Journal of Dynamic Systems, Measurement and Control

Discussion

Inverse Dynamics of Flexible Robot Arms: Modeling and Computation for Trajectory Control[1]

H. C. Moulin[2] and E. Bayo.[2] A method generating causal solution to the inverse dynamics of planar multi-link flexible manipulators has been proposed in the discussed paper.

A very complete nonlinear model of planar multi-link flexible arms has been developed, then simplified by the use of a Virtual Link Coordinate System, of orthogonal Ritz functions, and in the simulation part by appropriately identifying negligible terms, resulting in a more tractable nonlinear inverse problem. However, there seems to be no reasons why the proposed method for inverse dynamics computations should not be applicable to the simpler case in which linear models are used. In this case, as it has been reported time and again by many authors e.g., [1], the transfer function between hub torque and endpoint displacement is non minimum phase, and causal solutions to the inverse dynamics are unbounded, e.g., [1]. Applied to a single-link arm moving in the horizontal plane modelled by a linear system and using the method proposed in the article with m Ritz functions, the following second order system of $m+1$ ordinary differential equations is obtained:

$$\begin{bmatrix} m_{\theta\theta} & \mathbf{m}_{\theta\mathbf{p}}{}^T \\ \mathbf{m}_{\theta\mathbf{p}} & \mathbf{M}_{\mathbf{pp}} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\mathbf{q}}_\mathbf{p} \end{bmatrix} + \begin{bmatrix} 0 & \mathbf{O} \\ \mathbf{O} & \mathbf{K}_{\mathbf{pp}} \end{bmatrix} \begin{bmatrix} \theta \\ \mathbf{q}_\mathbf{p} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{v}_\mathbf{p} \end{bmatrix} \tau \qquad (1)$$

where θ is the angle between a fixed horizontal axis and the horizontal line through the joint axis and the tip of the arm, $\mathbf{q}_\mathbf{p}$ is the $m \times 1$ column vector of modal coordinates corresponding to the m Ritz functions, $m_{\theta\theta}$ is a scalar $\mathbf{m}_{\theta\mathbf{p}}$ and $\mathbf{v}_\mathbf{p}$ are an $m \times 1$ column vector, $\mathbf{M}_{\mathbf{pp}}$ and $\mathbf{K}_{\mathbf{pp}}$ are $m \times m$ matrices which are diagonal if an orthogonal set of Ritz functions is selected, and τ is the actuator torque applied at the hub. As mentioned in the article damping can be incorporated in the model by appropriate means. Given a tip trajectory $\theta(t)$ is a known function of time, and the method proposed to compute the torque $\tau(t)$ that drives the tip along this trajectory is to:

(1) Estimate the actuator torque $\tau_r(t)$ assuming that the link is rigid, that is to use the first equation of system (1) setting $\mathbf{q}_\mathbf{p}$ to 0. This yields: $\tau_r(t) = m_{\theta\theta}\ddot{\theta}$.

(2) Rewrite the last m equations of system (1) as:

$$\mathbf{M}_{\mathbf{pp}}\ddot{\mathbf{q}}_\mathbf{p} + \mathbf{K}_{\mathbf{pp}}\mathbf{q}_\mathbf{p} = \mathbf{v}_\mathbf{p}\tau_r(t) - \mathbf{m}_{\theta\mathbf{p}}\ddot{\theta} \qquad (2)$$

which can be integrated to obtain $\mathbf{q}_\mathbf{p}(t)$ since the right-hand-side is a known function of time.

(3) Substitute the values of $\mathbf{q}_\mathbf{p}(t)$ computed in (2) in the first equation of system (1) to obtain a revised estimate of the torque:

$$\tau(t) = m_{\theta\theta}\ddot{\theta} + \mathbf{m}_\theta^T\mathbf{q}_\mathbf{p} \qquad (3)$$

It is reported in the article: "If one computes actuator torques with the algorithm described, the torque profiles will be oscillatory, [...]. As shown [...] both actuator torques become very oscillatory, where the frequencies are approximately the same as the fundamental frequencies of both links," However, equations (2) and (3) show that the computed torque contains sine and cosine terms with frequencies that are the solutions for ω of the eigenvalue problem $det|\mathbf{M}_{\mathbf{pp}}\,\omega^2 - \mathbf{K}_{\mathbf{pp}}| = 0$, which need not be the natural frequencies of the link. For example, the inclusion of hub inertia or of a tip mass does not change the matrices $\mathbf{M}_{\mathbf{pp}}$ and $\mathbf{K}_{\mathbf{pp}}$, so that the frequencies of oscillations in the torque do not change, yet the natural frequencies of the link do change. This remains true if damping is included. Is the fact that the computed torque is oscillatory an intrinsic property of the inverse dynamics approach, or is it a consequence of proposed numerical approach? Should the authors not be looking for noncausal solutions to the inversion of a nonminimal phase system, as was done in [2–5]?

It is recognized in the paper that oscillatory torques are undesirable and it is proposed to increase damping by some means and concluded: "The proposed inverse dynamics approach based on the flexible arm model needs to be incorporated with some vibration control methods."

It is certainly true that in an actual physical implementation of the inverse dynamics for control purposes it is desirable to have some damping, or even to increase it by some means, in part due to unmodelled phenomena (friction, stiction, parameter uncertainties, unmodelled dynamics in the arm and in the actuators actuator limitations, . . .). However, this last comment confuses the issues rather than provide insight in the problem. A position has to be taken, either the inverse dynamics can be solved and by definition a trajectory can be followed without deviation or residual oscillations (we are talking theory here, not actual physical implementation), or it cannot be solved and other means of control have to be devised. Again, references [2–5] show that trajectories can be followed exactly with non causal bounded non oscillatory inverse dynamics torques with no need for additional damping mechanisms.

Additional References

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3 Bayo, E., and Moulin, H., "An Efficient Computation of the Inverse Dynamics of Flexible Manipulators in the Time Domain," 1989 IEEE Conference on Robotics and Automation, Vol. 2, pp. 710–715.

[1]By H. Asada, Z.-D. Ma, and H. Tokumaru, published in the June, 1990 issue of the JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, Vol. 112, pp. 177–185.

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4 Kwon, D. S., and Book, W. J., "An Inverse Dynamics Method Yielding Flexible Manipulators State Trajectories," 1990 IEEE American Control Conference, Vol. 1, pp. 186–197.

5 Kokkinis and Sahraian, M., "Inverse Dynamics of a Flexible Robot Arm by Optimal Control," 1990, 21st Biennial ASME Mechanisms Conference, ASME DE Vol. 14, edited by S. Derby, M. McCarthy, and A. Pisano, pp. 497–502.

Authors' Closure

The authors appreciate the valuable comments made by Dr. H. C. Moulin and Dr. E. Bayo. We would like to clarify the issue raised by them: whether or not the oscillatory profiles of the computed torques were caused by the nonminimum-phase nature of the system.

In the numerical simulation, we dealt with a two-link arm having a special structure. As described at the beginning of Section 7 and in Fig. 7, the torque of each actuator is transmitted by a transmission to the distal end of the link. Therefore, the system is, in a sense, a collocated system, having no nonminimum phase zeros. In consequence, it is clear that the oscillation mentioned above was not caused by the nonminimum-phase zeros. The point we intended to make in the latter half of Section 7 is that, even though the system is of minimum phase, the results can be oscillatory, particularly for multi-link systems. Therefore,the dynamics inversion must be done with care, and needs further analysis.

In this paper, we do not claim to have developed a method generating causal solutions of the inverse dynamics problem. The main scope and contribution of this paper is the development of an efficient modeling and computation method, which reduce the extremely complicated problem to a simple, tractable one. Depending on the characteristics of a given system, the torque profiles obtained by the proposed method become oscillatory or diverging, but the theory of the modeling and computation is valid for all cases, including both minimum phase and nonminimum phase systems, as stated at the beginning of Section 5.