

At the interface, the stress is somewhat indeterminate so it is not too important which value of the effective viscosity is taken in modelling the interface. Chen and Chen chose to take  $\mu$  (the simplest expression) as the effective viscosity at the interface and in my view that is a reasonable choice.

Caution is needed in referring to the work of Brinkman (1947), Lundgren (1972), and Neale and Nader (1974). Brinkman used the equation which now bears his name for a purpose different from that of modern authors, and a similar remark applies to Lundgren's work. Neale and Nader were concerned with relating the Brinkman equation to the well known semi-empirical equation of Beavers and Joseph, and their work is restricted to unidirectional flows parallel to the interface. Thus, their work should not be expected to extend automatically to the more general flow treated by Chen and Chen (1992), so Chen and Chen were under no obligation to follow Brinkman, Lundgren, Neale, and Nader in their choice of effective viscosity.

For the case of media of small or moderate permeability, it probably does not matter much, from a purely empirical as distinct from a scientific viewpoint, whether or not one takes the effective viscosity to be equal to the fluid viscosity, or that divided by the porosity, in the differential equation or the boundary conditions. In that case, the Brinkman term plays a minor role in comparison with the Darcy term. Also, in the determination of the temperature field, the stress boundary conditions play a minor role (because higher derivatives are involved) in comparison with the conditions of continuous temperature, heat flux, and normal velocity. Consequently, the published heat transfer results of Chen and Chen, Vafai and coworkers, and other authors should all be satisfactory. However, for media of high permeability further investigation is necessary in order to determine the range of validity of the published results (the ratio of effective viscosity to fluid viscosity can be of order 10; for example, Givler and Altobelli (1994) reported the fact that one material yielded an experimental value of about 7.5 for this ratio).

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## Closure<sup>3</sup>

**K. Vafai<sup>4</sup> and S. J. Kim<sup>5</sup>**. We would like to thank Professor Nield for his comments and further discussion on this topic. However, it would have been quite beneficial if the author had read our statements, or even his, as related to this topic more carefully. Perhaps we can just quote Professor Nield here as we believe his own previous statements and our responses will further clarify what has already been a very clear description by us all along. First, it should be noted that in Nield (1995) the main subject of the discussion was about the fact that Vafai and Huang (1994) and Huang and Vafai (1994) are setting the fluid viscosity and the effective viscosity equal, i.e., they are using the same viscosity in front of both terms in both papers. In fact, Nield (1995) starts out his discussion about the works of Vafai and Huang (1994) and Huang and Vafai (1994) by saying, "In each of these papers the authors have modeled flow in a porous medium by a Brinkman-Forchheimer-extended Darcy equation (Eq. (5) of the first paper, Eq. (2) of the second) in which the coefficient of the Darcy term  $v/K$  is the same as the coefficient of the Brinkman term  $\nabla^2 v$ , and each is denoted by  $\nu_{eff}$ ." Therefore, the primary subject of the discussion and the closure (Vafai and Kim, 1995b) is centered around the fact that our coefficient for the Darcy term  $v/K$  is the same as our coefficient for the Brinkman term  $\nabla^2 v$ . We use different symbols for the fluid viscosity and the effective viscosity so as to make it clear that they are usually different (also to avoid another discussion on the same point), but then we make it clear that due to lack of definitive data we always use the same value of viscosity for both when dealing with porous-fluid interfaces.

These points were reiterated in our response in Vafai and Kim (1995b) which is reproduced here: "In general the coefficient of the Darcy term  $v/K$  is  $\mu_f$ , and the coefficient of the Brinkman term,  $\nabla^2 v$ , is  $\mu_{eff}$ , as shown in Eq. (5) of Vafai and Kim (1990). We are well aware that there are some situations where it is important to make a distinction as shown and discussed (for example, Vafai and Kim, 1990; Etefagh, Vafai, and Kim, 1991; Huang and Vafai, 1994). But Lundgren (1972) and Neale and Nader (1974) have shown that setting the effective viscosity of the fluid-saturated porous medium equal to the fluid viscosity provides good agreement with experimental data. Hence, lacking definitive information on  $\mu_{eff}$ , it has become a common practice to set the effective viscosity equal to the fluid viscosity." This effective viscosity as explained by Vafai and Kim (1995a, 1995b) is taken to be the fluid viscosity. This has always been our de facto approach for problems dealing with the porous-fluid interface.

In addition, it would be instructive if our interactive joint discussion in Nield et al. (1996) is read more carefully. In this discussion, once again, Nield states that, "The effective viscosity . . . may differ substantially from the fluid viscosity," and also that, "Vafai and Kim's argument on the normal stress condition collapses because they . . . have confused tangential and normal coordinates." In the same interactive discussion we once again have responded to these statements by stating, "The reason we always set the effective viscosity equal to the fluid viscosity is, as we had mentioned at various times, due to the lack of rigorous data and that it provides good agreement with past experimental data (Lundgren, 1972; Neale and Nader, 1974)," and, "With respect to the porous/fluid interface, the arguments stand. The confusion appears to be Nield's as we have not 'confused tangential and normal coordinates.'" In Vafai and Kim (1995) and our other porous/fluid interface works, always a two-dimensional, incompressible, and isotropic porous medium, in which the effective and the fluid viscosities are

<sup>3</sup> Only references which are not given, if any, in the discussion are cited.

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equal, is considered. The equations are correct as they are for the cited conditions. We cannot emphasize this point any stronger. Once again, it would be good if our previous discussions are read more carefully.

With respect to, "There is one further feature of the Vafai and Huang (1994) paper which requires comment . . .," we are thankful for Professor Nield for clarifying the positions that we have always invariably held. However, it is ironic that he is now basing his discussion on a term that he has been stating all along should not even be there. This is a term that we have clearly shown to be negligible "for most practical situations", Vafai and Tien, (1981), and various other works have also shown it to be negligible except for rare cases discussed in Vafai and Tien (1981). In the work presented in Vafai and Huang (1994), once again we have found that the convective term has essentially no effect on the results. We had found early that it makes no difference in our runs whether the epsilon-related coefficient in front of the convective term is present or not. This is because this term itself has no impact on the results. Therefore, in lieu of the fact that it has literally no impact, we had decided to just leave it out. This has no effect on any of the results. It is good, however, to see that Nield is moving ever more closely to the position that we have invariably held from the very beginning of proposing the generalized formulation. With respect to the thermal conductivity, we are thankful to Professor Nield for raising the question. However, it should be noted that the porosity in Eq. (6) of Vafai and Huang (1994) is embedded in the effective thermal conductivity. It should be noted that the effective thermal conductivity is already a function of the porosity. Therefore, the effective thermal conductivity is just presented in a different form. In essence, this is like representing  $f_1(x)/x$  by  $f_2(x)$ . Conceptually, there is no difference between the two formulations. We are also thankful for Professor Nield's attention to this paper which is quite rich in physical and phenomenological descriptions that set a very innovative approach for heat transfer regulation and modification.

With respect to the comments on the vorticity-stream function formulation, here, once again, it appears that there is misunderstanding of the numerical procedure for handling a porous/fluid composite layer. As pointed out in Vafai and Kim (1990), one can use separate calculation schemes for the porous and fluid regions which would require an involved iterative procedure for matching the interface conditions. A more efficient approach is to combine the two sets of governing equations for the fluid region and the porous region into one set of conservation equations. In other words, the porous substrate and the fluid region can be modeled as a single domain governed by one set of equations, the solution of which satisfies the continuity of the velocities, stresses, temperature, and the heat fluxes across the porous/fluid interface. Hence, the inclusion of derivatives of the Darcy number and inertial coefficient in Eq. (17) of the first paper plays a role only across the interface. The same approach has been used by other investigators. It is good to hear about Professor Lage's personal feelings about this approach and we are thankful for his extra attention to our works. However, it should be noted that the one domain approach used by us is based on physical principles and it is not at all dependent on how our code is structured, as long as it is written correctly.

As for our statements regarding Chen and Chen's failure to set these coefficients equal, our statements and position stand as they are. Unfortunately, the statements by Professor Nield regarding the work of Brinkman (1947) stem from a confusion of the very issue that he is trying to discuss. Therefore, to clarify this issue once again we will briefly go through the explanation of the factors that must be considered and the clarifications regarding the effective viscosity and the fluid viscosity coefficients. For these coefficients to be equal, the coefficient for the Darcy term  $\nu/K$  and the coefficient for the Brinkman term  $\nabla^2 \nu$  need to be set equal. This means that the effective viscosity,

which refers to the coefficient in front of the Laplacian of the velocity of the fluid-saturated porous medium, should be equal to the fluid-viscosity. This point is directly and exactly consistent with the original formulation by Brinkman (1947) as displayed in his Eq. (5), Neale and Nader (1974) as displayed in their Eq. (1.7), as well as various other investigators. This is precisely what Vafai and Huang (1994) and Huang and Vafai (1994) have done. This is not a matter of style.

Finally, it is quite interesting to read after all these discussions that Nield is stating, "For the case of media of small or moderate permeability, it probably does not matter much, from a purely empirical as distinct from a scientific viewpoint, whether or not one takes the effective viscosity to be equal to the fluid viscosity, or that divided by the porosity, in the differential equation or the boundary conditions." We cannot help but think why we had to spend so much time on this issue.

In summary, we would like to thank Professor Nield for an extension of an extension of this discussion. However, we have already discussed these issues at length and with clarity with respect to several inapplicable points. As such, we believe any further discussion on what has already been presented at length would not serve any technical need.

### Additional Reference

Nield, D. A., Vafai, K., and Kim, S. J., 1996, "Closure Statements on the Brinkman-Forchheimer Extended Darcy Model," *International Journal of Heat and Fluid Flow*, Vol. 17, pp. 34-35.

### Discussion of a Discussion by K. Vafai and S. J. Kim<sup>1</sup>

**D. A. Nield<sup>2</sup> and J. L. Lage.<sup>3</sup>** In their discussion of a recent paper, Vafai and Kim (1995, p. 1097) stated that they had shown that, "the solution of Vafai and Kim (1989) . . . is valid for  $Da < 1$ , which covers the entire range of all porous media." In making this statement they have overlooked the fact that there is a class of materials of practical importance which are appropriately modeled by the Brinkman-Forchheimer equation and for which the Darcy number can exceed unity. For example, Weinert and Lage (1994) reported permeabilities of compressed aluminum-alloy foams as high as  $8 \times 10^{-6} \text{ m}^2$ , and for a 1.0 mm thick layer of such material the Darcy number is equal to about eight. Materials in this class have a connected solid matrix and a connected void space.

For this type of porous medium that we propose be called a "hyperporous medium," the Brinkman term has order of magnitude comparable with that of the Darcy term throughout the medium (rather than just in boundary layers near solid walls). The "permeability"  $K$  which appears in the Darcy term of the general momentum equation is no longer determined, in a Darcy type experiment, as simply the fluid viscosity times the mean Darcy velocity divided by the pressure gradient. Rather, one has to allow for the contribution of the Brinkman term to the pressure gradient. As the porosity becomes closer to unity, the Brinkman term dominates over the Darcy term, and the value of  $K$  (and hence the Darcy number) can increase without limit (other than that imposed by the strength of the solid material from which the matrix is constructed).

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