INTRODUCTION

The geomorphic surfaces of depositional basins represent a balance between the construction of topography by sediment accumulation and the loss of topography by load-driven subsidence and deformation of the surface. In large measure, this balance is a function of the rheology of the substrate. Previous studies have investigated how the strength of rock (substrate) exerts control on erosion rates and whole-sale collapse of mountain belts (Montgomery, 2004; Selby, 1980), but the control of substrate strength on the topographic development over relatively short time and length scales has not been fully explored. This is a part of the geomorphic system that requires more attention in order to understand the influence of substrate rheology on the geomorphology of a depositional landscape. Here, our research is motivated by the loss of surface topography in the Upper Jurassic Norphlet Formation due to load-driven deformation of the Louann Salt substrate.

The Norphlet Formation is a major hydrocarbon reservoir in the northeastern Gulf of Mexico basin (Mancini et al., 1985; Story, 1998; Kugler and Mink, 1999; Ajdukiewicz et al., 2010). In three-dimensional (3-D) seismic data from the Mobile Bay region, the Norphlet Formation exists as subparallel, NNW-trending lenticular sandstone bodies up to 250 m thick and 1.5 km across, which are characterized by both concave-downward upper surfaces and concave-upward lower surfaces (Fig. 1). These lenticular features with varying degrees of preserved relief have a spacing of ~2.5 km (fig. 8 in Story, 1998; fig. 2 in Ajdukiewicz et al., 2010) and are separated by smaller sand masses with thicknesses less than that of seismic resolution (~100 m). The morphology of the Norphlet lenticular bodies has been interpreted to represent partly preserved eolian large linear dunes (Story, 1998; Ajdukiewicz et al., 2010), which are rare in the geologic record (Eschner and Kocurek, 1988). The unique geologic configu-
ration of the Middle and Late Jurassic Gulf of Mexico basin that gave rise to the preservation of Norphlet dunes includes: (1) Callovian-Oxfordian deposition of a thick (up to 600 m) evaporite sequence, the Louann Salt, and the locally present Pine Hill Anhydrite within a shallow, restricted sea with a narrow opening to the paleo-Pacific Ocean; (2) development of the Norphlet dunes from fluvial sediment sources and subsequent dune progradation over the bedded salt during a regression; and (3) a rapid, low-energy marine transgression during the Oxfordian that occurred with opening of the basin to the Atlantic, represented by the Smackover Formation (Mancini et al., 1985; Salvador, 1991; Prather, 1992; Story, 1998; Ajdukiewicz et al., 2010). The rapid transgression with minor reworking of the Norphlet dunes facilitated this rare preservation of linear dunes in the stratigraphy. Variability in the amount of reworking during the marine transgression has been previously hypothesized as the cause of the observed local variations in dune surface relief (Prather, 1992), but the nonuniformity of preserved relief has not been rigorously studied.

This study uses physical experiments and a mathematical model to (1) investigate the loading of salt by linear dunes, (2) provide a physical reasoning for the observed variability in preserved dune topography, and (3) create a method to approximate salt thickness. We hypothesize that dune subsidence into salt can occur with unequal loading of the salt (caused by natural dune geometry [see Fig. 2] and/or variations in dune spacing), and that the ratio of sand to salt thickness controls the subsidence behavior, thus controlling the final preserved topography of the subsiding dunes.

**EXPERIMENTAL DESIGN AND RESULTS**

We conducted a set of two experiments in a flume with dimensions of 9 cm in width, 82 cm in length, and 46 cm in height. A cross section of a linear sand dune with varying heights was modeled by a triangular sand pile constructed over a layer of polydimethylsiloxane (PDMS), a proxy for salt (Weijermars, 1986; Weijermars et al., 1993) (Fig. 2). We used 100 μm quartz sand with a density of 2600 kg/m³. The polymer had a viscosity of 2.5e4 Pa·s (Dooley et al., 2007) and a density of 965 kg/m³. Although the density of the polymer is less than that of natural salt, it has been shown that the ratio of densities between the substrate and the overburden material in laboratory models is often higher than in natural systems (Weijermars et al., 1993). The increased density ratio may result in faster deformation rates, but it does not affect our investigation of the main controls on subsidence. Both experiments had the same initial salt layer thickness (η_s = 14.5 cm). The sand pile was made by loading sand onto the polymer from a point source at the beginning of the experiment, and no sediment was added during the experiments. Thus, we essentially modeled a static dune in order to isolate the subsidence behavior. The Norphlet dunes were very large, so their migration rates were likely very small. Once they began to subside into the salt, they were likely much less mobile, so we do not address downwind dune migration in this study but rather focus on subsidence dynamics.

The first experiment (henceforth run 1) had a maximum sand thickness at the center of the pile equal to salt thickness (η_s = 14.5 cm), whereas the second experiment (run 2) had a sand thickness equal to half of the salt thickness (η_s = 7.5 cm) (Fig. 2).

In run 1, in which η_s = η_d, the “dune” fully subsided and was surrounded by salt on all sides. There was a short time lag (<5 min) between deposition of the dune and the start of deformation, but the temporal resolution of the time-lapse photographs was not high enough to capture the exact time lag. As the deformation began, the bottom of the deposit created a bowl-shaped basin under extension over time, causing small fractures to develop at the base of the deposit. The top of the dune began with straight sloping sides that became concave up as the overall slope decreased (Fig. 2A). Once the top of the dune was approximately level with the surrounding salt layer (t ~ 120 min), the polymer flow slowed, and the dune shape became fairly stable. As the dune continued to subside, the salt flowed out from under the dune toward the sides of the flume, creating bulges on both sides of the dune. The dune did not reach the bottom of the flume, as there was still a thin layer of salt (~2 cm) between the sand and the flume bottom. For simplicity, here we only consider data for the first 2 h of run time, such that we do not address the data for the time when the polymer began to cover the dune (t > 2 h).

Run 2 had similar results to those of run 1, but run 2 had a longer initial time lag between deposition and deformation (t > 5 min). The bottom of the deposit again formed a bowl-shaped basin that widened and fractured. Basal extension slowed over time, but the dune shape did not fully stabilize, as it did in run 1. Instead, the dune flattened more in run 2 than the dune in run 1, creating less relief in the final dune geometry.

The maximum amount of dune subsidence over time (w), measured at the center of the dune, for both experiments is shown in Figure 3. Both dunes showed an initial rapid subsidence, followed by a deceleration after ~30 min. Run 2 showed a faster deceleration than did run 1.

**MATHEMATICAL MODEL**

Here we develop a model to capture the dune subsidence over time in our experiments. We assume that the polymer is not compressible, and thus it maintains a constant density through time. To determine the dune subsidence, we must quantify the displacement of polymer from beneath the dune as it flows laterally. Consider a dune cross section on a salt layer of uniform thickness (Fig. 2). The triangular-shaped dune subsides into the underlying salt layer without changes in the initial width of the deposit. Changes in salt volume underneath the dune occur by outflow of the salt from beneath the dune. To solve for this salt flow, we apply a parabolic horizontal velocity profile for viscous flow.
between two vertical rigid plates. The no-slip condition is applied to the boundaries between the salt layer and the plates. By integrating over the width of the flume, we can determine the vertical profile of outflow velocity. We apply a pressure gradient \( d p / dx \) that addresses the unequal loading of the salt as a result of the triangular dune geometry to arrive at the equation for subsidence rate (see a full derivation in supplemental material 1):

\[
\frac{dw}{dt} = \Phi_d \eta_d - \Phi_w,
\]

where

\[
\Phi_d = \frac{\rho g}{9 \mu 2 B_x^2} b^3,
\]

\[
\Phi_w = \frac{\rho g}{9 \mu 2 B_x^2} b^3 \left[ 1 + 2 \left( \frac{B_y}{L - B_x} \right)^3 \right].
\]

and \( w \) is the amount of vertical deflection of the dune into the salt layer, \( \eta_d \) is the initial dune height, \( \mu \) is the viscosity of the polymer, \( \rho_d \) is the density of the dune deposit, \( \rho \) is the density of the polymer, \( g \) is the gravitational constant, \( b \) is the width of the flume in the \( y \) direction, \( B_x \) is the width of the dune in the \( x \) direction, and \( L \) is the length of the flume in the \( x \) direction, so \( L - B_x \) scales with interdune distance (Fig. 2).

Equations 1–3 indicate that the subsidence rate of the dune is most strongly dependent on the flume width (\( b \)). The initial dune height (\( \eta_d \)) and dune spacing (\( L - B_x \)) are also important factors in controlling subsidence rate, as seen in Equation 1. To apply the model to the experiments, all length scales and material properties from run 1 and run 2 (dune height, polymer height, polymer viscosity, etc.) were used. We assumed 30% porosity for the dune.

The modeling results slightly underestimate the subsidence rates of the dunes in the experiments (Fig. 3). However, model 1 shows a faster initial subsidence rate than model 2, which matches the results from the experimental data. A correction of decreasing dune width \( B_x \) by 5% yields model results more similar to the experimental data. The dune width measured from experimental images represents the maximum lateral extent of the dune. We assumed in the model that the dune was a perfect triangle, but we know from observation that there is some irregularity to the dune geometry in the experimental data. The dune in run 2 had a smaller pressure gradient that resulted in a lower salt outflow velocity. In general, smaller dunes will subside less, as the maximum subsidence is limited by the initial dune height and strongly controlled by dune width. Small dunes create a small pressure gradient, so the force driving the expulsion of material from beneath the dune is much less than that created by larger dunes. There is not necessarily a minimum size for dune subsidence to occur, but as the pressure gradient approaches zero, the subsidence likewise approaches zero.

Thus far, our model has not shown a relation between subsidence and initial salt thickness (\( \eta_d \)). However, the model is designed specifically to capture the experiments and demonstrates that the subsidence rate in the experiments is strongly dependent upon the width of the experimental apparatus, such that \( d \eta / dt \sim b^3 \). A narrower tank leads to slower subsidence rates due to the friction with flume walls, which slows down salt flow. It is important to note that this relation is specific to our experimental system and the intention to model the experiments rather than a natural system.

We can expand the model to natural systems by reconsidering the vertical velocity profile of salt flow. For a natural system, the basin width (\( b \)) can be approximated to be infinitely large. While some dune fields do exist in enclosed basins (e.g., White Sands National Monument in the Tularosa Basin), the basin width is still infinitely large compared to the size of individual dunes (\( b \gg B_x \)), so we believe this assumption to be valid.

The velocity profile in the \( y \) direction in an infinitely large basin is therefore not controlled by basin width \( b \). Instead, we use the parabolic...
vertical profile of outflow velocity to calculate viscous flow between a horizontal plate and a free surface, in which velocity is zero at the basement and maximum at the free surface. Integrating this velocity profile through the vertical direction produces the flux of salt outflow beneath the dune.

While we do expand the model to three dimensions with this approximation, we are still modeling subsidence apparent only in the lateral cross section rather than the downwind direction. We do not model downwind dune migration. We instead present an additional hypothesis to that of Prather (1992) to address lateral cross section rather than the downwind modeling subsidence apparent only in the beneath the dune.

Integrating this velocity profile through the vertical profile of outflow velocity to calculate the flux of salt outflow $Q_{s}$, the subsidence rate will be relatively low. A small length $L$ will lead to a higher $\Phi$ term in Equation 5, thus decreasing the subsidence rate $d\eta/dt$.

Figure 4 shows the amount of subsidence over time for five different salt substrate thicknesses $\eta$ and two lengths $L$. Here we prescribed an initial dune thickness of 130 m, which was approximated from isopach maps of the Norphlet Sandstone. The dune width was set such that the dunes maintained a constant ratio of width to height of $B_{d}/\eta_{s} \sim 13$ ($B_{d} = 1700$ m). This ratio was determined from seismic data showing Norphlet dune geometries. Keeping dune size constant, we ran the same model for different salt thicknesses from 50 m to 350 m over a period of 2 m.y., assuming that the Norphlet Sandstone was developed over ~2 m.y. (Ajdukiewicz et al., 2010). Sand was assumed to be quartz sand with a density of 2600 kg/m$^3$ and a porosity of 45% (McBride et al., 1987), and salt had a density of ~2100 kg/m$^3$. The modeling results indicate that thicker salt substrates allow for higher subsidence rates. A comparison of the modeling results for the case of small interdune width (Fig. 4A) with the case of large interdune width (Fig. 4B) highlights the control of interdune width on the amount of subsidence that can occur. The larger interdune spacing (Fig. 4B) allowed for more overall dune subsidence during the 2 m.y. run. The application of our model to the Norphlet Sandstone also allows for approximation of salt substrate thickness, which is typically difficult to determine due to seismic imaging inaccuracies and signal attenuation (Montgomery and Moore, 1997; Hardage et al., 1999; Hoxha et al., 2011; Frekers et al., 2012). Given a particular preserved geometry and timing, we can approximate salt thickness by modeling multiple thicknesses (Fig. 4A). For example, a system with interdune spacing of 3000 m and ~30 m of preserved subsidence that developed over ~0.5 m.y. yields an approximate salt thickness of 150 m (Fig. 4A).

We further investigated the control of dune height on subsidence using an interdune spacing of 3000 m with two different salt thicknesses (100 m and 200 m) (Fig. 5). We again maintained a constant ratio of width to height of $B_{d}/\eta_{s} = 13$. In this model, we prescribed dune heights ($\eta$) ranging from 50 m to 150 m such that dune width ranged from 650 m to ~2000 m. Initially, smaller dunes subside faster than larger dunes, but the maximum amount of subsidence is set by the initial dune height. For a given salt thickness, the reduction of subsidence rate occurs more quickly for smaller dunes as the pressure gradient more rapidly decreases, with only small amounts of dune deformation. As the dune height increases, the dune also widens, thus diminishing the interdune distance. The decreased interdune spacing enhances salt upwelling between dunes but also creates resistance to the outflow of salt from beneath the dune. Hence, larger dunes subside slower than smaller dunes but have more overall subsidence. We can infer from these modeling results that dune size is limiting the maximum potential subsidence but that interdune distance is also a strong control on the amount of total subsidence.

$$\Phi_1 = \frac{1}{3} \rho_{s} g \eta_{s}^{-3} L B_{d}$$

$$\Phi_2 = \frac{1}{3} \rho_{s} g \eta_{s}^{-3} \left[ 1 + \left( \frac{B_{d}}{L-B_{d}} \right)^{2} \right]$$

Equations 4 and 5 indicate that the strongest control on subsidence rate in non-width-limited systems is the initial salt thickness, as $d\eta/dt \sim \eta_{s}^{-3}$. For an initially thick salt layer, the dune would subside quite rapidly. The initial dune thickness ($\eta_{d}$) sets the maximum amount of deflection into the salt that can occur for a given amount of time while $w \leq \eta_{d}$.

However, we must also note that there is still a strong dependence on dune width ($B_{d}$) and interdune spacing ($L$). Dune spacing in a natural field is likely controlled by sediment supply, wind speed, and the distribution of wind directions that lead to dune interactions. By decreasing the interdune spacing, the upwelling salt is forced into a smaller interdune area, thus more rapidly decreasing the lateral pressure gradient. Since unequal loading leading to a lateral pressure gradient drives dune subsidence into the salt, homogenization of the overburden pressure diminishes dune subsidence. For cases in which the interdune spacing is very large compared to the individual dune width ($L \gg B_{d}$), the subsidence rate (and total amount of subsidence) is relatively high. We can see this mathematically in Equation 4, where a large length $L$ will decrease the $\Phi$ term, making the subsidence rate $d\eta/dt$ relatively high. On the other hand, for the case of closely spaced dunes with a small interdune length ($L$) compared to dune width ($B_{d}$), the subsidence rate will be relatively low. A small length $L$ will lead to a higher $\Phi$ term in Equation 5, thus decreasing the subsidence rate $d\eta/dt$.

**Figure 4.** Modeling results showing the evolution of subsidence over time for five different salt substrate thicknesses ($\eta$) and two interdune lengths $L$: (A) $L = 3000$ m, (B) $L = 4500$ m. The lines on both graphs correspond to the same legend in A.
that can occur. Thus, dune spacing (i.e., $B_j/L - B_j$) is a strong control on subsidence behavior. Relative salt thickness, on the other hand, is a strong control on the initial subsidence rate, as thicker salts allow for faster dune subsidence.

We can see in Figure 1A that the lenticular Norphlet dunes have an irregular spacing. Salt thickness in the region is also somewhat variable (Story, 1998). We therefore suggest that variability in the preserved lenticular dune topography (i.e., flattened vs. unflattened) in the Norphlet Sandstone likely reflects the variability in dune spacing throughout the region, as seen in Figure 1A, and also possible spatial variation in salt substrate thickness up to a few hundred meters.

**CONCLUSIONS**

The subsidence of dunes into a salt substrate in the natural environment occurs due to unequal loading of sand on the salt. The lateral pressure gradient drives dune subsidence via expulsion of salt from beneath the dune. The salt provides an accommodation space for dune deformation, and salt thickness strongly controls dune subsidence rate.

The maximum amount of dune deflection into the salt is set by the initial dune deposit thickness, but the final amount of dune subsidence is largely controlled by interdune spacing. This interdune spacing sets the larger-scale lateral pressure gradient that drives dune subsidence. Experimental systems may be limited by basin size, which increases boundary layer friction and decreases subsidence rates, but natural systems have comparatively infinite basin width and are therefore controlled by local dune geometries.

Our model of dune subsidence into underlying salt may be useful for determining salt substrate thickness in a dune field. Seismic data can be used to show sand bodies present in the subsurface, but signal attenuation near salt may complicate full data interpretation. Our new model can be used to calculate salt thicknesses given an approximate time scale of deformation and by imposing simple dune geometry, which may be determined from seismic data.

While this study is not exhaustive, it does shed light on the relations between dune geometries and dune subsidence. Given the static dune modeling presented here, we do not capture the full dynamic behavior of dunes, which involves downwind dune migration and the feedbacks that occur between the migrating dunes and deformable substrate. Future research could help to validate or invalidate some of the hypotheses presented here and allow for a more complete understanding of the dynamics and interactions between sand dunes and a deformable substrate.

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**REFERENCES CITED**


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