5.6	23.3
X_m, %	8.5	2.6	4.4	22.0
m	11.9	3.7	4	23
X_Q (4), %	9.1	4.8	2.3	36.6
X_Q (8), %	9.3	4.5	2.7	35.5
X_N,%	7.5	3.9	2.0	32.5
N	30.8	14.5	5	70
f_c	1.29	0.49	1.00	4.02
f_f	1.34	0.39	1.05	3.12
dif (14, 13) / X_Q (14)	0.07	0.05	−0.12	0.27
X_m / X_Q (13)	0.89	0.13	0.39	0.98
X_m / X_Q (14)	0.83	0.14	0.34	0.95
dif (8, 4) / X_Q (8)	0.04	0.08	−0.18	0.34
sqrt (f_c) X_N / X_Q (4)	0.93	0.09	0.50	0.99
sqrt (f_f) X_N / X_Q (8)	0.92	0.10	0.47	0.99

Test Results

The methods just outlined have been applied to an arbitrary selection of 154 Danish measurements from the period 1986–1988. Herschy's suggested values for the contributory uncertainties were used in computing X_Q Eq. (13) and X_Q Eq. (14). For computation of X_Q Eq. (4) and X_Q Eq. (8), the same values were used for X_B, X_d, X_c and X_e. X_N was found as $X_m(m = N/2)$ and P_o was set to 0.4. For X_N and P_o these relations are of course very approximate!

Main statistics for the distribution of the number of verticals, the total number of measuring points, and 15 derived functions of measurement parameters are given in Table 1.

The mean relative importance of X_m in Eqs. (13) and (14) and sqrt(f)X_N in Eqs. (4) and (8) is apparent: 0.89, 0.83 and 0.92, 0.92, respectively. Considering that the former two only account for the horisontal lack of definition of the velocity profile the latter two are only slightly greater.

Table 2 – Selected correlations between uncertainties, correction factors, the number of verticals and number of measuring points in Danish testset, (154 measurements 1986-88).

r	$X_Q(14)$	X_m	m	$X_Q(4)$	$X_Q(8)$	X_N	N	f_c	f_f
X_Q (13)	0.98	0.78	−0.74	0.78			−0.70		
X_Q (14)	1	0.70	−0.67		0.74		−0.64		
m			1	−0.67	−0.66				
X_Q (4)				1	0.99	0.94	−0.78	0.20	
X_Q (8)					1	0.95	−0.78		0.01
f_c								1	0.94

With the Flex method, the uncertainties are seen to cover a greater range. The suggested functions f_c and f_f perform consistently.

Selected correlations between parameters and functions are given in Table 2.

X_Q Eq. (4) and X_Q Eq. (8) are seen to perform *at least* as consistently as X_Q Eq. (13) and X_Q Eq. (14).

In particular it is noted that X_Q Eq. (13) and X_Q Eq. (14) are more strongly correlated to m than to N while the reverse is true for X_Q Eq. (4) and X_Q Eq. (8). The latter is clearly the more reasonable behavior.

Applying the same test procedures to a set of 23 Scandinavian measurements ($m_{max} = 36$, $N_{max} = 179$ and $Q_{max} = 115$ m^3/s) yielded qualitatively similar results.

Discussion

A new method has been proposed for computation of the random uncertainty of a current meter measurement. Existing methods prescribe individual investigation of a stream profile to determine the contributory structure of certain elements to the total uncertainty, whereas the new method incorporates information about profile geometry directly into its computational structure. Properly calibrated and verified it is therefore a potentially efficient and flexible tool. Individual profile investigations might be avoided and reliable uncertainties might be computed even for measurements at irregular sites.

Particularly important to investigate would be the magnitude of an aspect ratio describing the optimal relation between a measurements horisontal and average vertical resolution P_0 the basic measurement uncertainty depending on the total number of measuring points X_N and a correction factor f as a function of certain dimensionless ratios. It is recommended that the proposed method be tested for its ability to accurately reproduce calculated uncertainties of measurements performed according to traditional recommendations.

In incorporating more information directly into the uncertainty computation the suggested method may be seen as a logical extension of existing theory. An obvious further extension would be to include information about the velocity distribution over the profile. One might at first expect kriging techniques to be a good way of doing this by way of estimating the semivariogram for the velocity. Indeed, one might be able to obtain an estimate of the profile average velocity which was optimal in a certain sense. But it must be noted that existing theory does not permit calculation of the standard error of such an estimate (Journel and Huijbregts 1978, – as explained in Istok and Cooper 1988).

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Notation

A	– The area of a stream profile
B	– The breadth of a stream profile in the water-line
D, D	– The maximum (average) depth of a stream profile
N	– The total number of points in a measurement
Q	– The computed discharge of a measured stream profile
X	– A relative random uncertainty corresponding to the 95 % significance level (app. 2 std.)
$X_{(.)}$	– X associated with (.)
b, b_i	– The breadth of a (the i'th) vertical
c	– Index referring to the calibration of a current meter
d, d_i	– The depth of a (the i'th) vertical
e, e_j	– Index referring to exposure time and through that to pulsation of flow in a (the j'th) measuring point
f	– A correction factor accounting for improper distribution of points in a stream profile
f_c, f_f	– c – coarse, f – fine
i	– Summation index of verticals: $i=1...m$
j	– Summation index of points: $j=1...N$
k	– Summation index
m	– The number of verticals in a measurement
n, n_j	– The number of revolutions of the current meter propeller counted in a (the j'th) measuring point
$p, \bar{p}, p_i$	– The (average) number of points in a (the i'th vertical
q, q_i	– The computed discharge of a (the i'th) vertical
v, v_j	– The stream velocity in a (the j'th) point
v, v_i	– The average stream velocity of a (the i'th) vertical
P_1	– A double dimensionless ratio defined in the text. – Dependent on N and m. P_1 is the independent variable of f_c
P_o	– An aspect ratio (the ratio of an optimal measurements horisontal definition to its average vertical definition)
g, g_i	– A double dimensionless ratio (defined in the text) of a (the i'th) vertical. – Dependent on b_i and p_i and general characteristics of the measurement. g_i and r_i, $i=1..m$ constitute the independent variables of f_f
r, r_i	– As above but only dependent on vertical characteristics
g_i^*, r_i^*	– Simple functions of g_i and r_i, resp. Introduced to avoid notational complexity in the expression of the relationship between f_f and g_i and r_i, $i=1..m$
dif(.,-)	– $X_{(.)} - X_{(-)}$

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