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DEVELOPMENT OF A STORM RUN-OFF PREDICTION MODEL WITH SIMULATED TEMPORAL RAINFALL DISTRIBUTION

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The Yamuna catchment up to Kalanur is studied. The representative character of limited pluviograph data was determined using the storm index concept. After establishing a relationship between the limited pluviograph data and areal rainfall data, the possible effects of areal rainfall variation on the use of limited data was studied. A simulated temporal rainfall pattern for the area was then worked out and applied in the unit graph analysis, which led to the development of interesting relationships between rainfall intensity, Antecedent Precipitation Index and Phi-Index on the one hand and the API-IL relationships on the other. These two put together have given rise to a reasonable peak trend prediction diagram for the Yamuna catchment up to Kalanur.

It is well recognised that in any flood forecasting system based on either rainfall or riverstage relationships or hydrograph methods, one of the difficulties encountered is in the estimation of areal average rainfall with reference to a catchment area. The second important factor that is difficult to estimate accurately is the initial catchment condition, and also the quantitative accounting of the manner in which the catchment releases water as surface runoff. Hewlett (1967) suggests the use of what are called the hydrologic response maps to show how land areas release rainfall during a storm event. The third important factor is the assumption of uniform rainfall distribution over a given area.

As a result of the above limitations it is not very surprising if one finds that the particular hydrological prediction model or a simulation model is ineffective in as much as the various factors could not be taken into account in a quantitative sense. The problem gets particularly complicated when we deal, as we often do, with the rainfall patterns which exhibit considerable areal variability. In other words, the variations in areal rainfall can manifest themselves in different ways: they affect the shape of the unit hydrographs, the value of the predicted peak discharges, antecedent precipitation index - initial loss relationships, and a number of other parameters used in flood forecasting. For instance, the inconsistencies noticed in the antecedent precipitation index (A.P.I.) initial loss relationship with reference to a given catchment area, can be largely attributed to the skewness of rainfall distribution in space as well as in time.

Special mention, therefore, needs to be made here of the assumption of uniform rainfall distribution. While there is no doubt that the basic assumption continues to remain the same, in practice we take into account an implied non-uniform distribution by adjusting the predicted flood peak value and its time of occurrence with the help of peak trend diagrams, and by developing relationships between the duration of effective rainfall and Φ-index and so on. Further, the concept of average unit graph seems primarily to take into account the non-uniformities in rainfall distribution. The question that arises from the above, therefore, is whether or not we are already taking into account a built-in non-uniformity of areal rainfall distribution while tacitly assuming a uniform rainfall pattern in unit graph analysis.

Furthermore, Laurenson (1973) makes the point that it is not strictly necessary that there be a uniform rainfall pattern but rather, that the condition that is necessary to be fulfilled for application of the unit graph is merely that the excess rainfall patterns should be consistent from storm to storm.

Suffice it to say at the present state of development, that the concepts in application of various methods including unit graph methods are fast changing. For instance, it is possible to think of rainfall data of each representative rain gauge as denoting one unit graph pattern, and these various unit graphs derived from individual rainfall station data can be averaged by adopting the same procedure as is done for an average unit graph based on average areal rainfall pattern. Another improvement that can be effected is deducting the contribution to initial losses from the individual station rainfall patterns itself, instead of deducting from the average pattern, and then averaging the equivalent effective rainfall of individual stations. The disadvantage in adopting the procedure of deducting losses from average pattern is obvious, in the sense that we can get unrealistic rainfall patterns which do not have strict correspondence with the hydrograph patterns because, as already stated, when the initial rainfall is high

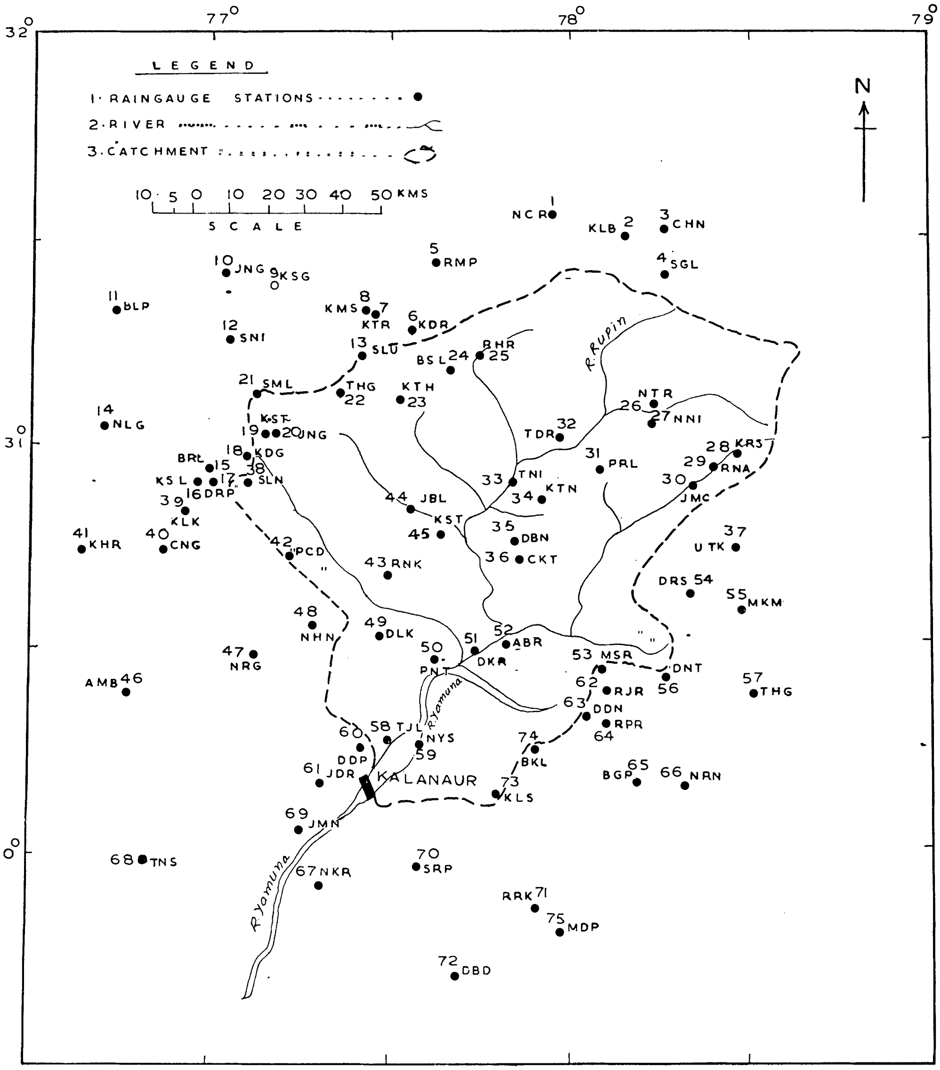

Fig. 1.
Raingauge stations in Yamuna Catchment up to Kalanur.

or when there is skewness in the rainfall distributions, surface runoff can occur even before the total rainfall values of the catchment exceed the estimated initial loss value.

In the present study, an attempt has been made to apply the unit graph method for forecasting the flood hydrograph pattern at Kalanur based on the representative rainfall data of two recording rain gauges in the Yamuna catchment (Fig. 1). For the purpose of determination of areal rainfall pattern, the representative rain gauge network is determined using the "storm index" approach. At present, the flood forecasting for Yamuna river does not take into account the forecast flood at Kalanur, but is based on the actual observed hydrograph at Kalanur and forecasts downstream points as worked out by a flood routing procedure. If on the other hand, one is able to forecast flood hydrographs at Kalanur itself from upstream, it will result in an increase in warning time; hence the advantage in attempting the present method.

RAINFALL REPORTING NETWORK IN YAMUNA CATCHMENT

There are 62 rain gauges existing within the Yamuna catchment. This network works out to one rain gauge for every 300 km^2. There are 18 recording rain gauges in the catchment. Out of these eighteen, only three recording rain gauges, viz. Simla, Mussorie and Dehradun, fall in the upper catchment above Kalanur, for which long term data are available. The present study is confined to the Yamuna catchment, whose area is 12,900 km^2. The special problem in this area has been the inadequacy of the rain gauge network of the recording type and hence studies have initially been made to determine if the existing three recording rain gauges in the upper Yamuna catchment are representative in character for the purpose of obtaining average areal rainfall pattern of this area.

REPRESENTATIVE RAINGAUGES IN UPPER YAMUNA CATCHMENT

About 40 rainfalls that occurred in the Yamuna catchment up to Kalanur during the monsoon season (June to September) 1951–1958 were scrutinised and whenever the average rainfall was 1.0 cm or more, those storms were considered as being of 1-day duration. It was noticed that whenever heavy rainstorms occur whose areal extent covers the catchment up to Kalanur, there are certain stations which are prone to recording the highest storm point rainfall in the area. It was also observed that in the case of 1-day storms, the storm centres generally were located at either Simla or Mussorie which are pluviograph stations. The average of the highest storm point rainfalls of the stations near Simla and Mussorie has been found to be pivotal and reflects the average catchment rainfall. It has been further noticed from the locations at Simla and Mussorie that they are situated at the extreme periphery opposite each other and hence the average of the highest storm point rainfall should depict the dispersion of the storm rainfall over the catchment. The movement of rainstorm with reference to the catchment is in the general direction of north-northwest.

Since the area under study is a fixed one, the relation of arithmetic average rainfall depth of the catchment to the highest storm point rainfall located in the catchment has been worked out and is referred to as the "Storm Index". For the purpose of this study, storm indices have been prepared for all the 40 one-day storms that occurred in the area. The normal seasonal rainfall maps of Yamuna catchment up to Kalanur were also prepared, and it was seen that the principal rainy months are July to September. Simla, Mussorie and its neighbourhood stations record the highest monsoon rainfall and are also located at storm rainfall centres.

EQUATION FOR THE REGRESSION LINE BETWEEN STORM INDEX AND CENTRAL RAINFALL

The correlation coefficient for the monsoon months (June-September) between storm index values and point rainfall values of the stations gives a value of 0.62. The storm indices were then plotted against corresponding central storm rainfall as shown in Fig. 2. From the scatter of points on the graph, a straight line relationship was obtained, with the regression equation as

$$Y = 0.52 - 0.0013\ x$$

where Y stands for storm index, and x for the central rainfall in mm. This equation becomes valid for all 1-day storms with average catchment rainfall of 1.0 cm or more.

It will be seen that Simla and Mussorie are quite close to the regression line of best fit. This graph also gives an indication as to the most representative locations of rain gauges, and serves as a "filter" for non-representative locations. Thus Simla and Mussorie individually have a correlation with the average rainfall depth reflected in the same index.

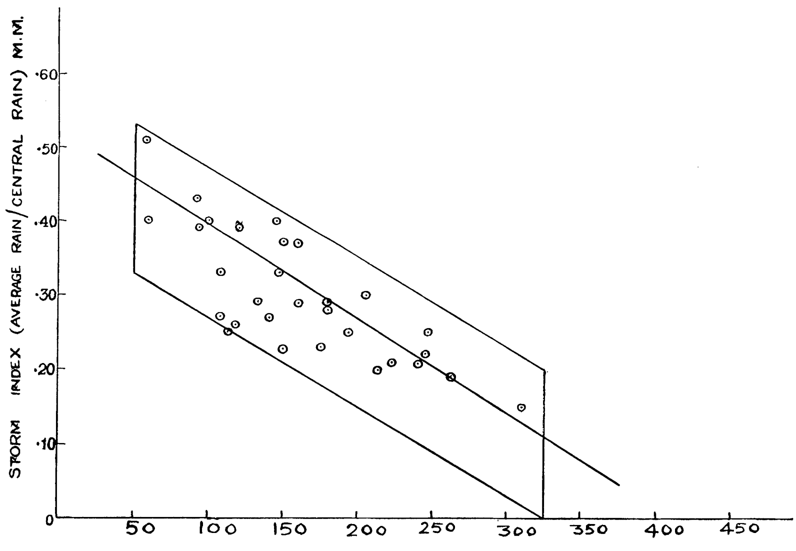

Fig. 2.
Yamuna River at Kalanur.
Diagram showing relationship between storm index and central rainfall. Equation used: $y = 0.52 - .0013x$.

CATCHMENT TEMPORAL RAINFALL DISTRIBUTION

Out of three pluviograph stations, the storm rainfall data in the case of Dehradun was incomplete; hence the consideration of pluviograph data was reduced to two stations, viz. Simla and Mussorie. These stations, Simla and Mussorie, which are seen to be representative of the catchment rainfall and which are pluviograph stations have been used in simulating the catchment rainfall distribution with respect to time. The general requirement in a computer derivation of unit graphs is the rainfall pattern that represents the average catchment rainfall in both area and time. In so far as areal rainfall average is concerned, this is obtained by averaging about 24 rain gauge stations up to Kalanur and using the average areal rainfall estimate, which is a result of a reasonably good network of stations. Thereafter, for obtaining the temporal rainfall distribution the following procedure was adopted. An examination is made of the time distribution of the rainfall of the two stations Simla and Mussorie and a most probable time distribution for the whole catchment has been adopted. This distribution is superimposed on the areal average rainfall earlier obtained for the catchment up to Kalanur and 3-hour rainfall values, representing the average rainfall conditions over the entire area, have been worked out. The average temporal rainfall patterns for the storms studied are shown in Table 1. It has been concluded that, as a first step, the areal rainfall over this area can be classified into a single system. It would perhaps be worthwhile to caution at this stage that the entire unit graph analysis making use of rainfall as input is sensitive to the temporal rainfall pattern and, therefore, it is always advisable not to damp this sensitivity by taking enlarged unit periods, as some do, because in such circumstances, any real deficiencies in determinations of temporal rainfall distribution are not brought to light. With this in view, the authors have chosen 3 hours as a unit period.

UNIT GRAPH ANALYSIS

Individual and average unit graph

Following the methods adopted by the Bureau of Meteorology, Australia (Heatherwick, 1969; Rao, 1971) the discharge hydrographs, after separation of base flow, and computation of recession constants (see Fig. 3) have been subjected to individual unit graph analysis, with the simulated average temporal rainfall distribution, as obtained above, as input. The computed recession constants and excess rain for the storm studies are shown in Table 2. The derivation consists of determining, for different sets of conditions of initial loss and Φ-index, the reproduced hydrograph. Quality of fit between the reproduced hydrograph and the original hydrograph is obtained with the method of least squares. The individual unit graphs for a few storms, which are programmed on an IBM-360/44 computer at Delhi University, may be seen in Fig. 4, making use of the simulated temporal rainfall distribution.

An average unit graph is derived by computer from the individual unit graphs by averaging the response functions of each individual unit graph. The average unit graph adopted for this study is shown in Fig. 5.

Duration of excess rain-loss rate diagram and API-IL diagram

From the selected unit graphs used in the derivation of the average unit graph at Kalanur, it is possible to obtain sets of conditions of initial loss and continuous loss and, together with A.P.I., form the three most important parameters to be solved on the development of a model for an operational rainfall run-off relationship. One of the requirements for operational flood forecasting is the determination of effective rainfall over the area which results in direct runoff.

Table 1.
Average rainfall pattern of the Yamuna River at Kalanur for the storms investigated.

August 1971		July 1971		July 1971	
Date and Time	Average Pattern of Rainfall	Date and Time	Average Pattern of Rainfall	Date and Time	Average Pattern of Rainfall
	(mm)		(mm)		(mm)
5th		25th		5th	
09	1	09	0	09	0
12	0	12	2	12	0
15	1	15	0	15	0
18	0	18	0	18	9
21	0	21	0	21	2
24	1	24	0	24	2
6th		26th.		6th	
03	0	03	0	03	0
06	4	06	0	06	0
09	45	09	13	09	1
12	28	12	15	12	2
15	8	15	2	15	2
18	8	18	0	18	0
21	8	21	0	21	3
24	2	24	1	24	6
7th		27th		7th	
03	5	03	1	03	0
06	2			06	2
09	5			09	11
12	3			12	3
15	1			15	1
18	0			18	7
21	4			21	0
24	8			24	0
8th				8th	
03	9			03	2
06	2				
09	1				
12	5				
15	10				
18	1				
21	4				
24	1				
9th					
03	5				
06	4				
09	1				
12	2				

Table 1. (cont.)

August 1969		July 1968		September 1964	
Date and Time	Average Pattern of Rainfall	Date and Time	Average Pattern of Rainfall	Date and Time	Average Pattern of Rainfall
	(mm)		(mm)		(mm)
5th		17th		24th	
09	0	09	0	09	0
12	1	12	0	12	17
15	3	15	9	15	8
18	1	18	8	18	8
21	0	21	0	21	0
24	0	24	3	24	0
6th		18th		25th	
03	6	03	4	03	7
06	15	06	4	06	15
09	11	09	5	09	30
12	15	12	7	12	0
15	6	15	2	15	0
18	2	18	13	18	5
21	1	21	0	21	6
		24	0	24	13
		19th		26th	
		03	2	03	9
		06	6	06	5
		09	1	09	3
		12	6	12	2
		15	7		

The A.P.I., which represents the antecedent soil moisture conditions in the catchment, provides an important rational tool in the methodology relating to the determinations of effective rainfall or excess rainfall. The computed A.P.I. value and the initial loss condition for the selected unit graph for each storm are shown in Table 3. A plot of A.P.I. – initial loss values from Table 3 shows a marked straight line relationship and may be seen in Fig. 6. A regression

Table 1. (cont.)

August 1967		September 1969		September 1970	
Date and Time	Average Pattern of Rainfall	Date and Time	Average Pattern of Rainfall	Date and Time	Average Pattern of Rainfall
	(mm)		(mm)		(mm)
26th		13th		12th	
09	2	09	0	09	0
12	2	12	0	12	0
15	2	15	0	15	0
18	10	18	13	18	0
21	4	21	2	21	1
24	0	24	0	24	0
27th		14th		13th	
03	0	03	0	03	0
06	1	06	0	06	0
09	1	09	1	09	0
12	8	12	0	12	0
15	2	15	2	15	0
18	0	18	1	18	5
21	0	21	1	21	5
24	0	24	3	24	3
28th		15th		14th	
03	0	03	0	03	2
06	0	06	3	06	5
09	0	09	4	09	0
12	0	12	0	12	0
15	1	15	2	15	5
18	0	18	0		
21	0	21	1		
24	0				
29th					
03	0				
06	3				
09	7				
12	2				
15	2				

Table 2.
Excess rain and recession constants for Yamuna River at Kalanur.

Storm No.	Flood Date (Peaks)	Recession Constant	Excess Rain (mm)
1	6.8.71	.79	31
2	27.7.71	.78	7
3	7.7.71	.84	18
4	6.8.69	.80	16
5	18.7.68	.83	10
6	29.6.64	.71	48
7	27.8.67	.74	18
8	14.9.69	.80	4

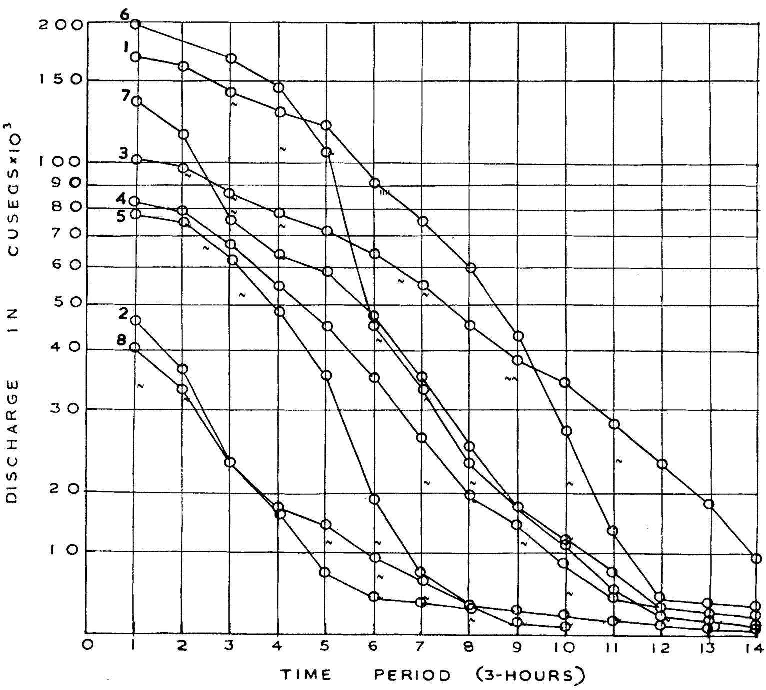

Fig. 3.
Yamuna Catchment up to Kalanur. Recession curves. (Editors note: cusecs = ft³/sec.

Table 3.
Operational relationship of A.P.I. initial loss for Yamuna River at Kalanur.

Storm No.	Date	A.P.I. (mm)	Initial Loss (mm)
1	6.8.71	126	7
2	26.7.71	81	15
3	5.7.71	114	18
4	6.8.69	150	12
5	18.7.68	151	9
6	25.9.64	42	35
7	24.8.67	139	6
8	14.9.69	97	32

equation has been fitted after actually establishing a high correlation between A.P.I. and initial loss. The regression equation obtained is given below:

$$I = 39.3 - 0.2\ A$$

where A is A.P.I. and I is the initial loss.

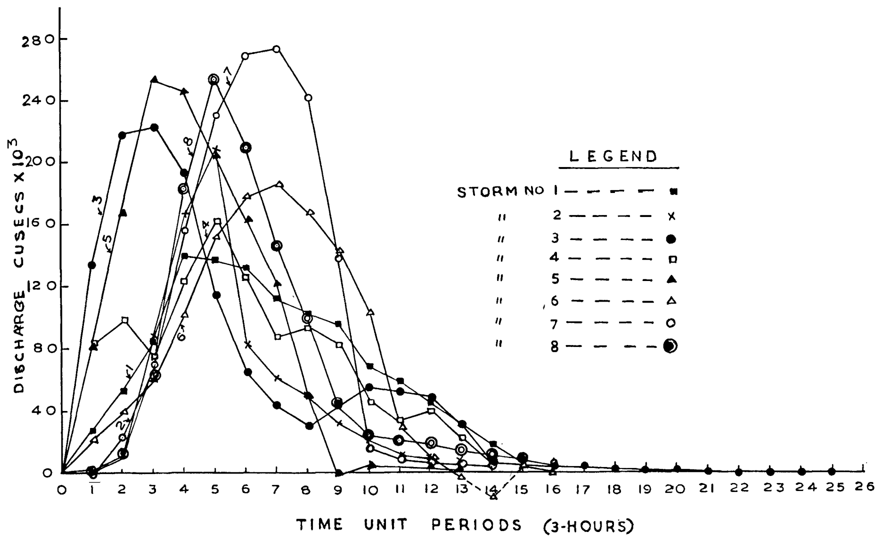

Fig. 4.
Individual unit graphs for Yamuna River up to Kalanur.

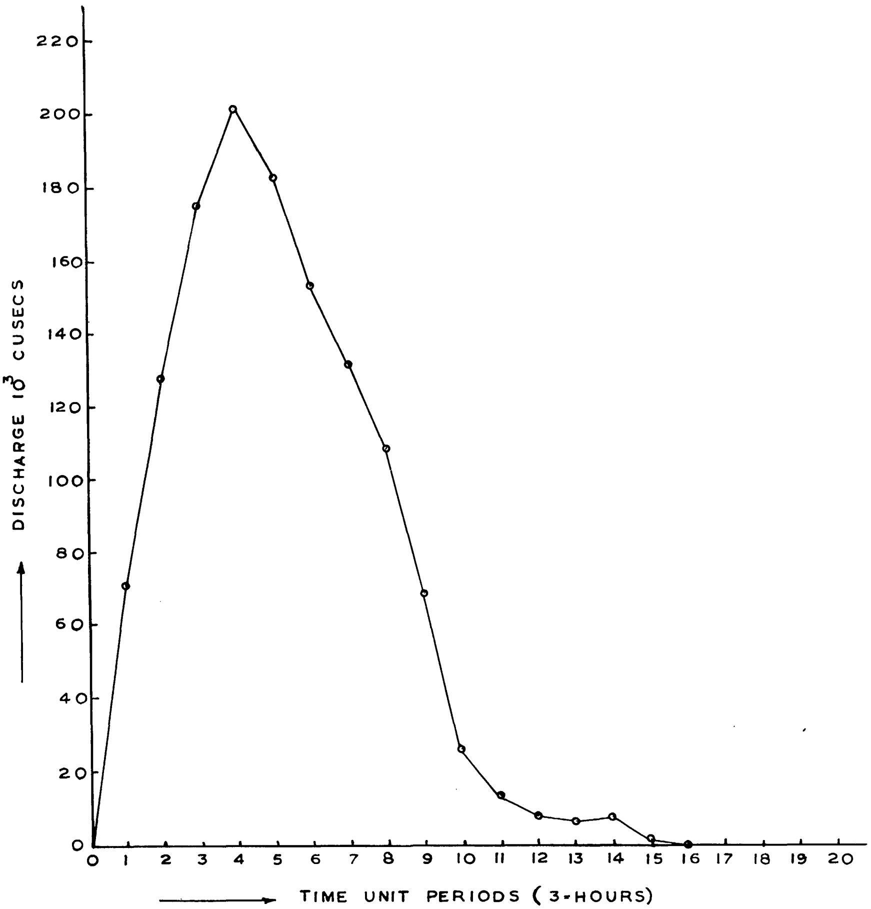

Fig. 5.
Average unit graph for River Yamuna up to Kalanur (computer produced).

During the process of transformation of rainfall into runoff, the instantaneous losses from the rainfall will be rendered small as the duration of storm increases and hence a theoretical relationship is expected to exist between loss rate and duration of excess rainfall. A plot of this may be seen in Fig. 7.

The fact that the duration of excess rainfall and loss rate has given good correlation in this study lends sufficient support to the reasonableness of rainfall analysis carried out earlier and also indirectly proves the plausibility of

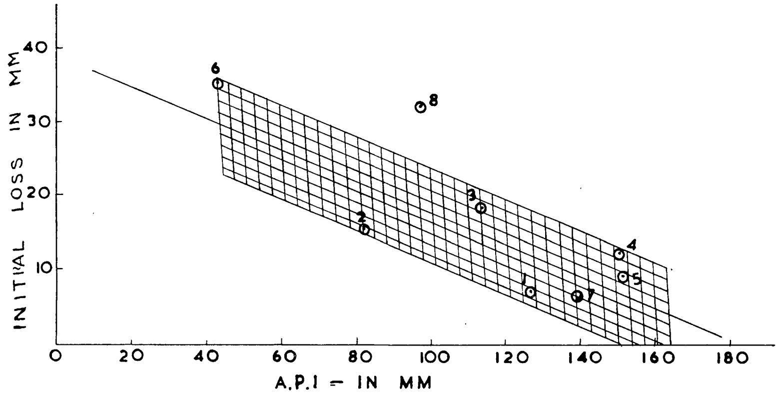

Fig. 6.
Yamuna River at Kalanur. A.P.I. Initial loss diagram.

the computed excess rainfall pattern. However, it may be emphasised that in this particular case the relationship might possibly have been due to the existence of relatively low residual errors which are built into the Φ-index value.

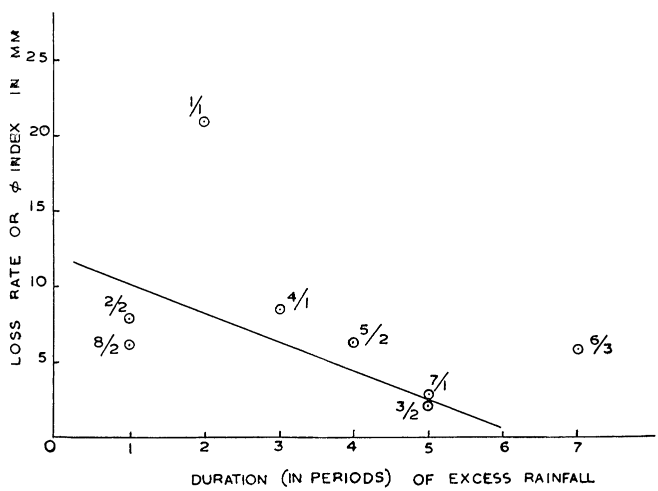

Fig. 7.
Yamuna at Kalanur.

Table 4.
Summary of relevant data of Yamuna River up to Kalanur.

Storm No.	Rainfall Commences	Runoff commen-ces	Initial Loss from start, gross rainfall	Φ-index, inches/mm/ 3 hr.	Excess Rainfall	Hydrograph Peak, Cusecs	Unitgraph Peak, Cusecs	Time to Peak. 3 hr. unit periods
1	Gross 12 PM 6.8.71 Excess 12 AM 7.8.71	3 A.M. 7.8.71	0.28 in. (7.1 mm) 6 periods	0.827 in. (21.1 mm)	1.22 in. (31.0 mm) 2 periods	168,000	139,304	8th Period
2	Gross 9 PM 26.7.71 Excess 3 AM 27.7.71	3 A.M. 27.7.71	0.59 in. (15.0 mm) 7 periods	0.315 in. (7.9 mm)	0.28 in. (7.1 mm) 1 period	63,000	228,600	5th Period
3	Gross 6 AM 5.7.71 Excess 3 AM 6.7.71	3 A.M. 6.7.71	0.83 in. (21.1 mm) 7 periods	0.089 in. (2.3 mm)	0.71 in. (18.0 mm) 4 periods	102,000	222,708	3rd Period
4	Gross 9 AM 6.8.69 Excess 12 AM 7.8.69	3 A.M. 7.8.69	0.47 in. (11.9 mm) 6 periods	0.328 in. (8.4 mm)	0.63 in. (16.0 mm) 3 periods	83,000	147,116	7th Period
5	Gross 9 AM 18.7.68 Excess 12 PM 18.7.68	3 A.M. 19.7.68	0.35 in. (8.9 mm) 7 periods	0.246 in. (6.3 mm)	0.39 in. (9.9 mm) 4 periods	78,000	253,188	3rd Period
6	Gross 6 AM 26.9.64 Excess 6 AM 26.9.64	9 A.M. 26.9.64	0.0 in. (0.0 mm) 8 periods	0.354 in. (8.9 mm)	1.89 in. (48.0 mm) 5 periods	198,000	186,710	7th Period
7	Gross 3 PM 24.8.67 Excess 12 AM 25.8.67	3 A.M. 25.8.67	0.24 in. (6.1 mm) 6 periods	0.110 in. (2.8 mm)	0.71 in. (18.0 mm) 5 periods	135,000	274,186	7th Period
8	Gross 3 AM 15.9.69 Excess 3 AM 15.9.69	3 A.M. 15.9.69	0.0 in. (0.0 mm) 7 periods	0.433 in. (10.9 mm)	0.16 in. (4.1 mm) 1 period	40,000	254,000	5th Period

Rainfall intensity and Φ-index relationship

Since Φ-index depends upon the intensity of rainfall, and on the wetness of the catchment, an attempt was made to obtain a relationship of Φ-indices with rainfall intensities for two A.P.I. values, representing two typical wetness conditions. It has been possible to fit an exponential curve between Φ-index and intensity of rainfall for the two A.P.I. conditions chosen. A similar result seems to have been obtained by Clark and Hebbert (1971) between A.P.I. and rainfall intensity in their rainfall variability model studies for predicting storm runoff. The fact that the exponential curve has been obtained shows that the loss rate becomes an independent parameter beyond a certain rainfall intensity (Fig. 8) for a given A.P.I condition. The simultaneous equation form of the curve obtained for this relation is as follows:

For A.P.I. $\equiv$ 150 mm

$$P = \frac{R - 2.0}{1.197 + 0.066\,R} + 0.7$$

Simplification of this becomes

$$P = \frac{1.046\,R - 1.162}{0.066\,R + 1.197}$$

For A.P.I. $\equiv$ 100 mm

$$P = \frac{R - 2.0}{1.064 + 0.049\,R} + 1.7$$

This can be simplified for A.P.I. $\equiv$ 100, to

$$P = \frac{1.083\,R + 16.088}{.049\,R + 1.064}$$

Where P is the Φ-index and R is the rainfall intensity in mm/hour.

Relation between excess rainfall Q-max

The Q_{max} or peak discharge with respect to eight storms has been plotted against the excess rainfall value (see Fig. 9). The plot shows a good fit. This relationship would be of great operational significance in two ways. First, it shows the consistency of the estimated excess rainfall, and second, once the total excess rainfall is determined, this diagram enables one to find the expected peak discharge which is one of the important requirements in flood warning.

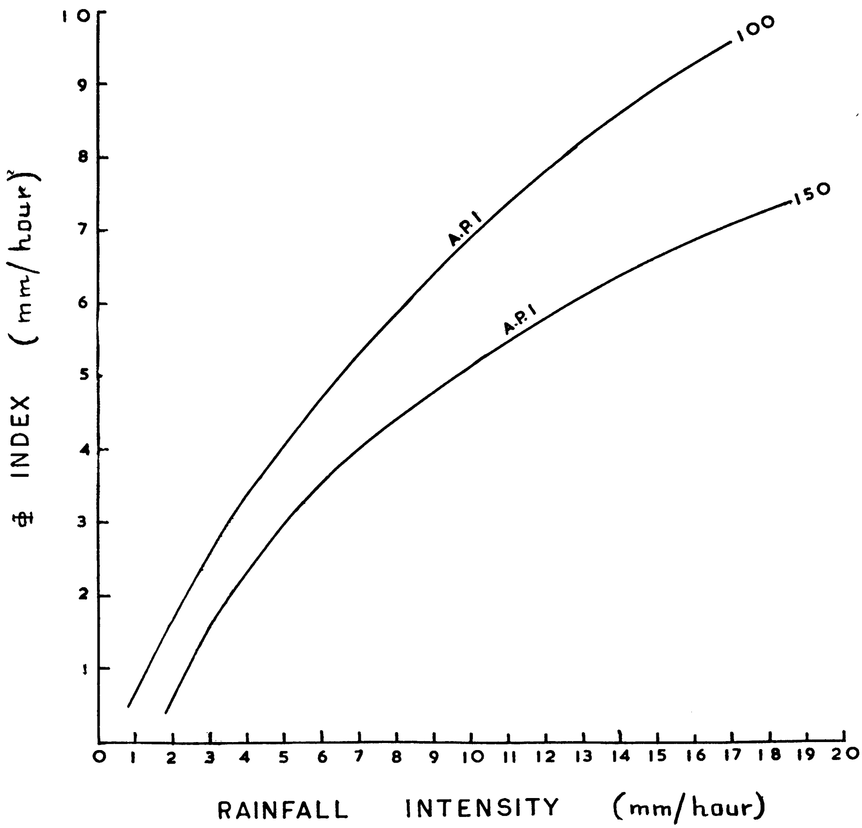

Fig. 8.
Rainfall intensity – Φ Index for Yamuna Catchment up to Kalanur.

Peak trend diagram for the forecast peak discharge

The computed peaks using the adopted average unit hydrograph and the observed peak discharge values together with their times of occurrences and time differences are shown in Table 5. A peak trend diagram by plotting the observed peaks and computed peaks against each other is prepared. In this diagram the corresponding time differences between observed and computed peaks are also marked (see Fig. 10). The peak trend diagram is expected to take into account most of the residual errors and effective areal variations in rainfall.

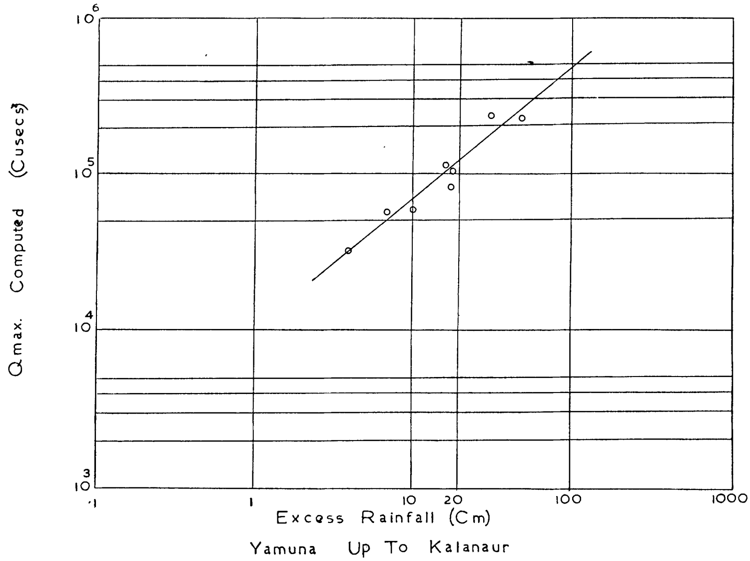

Fig. 9.
Relationship between Q_{max} and excess rainfall.

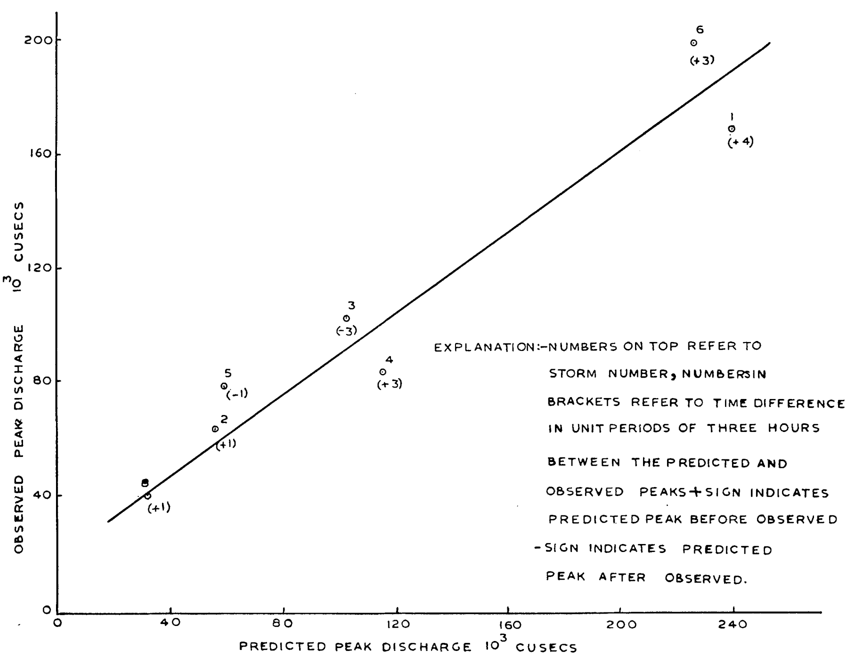

Fig. 10.
Peak trend diagram Yamuna River at Kalanur.

Table 5.
Predicted and observed peaks and their times of occurrence for Yamuna River up to Kalanur.

Storm No.	Observed Peak (Cusecs)	Time of Observed Peak	Predicted Peak (Cusecs)	Time of Predicted Peak	Time Difference (Units)
1	168,000	12 A.M. 8.8.71	238,859	12 P.M. 7.8.71	+ 4
2	63,000	6 P.M. 27.7.71	55,571	3 P.M. 27.7.71	+ 1
3	102,000	3 P.M. 6.7.71	101,857	12 A.M. 7.7.71	− 3
4	83,000	12 A.M. 8.8.69	115,284	3 P.M. 7.8.69	+ 3
5	78,000	6 P.M. 19.7.68	59,087	9 P.M. 19.7.68	− 1
6	198,000	9 P.M. 27.9.64	225,438	12 P.M. 27.9.64	+ 3
7	135,000	12 A.M. 26.8.67	80,641	9 P.M. 25.8.67	+ 1
8	40,000	6 P.M. 15.9.69	31,755	3 P.M. 15.9.69	+ 1

CONCLUSIONS

In the present study it has been shown that it is possible to obtain important relationships based upon the simulated temporal rainfall distribution. There are, however, two important factors that determine the effectiveness of this approach. One is to be able to determine the temporal rainfall pattern for the entire area. The second and perhaps more important is the determination of the most representative excess rainfall pattern. It seems that further studies will be necessary on this aspect for achieving complete reliability and objectivity in using the method for flood forecasting purposes.

The rainfall intensity and loss rate relationship in particular will be a useful concept. This, together with the relationship obtained between A.P.I. and initial loss, will enable one to determine the effective rainfall for each unit period from the gross rainfall. It is seen from the above analysis that it should be possible to give an advance additional warning time of 15 to 18 hours of expected peak discharge for the Yamuna river at Kalanur.

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