13 Robinson, C. L., and Cameron, A., "Studies in Hydrodynamic Thrust Bearings," *Phil. Trans.*, Vol. 278, No. 1283, Apr. 1975, pp. 351–395.
14 Ettles, C., "The Development of a Generalized Computer Analysis for Sector Shaped Tilting Pad Thrust Bearings," ASLE Paper No. 75-AM-8A-2, 1975.
15 Dowson, D., Hudson, J. D., Hunter, B., and March, C. N., "An Experimental Investigation of the Thermal Equilibrium of Steadily-Loaded Journal Bearings," *IME Proc.*, Vol. 181, 3B, 1966, pp. 70–80.
16 Malcher, L., "Die Federungs und Dämpfungseigenschaften von Gleitlagern für Turbomaschinen," Dipl.-Ing. Thesis, Karlsruhe Technische Hochschüle, 1975.
17 Klumpp, R., "Beitrag zur Theorie von Kippsegment-Radiallagern," Dissertation, Karlsruhe Technische Hochschüle, 1975.

Appendix

The Tridiagonal Algorithm

Beginning with equation (3), the arrays p_j and q_j are defined as:

$$p_j = b_j - \frac{a_j c_{j-1}}{p_{j-1}}$$

$$q_j = d_j - \frac{a_j q_{j-1}}{p_{j-1}}$$

The pressures P_j can then be determined by back substitution:

$$P_J = \frac{q_n}{p_n}$$

$$P_{j-1} = \frac{(q_{j-1} - c_{j-1}P_j)}{p_{j-1}}$$

$$P_1 = \frac{(q_1 - c_1 P_2)}{p_1}$$

where J is the number of equations, $J = M - 2$.

DISCUSSION

R. C. Elwell[1]

This is an important paper on an important subject. Industry uses so many bearings of this type that the bearing vendors publish elaborate catalogs offering a wide variety of sizes and geometries. In addition, equipment manufacturers often make their own in still other geometries specialized to particular kinds of machines.

This bearing type is used mostly for rotor stability. As in all things, there is a price to pay for this advantage. In reference [1], Fig. 10 shows that a pivoted-pad journal bearing may operate with higher babbitt temperatures than an equivalent elliptical bearing. Furthermore, reference [1] estimates that babbitt creep will occur in the temperature range of 132°C (270°F) to 149°C (300°F). It is, therefore, important to be able to predict maximum babbitt temperatures in new designs.

Dr. Ettles should be commended for comparing his analytical results to measured data, particularly other people's data. In his Fig. 10, he compares calculated *film* temperatures with measured *metal* temperatures. Because of the complexity of the problem, it is impressive that his values come as close as they do.

The experimental results in Fig. 10 come from the author's reference [1], a paper by Messrs. Booser, Missana, and Ryan about bearings for large steam turbines. Figure 16 reproduces part of the figure in that paper from which the Author took the data in his Fig. 10. Curves of this shape have been published before for thrust bearings and show the same characteristic—beyond a certain point, the babbitt temperature cannot be reduced further by increasing flow. The scale of this data should be noted; the bearing is 508mm (20 in.) in diameter, running at 3,600 rpm. 12 liters/second oil flow is 190 gallons per minute.

Figure 17 reproduces another figure from reference [1], with two additions; the oil inlet temperature and babbitt creep range quoted in that paper. This is an extreme condition achieved in a large laboratory test—the bearing load of 5.93 MPa (860 PSI) is many times a typical design load. It illustrates several significant points about the performance of large pivoted-pad journal bearings:

1. The maximum babbitt temperature in this type of bearing is much higher than any other temperature in the bearing and occurs just past the load line.
2. The temperature gradient along Pad 4 (directly under the load) is much steeper than in the other five pads, leading one to expect more thermal distortion in this pad than in the rest of the set.

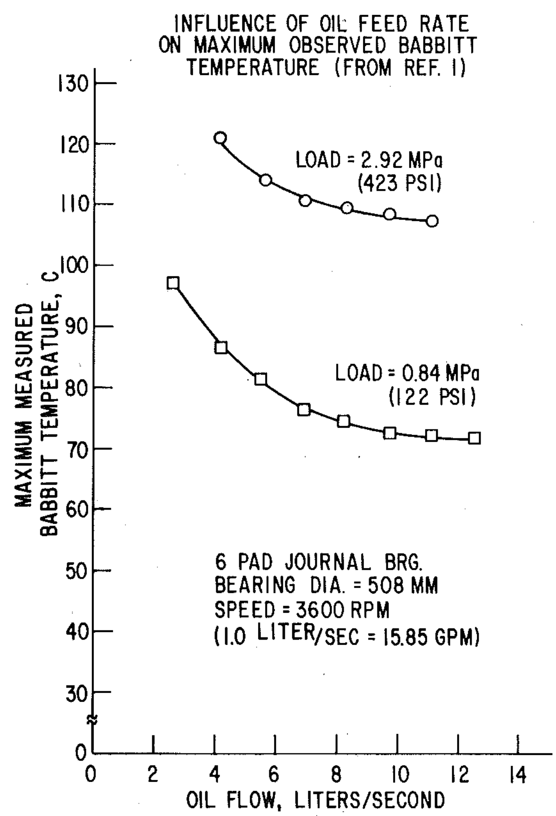

Fig. 16

3. The unloaded pads are all 25C (45F) above the oil inlet temperature.

The discusser has come to the conclusion in recent years that the predominant influence on babbitt temperatures is the temperature of the journal rather than the amount of oil being pumped through the supply grooves.

In the case illustrated here, the journal in this bearing is a steel cylinder weighing about 120 kg (260 lb). Once this reaches thermal equilibrium, after a period of some hours, any thin oil film on its surface will instantly come up to its temperature. In the bearing data in Figure 17 for instance, the shaft journal is probably at a temperature around 70°C (158°F). The oil flow effects shown earlier in Fig. 16 reinforce this conclusion.

[1] Materials and Processes Laboratory, General Electric Co., Schenectady, N. Y. 12345. Assoc. Mem. ASME.

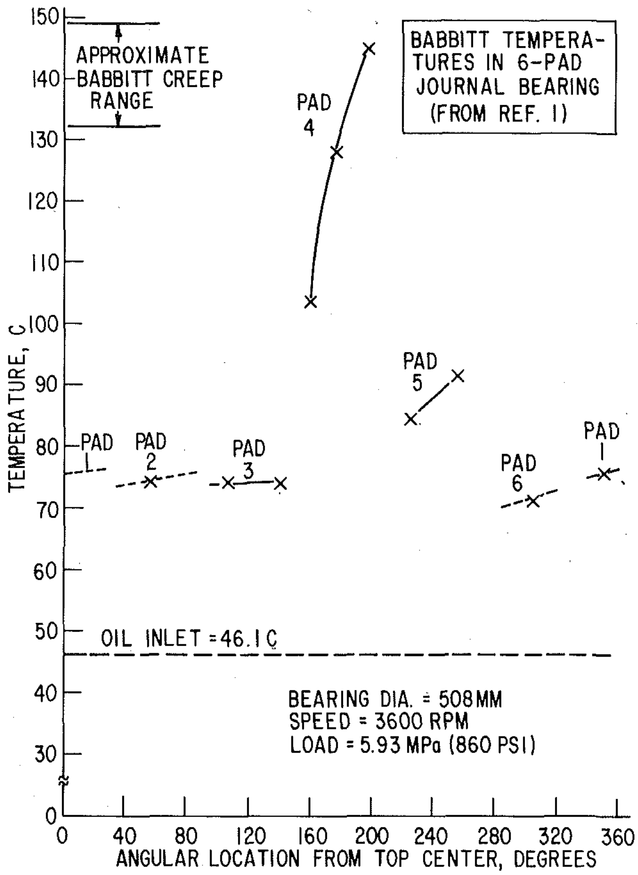

Fig. 17

The question to the author is: how can we be certain of the fundamental physics of this important part of the problem without knowing what the shaft temperatures are at the same time as the bearing temperatures we have measured and analyzed?

In closing, the discusser wishes to point out another significant recent paper which presents analytical results for this type of bearing. It is "Geometry Effects In Tilting-Pad Journal Bearings," by G. J. Jones and F. A. Martin in *ASLE Transactions*, Vol. 22, No. 3, July 1979, pp. 227–244. It will be useful for the physical understanding it gives to many aspects of the design problem.

Author's Closure

The discussion raises two sensitive questions relating to thermohydrodynamic analyses. The first concerns the fundamental matter of temperature differences across the thickness of the oil film. The analysis presented in the paper considers the film in plan and assumes the temperature to be uniform locally through the thickness of the film. The second question concerns the effect of supplied flow to the bearing and its effect on bearing temperature.

It is clear that the temperature distribution within the film of an actual bearing must vary in three dimensions. A three-dimensional analysis is within the state of the art, but, as will be shown later, there are difficulties in selecting realistic boundary conditions for heat transfer. Several three-dimensional analyses have appeared in the literature but, in the author's view, none of these are of practical use in assessing bearing performance.

For the sake of pragmatism and to avoid decisions as to an excessive number of input parameters, the analyst would prefer a two-dimensional analysis. Given this, there is a choice of viewing the film in elevation or plan. These choices are discussed in detail in [18].

It has been shown by theory and experiment that the temperature of the moving surface in a journal or thrust bearing is almost constant, while the temperature on the static parts of the bearing varies considerably. This gives rise to large temperature gradients across the thickness of the film and implies that the film should be considered in elevation. The analysis initially appears to be fairly simple. The method begins with the stress equation:

$$\frac{dp}{dx} = \frac{\partial}{\partial z}\left(\eta\,\frac{\partial u}{\partial z}\right)$$

where u is the local velocity in the x direction and z is the coordinate through the thickness of the film. Integrating twice and applying the boundary conditions for u at $z = 0, h$ gives the local velocity, u:

$$u = \frac{\partial p}{\partial x}\left[\int \frac{z}{\eta}\,dz - A\int \frac{1}{\eta}\,dz\right] + B\int \frac{1}{\eta}\,dz$$

where A and B are further integral terms across the film. Mass continuity can then be applied to give a modified form of the Reynolds equation and the pressure can be found provided the distribution $\eta = f(x,z)$ is known. This method can be extended to three dimensions and appears to be straightforward until a choice must be made for actual values involved with the boundary conditions.

For example, a numerical value of temperature must be assigned to the moving surface. To allow for heat transfer between the static surfaces and the film, the temperature distribution within the pads must be considered. This is not difficult until values of surface heat transfer coefficient must be chosen for all exposed surfaces of the pads.

The method of viewing the film in plan avoids such difficulties with the boundary conditions but uses the invalid assumption that the temperature is uniform locally through the thickness of the film. The question arises as to whether temperature variations through the film thickness affect the most important aspect of fluid film bearings which is the generation of pressure to support a load. The example of the parallel surface bearing shows that this effect is small. In a truly parallel surface bearing, the pressure developed has been shown to be negative. This effect is counteracted by even very slight film convergences due to thermal distortion.

Viewing the film in plan can be said to be satisfactory provided the "correct" values of local viscosity are used. It is certain, as the discussor points out, that the temperature for these effective values of viscosity are not those of the static surfaces. The analysis uses a form of empiricism to give better agreement with static surface temperatures than might be expected. This appears in the application of hot oil carry over theory, which attempts to predict the inlet edge temperature, T_1, in terms of the trailing edge temperature, T_2, the supply temperature, T_s and T_1 itself. The application of the theory is iterative. The numerical coefficients relating to hot oil carry over appear to be a function of sliding speed and the gap size between pads. The choice of coefficient is made from experience and is such that *bearing* temperatures measured under test in a range of different test rigs can be calculated using the analysis.

It is inevitable that some empiricism must enter the analysis of a practical situation. If an analysis is extended to cover (and remove) empiricism, the author's experience is that, usually, more empirical constants appear at the periphery of the analysis rather than less. This is undoubtedly true for a three-dimensional analysis compared to the method described in the paper.

The question of the supply flow and its effect on pad temperature bears on some of the factors already mentioned. The analysis, through some empiricism, is able to treat only the case of a "typical" supply flow as dictated by codes of practice. For the calculation of the required flow to the bearing assembly, it seems that the swept couette flow, $\frac{1}{2}\,Uc_rL$, gives satisfactory results. In the discusser's Fig. 16, this gives a value of 7.4 litres/second which is in the middle of the range of test flows. Flow above this figure does not give a worthwhile drop in bearing temperature.

Additional Reference

18 Ettles, C., and Advani, S., "The Control of Thermal and Elastic Effects in Thrust Bearings," *VIth Leeds-Lyon Conference (Lyon)*, Sept 1979, M.E.P. Publications (I.Mech.E.), London (in press).