Nuclear Geometry and Number of Collisions:
Glauber Model and Beyond

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The number of collisions, which is the factor used to normalize the \( pA \) over \( pp \) cross section ratio for hard processes, is calculated within: (i) the Glauber single channel approximation; (ii) adding the Gribov inelastic shadowing corrections; (iii) adding the effects of the short-range \( NN \) correlations. The largest effect is found to be related to inclusion or exclusion of the diffractive channels in the trigger.

§1. Number of collisions

Nuclear effects in hard processes in \( pA \) collisions are usually characterized by the \( pA \) to \( pp \) ratio,

\[
R_{A/N} = \frac{\sigma_{\text{hard}}^{pA}}{A \sigma_{\text{hard}}^{pp}} .
\]

(1.1)

Absolute values of the cross sections are difficult to measure, only the fraction of the total inelastic cross section, \( N_{\text{hard}}^{pA} \), is known. Then, one has to normalize it multiplying the numerator and denominator by the total inelastic cross sections in \( pA \) and \( pp \) respectively,

\[
R_{A/N} = \frac{N_{\text{hard}}^{pA}}{A \sigma_{\text{in}}^{pA}} = \frac{1}{N_{\text{coll}}} \frac{N_{\text{hard}}^{hA}}{N_{\text{hard}}^{pp}} ,
\]

(1.2)

where

\[
N_{\text{coll}} = A \frac{\sigma_{\text{in}}^{pp}}{\sigma_{\text{in}}^{pA}} .
\]

(1.3)

Naively, one may expect that this factor \( T_A(b) \) is all one needs to normalize the hard process, and this normalization is independent of the soft cross section \( \sigma_{\text{in}}^{pp} \). However, \( N_{\text{coll}} \) is defined for events where inelastic collision did happen. Therefore, it must be properly normalized by the probability for the incoming hadron to make inelastic interaction at the given impact parameter,

\[
N_{\text{coll}}(b) = \frac{\sigma_{\text{in}}^{pp} T_A(b)}{P_{\text{in}}(b)} ,
\]

(1.4)

where the partial probability of inelastic interaction in the Glauber approximation reads,

\[
P_{\text{in}}^{Gi}(b) = \frac{d\sigma_{\text{in}}^{hA}}{d^2b} = \frac{d\sigma_{\text{in}}^{hA}}{d^2b} - \frac{d\sigma_{\text{el}}^{hA}}{d^2b} - \frac{d\sigma_{\text{qel}}^{hA}}{d^2b} = 1 - e^{-\sigma_{\text{in}}^{hA} T_A(b)} .
\]

(1.5)
Number of Collisions

Here the cross section $\sigma_{qel}^{hA}$ corresponds to quasielastic scattering with the recoil nucleus broken to fragments. Averaging (1.4) over inelastic collisions at different impact parameters one indeed arrives at the expression Eq. (1.1).

For hard QCD processes the nuclear ratio Eq. (1.2) is usually rather close to one, within about 10%. Therefore, the normalization factor $N_{coll}$ in Eq. (1.2) should be calculated with a high accuracy, of about $\sim 1\%$. However, the Glauber approximation misses several effects leading to the corrections to $N_{coll}$ which are too large to be neglected. Those affects are:

- $N_{coll}$ should be redefined if the experimental set up misses a part of inelastic cross section, for instance the large rapidity gap diffractive process usually escape detection.\(^2\)
- The single nucleon density approximation employed in Eq. (1.4) is subject to corrections related to short range $NN$ correlations.\(^3\) Such corrections usually lead to a higher opaqueness of the nuclear medium, so act in an opposite direction compared to the Gribov corrections.

In what follows we evaluate the corrections to $N_{coll}$ caused by these effects.

§2. Gribov inelastic shadowing

The Glauber model\(^4\) is a single-channel approximation. A multi-channel problem includes the off-diagonal diffractive transitions, Gribov inelastic corrections, as is illustrated in Fig. 1.

Only the lowest order correction, corresponding to a single intermediate excitation, as is illustrated in the top part of Fig. 1, can be calculated,\(^5\)

$$\Delta \sigma_{tot}^{hA} = -4\pi \int d^2b \ e^{-\frac{1}{2} \sigma_{tot}^{hN} T_A(b)} \int_{M_{\text{min}}^2} dM^2 \frac{d\sigma_{sd}^{hN}}{dM^2 dp_T^2}.$$
where the longitudinal momentum transfer \( q_L = (M_X^2 = m_h^2)/2E_h \), and \( \rho_A(b, z) \) is the nuclear density.

The higher order multiple excitation shown in the bottom part of Fig. 1 cannot be calculated directly because of the lack of experimental information about diffractive amplitudes for hadronic excitations.

At high energies \( q_L \to 0 \) and one can sum up the inelastic corrections in all orders switching to the eigenstates of interaction.\(^6\) Those eigenstates can experience only elastic scattering and do not have any inelastic corrections. In this representation the fractional total cross section reads,

\[
\frac{1}{2} \frac{d\sigma_{tot}^{hA}}{d^2 b} = 1 - \left\langle e^{-\frac{1}{2} \sigma_i T_A(b)} \right\rangle,
\]

where averaging is done summing up the eigen amplitudes \( \sigma_i \) weighted with the corresponding distribution amplitude of the eigenstates.

Correspondingly, the Gribov corrections summed up in all orders, get the simple form,\(^6\)

\[
\Delta\sigma_{tot}^{hA} = 2 \int d^2 b \left[ e^{-\frac{1}{2} \langle \sigma_i \rangle T_A(b)} - \left\langle e^{-\frac{1}{2} \sigma_i T_A(b)} \right\rangle \right].
\]

The first non-vanishing term in expansion of the exponential reproduces the lowest order Gribov correction Eq. (2.1).

§3. Missing diffractive channels

While diffractive channels cannot be properly treated within the Glauber single-channel approximation, they can be taken into account within the Gribov multi-channel approach. In this case the interaction probability gets new terms,\(^7\)

\[
P_{in}^{gr}(b) = \frac{d\sigma_{tot}^{hA}}{d^2 b} - \frac{d\sigma_{el}^{hA}}{d^2 b} - \frac{d\sigma_{qel}^{hA}}{d^2 b} - \frac{d\sigma_{diff}^{hA}}{d^2 b} - \frac{d\sigma_{qsd}^{hA}}{d^2 b} - \frac{d\sigma_{tsd}^{hA}}{d^2 b} - \frac{d\sigma_{dd}^{hA}}{d^2 b}.
\]

The new term with subscripts “diff”, “qsd”, “tsd”, and “dd” denote the processes \( hA \to XA \), \( hA \to XA^* \), \( hA \to hN^*(A - 1)^* \) and \( hA \to XN^*(A - 1)^* \) respectively. Usually the trigger used for selection of hard processes is blind to these reactions.

To be able to calculate all these cross section within the eigenstate representation, one should specify the set of eigenstates. In QCD the color dipoles are the eigenstates of interaction,\(^6\) namely a dipole with a given transverse separation \( |r_T| \) interacts with the eigen-amplitude, which is the dipole-nucleon total cross section \( \sigma_{dip}(r_T) \), known from the high-energy phenomenology.\(^8\)

Now one is in a position to calculate all the terms in Eq. (3.1).

\[
\frac{d(\sigma_{el}^{hA} + \sigma_{diff}^{hA})}{d^2 b} = \frac{d\sigma_{tot}^{hA}}{d^2 b} - 1 + \left\langle e^{-\sigma_{dip} T_A(b)} \right\rangle,
\]
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Table I. The cross sections in mb entering Eq. (3.1) for proton-lead collisions, calculated within the Glauber model and the dipole approach with two model for the hadronic distribution amplitudes, $q-2q$ and $3q$ corresponding to quark-diquark and symmetric 3-quark structure of the nucleon.

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
\sqrt{s} = 200 \text{ GeV} & \sigma_{pP}^{\text{tot}} & \sigma_{pP}^{\text{el}} & [\sigma_{sd}^{\text{P}}]_{3P} & \sigma_{qcl}^{\text{P}} & \sigma_{qs}^{\text{P}} & \sigma_{td}^{\text{P}} & \sigma_{dd}^{\text{P}} \\
\hline
\text{Glauber} & 3616.8 & 1446.8 & - & 5.1 & 98.6 & - & 42.3 & - \\
q-2q & 3457.5 & 1313.8 & 33.3 & 7.6 & 96.2 & 3.1 & 41.4 & 3.1 \\
3q & 3514.1 & 1362.3 & 8.9 & 6.3 & 98.9 & 0.6 & 42.53 & 0.6 \\
\hline
\text{Glauber} & 4241.5 & 1794.9 & - & 28.8 & 141.43 & - & 22.9 & - \\
q-2q & 4194.2 & 1755.6 & 5.8 & 33.4 & 141.8 & 0.0 & 23.0 & 0.0 \\
3q & 4207.1 & 1767.3 & 0.9 & 31.2 & 142.5 & 0.0 & 23.1 & 0.0 \\
\hline
\end{array}
\]

The numerical results of calculations are presented in Table I for two energies $\sqrt{s} = 200 \text{ GeV}$ and $5.5 \text{ TeV}$.

### §4. Short range $NN$ correlations

Inclusion of SRC leads to an effective modification of the nuclear profile,\(^3\),\(^9\),\(^11\)

\[
T_A^h(b) \Rightarrow \tilde{T}_A^h(b) = T_A^h(b) - \Delta T_A^h(b),
\]

where

\[
\Delta T_A^h(b) = \frac{1}{\sigma_{tot}^{1N}} \int d^2l_1 \int d^2l_2 \Delta_A^1(l_1, l_2) \Re \Gamma^{pN}(b - l_1) \Re \Gamma^{pN}(b - l_2),
\]

\[
\Delta_A^1(l_1, l_2) = A^2 \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{\infty} dz_2 \left[ \rho_A^{(2)}(r_1, r_2) - \rho_A(r_1) \rho_A(r_2) \right].
\]

Here the usual single-nucleon density and the 2-body density matrix are defined via the multi-nucleon wave function of the nucleus respectively,

\[
\rho_A(r_1) = \int |\psi_o(r_1, r_2, ..., r_A)|^2 \prod_{i=2}^{A} d^3r_i,
\]

\[
\rho_A^{(2)}(r_1, r_2) = \int |\psi_o(r_1, r_2, ..., r_A)|^2 \prod_{i=3}^{A} d^3r_i.
\]

The 2-body density matrix $\rho_A^{(2)}(r_1, r_2)$ was calculated in 3) and 10) using the realistic $NN$ potential including spin, isospin and spin-isospin correlations.
Table II. The cross sections of different large rapidity gap processes in proton-lead collisions calculated including Gribov corrections and $NN$ short-range correlations. The values of $N_{\text{coll}}$ in parentheses correspond to full expression (3.1), i.e. to the case when the trigger misses diffractive channels.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{\text{in}}^p (\sigma_{\text{in}}^p - \sigma_{\text{diff}}^{pN})$</th>
<th>$\sigma_{\text{el}}^p$</th>
<th>$\sigma_{\text{el}}^p (\sigma_{\text{el}}^p + \sigma_{\text{diff}}^{pA})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RHIC</td>
<td>42.1</td>
<td>3228.1</td>
<td>1314.0</td>
</tr>
<tr>
<td>LHC</td>
<td>68.3</td>
<td>3833.3</td>
<td>1655.7</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\text{el}}^p (\sigma_{\text{el}}^A + \sigma_{\text{diff}}^{pA})$</td>
<td>$\sigma_{\text{in}}^p$</td>
<td>$N_{\text{coll}}$</td>
</tr>
<tr>
<td>RHIC</td>
<td>72.0</td>
<td>1842.1</td>
<td>4.75(3.42)</td>
</tr>
<tr>
<td>LHC</td>
<td>113.4</td>
<td>2064.2</td>
<td>6.88(5.67)</td>
</tr>
</tbody>
</table>

As an example, the net effect of $NN$ short-range correlation on the value of $N_{\text{coll}}$ in lead at $\sqrt{s} = 200$ GeV is plotted vs impact parameter in Fig. 2. Numerical results demonstrating the influence of the $NN$ correlations including the Gribov corrections are presented in Table II.

Comparing Tables I and II we see that the effects of $NN$ correlation indeed partially compensate the Gribov inelastic shadowing corrections. At the same time, the largest effect on the value of the normalization factor $N_{\text{coll}}$ is due to inclusion or exclusion of the diffractive channels in the trigger for the hard reaction.

Acknowledgements

I am grateful to Claudio Ciofi degli Atti, Boris Kopeliovich and Ivan Schmidt for the fruitful collaboration. I also thank the Galileo Galilei Institute for Theoretical Physics for the hospitality during the completion of this work. This work was supported in part by Fondecyt (Chile) grant 1090236.

References