Journal of Heat Transfer

Discussion

Discussion of a Discussion by F. Chen, and C. F. Chen[1]

D. A. Nield.[2] The closures by Vafai and Kim (1995b, 1996) have been helpful in clarifying some matters, but they have raised some other issues which invite comment.

Vafai and Kim (1990) in their equation (10b) wrote two expressions "which together imply the matching of the total normal stress at the interface," and only later mentioned that they were adopting the approximation of setting the effective viscosity equal to the fluid viscosity. They did not then point out the implications of this. Vafai and Kim (1995a, b) presented an argument aimed at showing that the total normal stress boundary condition reduced to the continuity of pressure at the interface without any mention of the value of the effective viscosity, leading the reader to infer that the argument is meant to hold for all choices of the effective viscosity. It seems to me that this argument (for the general viscosity ratio) breaks down at the crucial point where they referred to Chen and Chen (1992) and that the basic conclusion of Chen and Chen (1996) is correct: the two expressions are equivalent when the effective viscosity is equal to the fluid viscosity (and the fluid is incompressible), but there is inconsistency for other choices of the effective viscosity. In order to remove possible confusion, it is necessary that Vafai and Kim now clarify their position regarding the situation where the effective viscosity is not equal to the fluid viscosity.

It is fortunate that Vafai and his colleagues have not implemented two expressions for the normal stress in their numerical work. Rather, they used a one-domain approach in terms ot the vorticity-stream function-temperature formulation. They believed that in this formulation, the normal stress condition was satisfied indirectly. For example, Vafai and Huang (1994) said nothing about how their normal stress boundary conditions (10b) were implemented, and the reader was left to guess that they probably implicitly assumed that the stream function and vorticity are continuous across the interface. It is true that the continuity of velocity and tangential stress (and viscosity) do imply the continuity of stream function and vorticity, but that is independent of the form of the normal stress condition.

It is interesting that Poulikakos et al. (1986) did not explicitly impose any normal stress boundary condition at the interface in their work, which also involved a vorticity-stream function formulation. It seems that in this formulation there is some sort of implicit numerical matching condition which replaces the physical boundary condition. Professor J. L. Lage has pointed out to me that there is a fundamental difficulty in translating from a two-media formulation (with interface boundary conditions) to a single medium, in the way in which Vafai and Huang (1994) have done. What defines the interface conditions is no longer the analytical model but rather the numerical scheme used to solve the equations. One or more general differential equations are taken to apply for the whole domain. What distinguishes one subdomain from the other are the physical properties of each subdomain (permeability, porosity, inertia coefficient, etc.); what happens at the interface depends on how the code is structured. A detailed investigation will be required in order to learn how the numerical solution, based on a particular code, relates to the physical boundary conditions which should be satisfied at the interface between a porous medium and a fluid. I would expect the numerical solution from a vorticity-streamfunction approach to overestimate the heat transfer as a result of the constraining effect of the physical normal stress condition not being fully taken into account.

There is one further feature of the Vafai and Huang (1994) paper which requires comment. It is not immediately clear if the symbol **v** in their Eq. (5) denotes the Darcy velocity or the intrinsic velocity; the reader is left to deduce that if the effective conductivity is defined in the usual manner, then their Eq. (6) implies that **v** must denote the Darcy velocity. If that is correct then the left hand side of Eq. (5) should be divided by a factor ϵ (the porosity). It is fortunate that Vafai and Huang have performed calculations for the case of small Darcy numbers only, so the effect on the numerical results is expected to be small. It is important to note that their results are limited to this case. It would have been less confusing if they had omitted the convective term completely, as recommended by Nield (1991). Incidentally, it also appears that eq. (6) of Vafai and Huang (1994) differs from eq. (16) of Vafai and Tien (1981) in that the symbol α_e is defined differently in the two papers (the porosity is explicitly involved in one expression but not in the other). An explanation for this difference would help the reader.

Vafai and Kim (1996) conclude a long paragraph with "it becomes clear in their Eqs. (16) and (17), Chen and Chen (1992) have failed to set these viscosity coefficients equal. This mistake affects the results of the analysis of Chen and Chen (1992)." It is not clear to me that Chen and Chen have made any mistake. Their Eqs. (16) and (17) are simply the nondimensional component forms of their Eq. (5), which indicates that they have taken the effective (dynamic) viscosity μ_{eff} equal to the fluid μ/ϕ, where μ denotes the fluid viscosity and ϕ the porosity. This is in agreement with the result obtained by local volume averaging by Vafai and Tien (1981) and several other authors. This appears to be the best choice of effective viscosity to model flow near the solid boundaries. The solid boundaries are more restrictive to the flow and are thus expected to have a greater effect on heat transfer than the interface, so it is important to deal accurately with the effect of the solid boundaries.

[1] Published in the February 1996 issue of the ASME Journal of Heat Transfer, Vol. 118, pp. 266–268.

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At the interface, the stress is somewhat indeterminant so it is not too important which value of the effective viscosity is taken in modelling the interface. Chen and Chen chose to take μ (the simplest expression) as the effective viscosity at the interface and in my view that is a reasonable choice.

Caution is needed in referring to the work of Brinkman (1947), Lundgren (1972), and Neale and Nader (1974). Brinkman used the equation which now bears his name for a purpose different from that of modern authors, and a similar remark applies to Lundgren's work. Neale and Nader were concerned with relating the Brinkman equation to the well known semi-empirical equation of Beavers and Joseph, and their work is restricted to unidirectional flows parallel to the interface. Thus, their work should not be expected to extend automatically to the more general flow treated by Chen and Chen (1992), so Chen and Chen were under no obligation to follow Brinkman, Lundgren, Neale, and Nader in their choice of effective viscosity.

For the case of media of small or moderate permeability, it probably does not matter much, from a purely empirical as distinct from a scientific viewpoint, whether or not one takes the effective viscosity to be equal to the fluid viscosity, or that divided by the porosity, in the differential equation or the boundary conditions. In that case, the Brinkman term plays a minor role in comparison with the Darcy term. Also, in the determination of the temperature field, the stress boundary conditions play a minor role (because higher derivatives are involved) in comparison with the conditions of continuous temperature, heat flux, and normal velocity. Consequently, the published heat transfer results of Chen and Chen, Vafai and coworkers, and other authors should all be satisfactory. However, for media of high permeability further investigation is necessary in order to determine the range of validity of the published results (the ratio of effective viscosity to fluid viscosity can be of order 10; for example, Givler and Altobelli (1994) reported the fact that one material yielded an experimental value of about 7.5 for this ratio).

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Closure[3]

K. Vafai[4] and S. J. Kim[5]. We would like to thank Professor Nield for his comments and further discussion on this topic. However, it would have been quite beneficial if the author had read our statements, or even his, as related to this topic more carefully. Perhaps we can just quote Professor Nield here as we believe his own previous statements and our responses will further clarify what has already been a very clear description by us all along. First, it should be noted that in Nield (1995) the main subject of the discussion was about the fact that Vafai and Huang (1994) and Huang and Vafai (1994) are setting the fluid viscosity and the effective viscosity equal, i.e., they are using the same viscosity in front of both terms in both papers. In fact, Nield (1995) starts out his discussion about the works of Vafai and Huang (1994) and Huang and Vafai (1994) by saying, "In each of these papers the authors have modeled flow in a porous medium by a Brinkman-Forchheimer-extended Darcy equation (Eq. (5) of the first paper, Eq. (2) of the second) in which the coefficient of the Darcy term v/K is the same as the coefficient of the Brinkman term $\nabla^2 v$, and each is denoted by ν_{eff}." Therefore, the primary subject of the discussion and the closure (Vafai and Kim, 1995b) is centered around the fact that our coefficient for the Darcy term v/K is the same as our coefficient for the Brinkman term $\nabla^2 v$. We use different symbols for the fluid viscosity and the effective viscosity so as to make it clear that they are usually different (also to avoid another discussion on the same point), but then we make it clear that due to lack of definitive data we always use the same value of viscosity for both when dealing with porous-fluid interfaces.

These points were reiterated in our response in Vafai and Kim (1995b) which is reproduced here: "In general the coefficient of the Darcy term v/K is μ_f, and the coefficient of the Brinkman term, $\nabla^2 v$, is μ_{eff}, as shown in Eq. (5) of Vafai and Kim (1990). We are well aware that there are some situations where it is important to make a distinction as shown and discussed (for example, Vafai and Kim, 1990; Ettefagh, Vafai, and Kim, 1991; Huang and Vafai, 1994). But Lundgren (1972) and Neal and Nader (1974) have shown that setting the effective viscosity of the fluid-saturated porous medium equal to the fluid viscosity provides good agreement with experimental data. Hence, lacking definitive information on μ_{eff}, it has become a common practice to set the effective viscosity equal to the fluid viscosity." This effective viscosity as explained by Vafai and Kim (1995a, 1995b) is taken to be the fluid viscosity. This has always been our de facto approach for problems dealing with the porous-fluid interface.

In addition, it would be instructive if our interactive joint discussion in Nield et al. (1996) is read more carefully. In this discussion, once again, Nield states that, "The effective viscosity . . . may differ substantially from the fluid viscosity," and also that, "Vafai and Kim's argument on the normal stress condition collapses because they . . . have confused tangential and normal coordinates." In the same interactive discussion we once again have responded to these statements by stating, "The reason we always set the effective viscosity equal to the fluid viscosity is, as we had mentioned at various times, due to the lack of rigorous data and that it provides good agreement with past experimental data (Lundgren, 1972; Neale and Nader, 1974)," and, "With respect to the porous/fluid interface, the arguments stand. The confusion appears to be Nield's as we have not "confused tangential and normal coordinates." In Vafai and Kim (1995) and our other porous/fluid interface works, always a two-dimensional, incompressible, and isotropic porous medium, in which the effective and the fluid viscosities are

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