equal, is considered. The equations are correct as they are for the cited conditions. We cannot emphasize this point any stronger. Once again, it would be good if our previous discussions are read more carefully.

With respect to, "There is one further feature of the Vafai and Huang (1994) paper which requires comment . . . ," we are thankful for Professor Nield for clarifying the positions that we have always invariably held. However, it is ironic that he is now basing his discussion on a term that he has been stating all along should not even be there. This is a term that we have clearly shown to be negligible "for most practical situations", Vafai and Tien, (1981), and various other works have also shown it to be negligible except for rare cases discussed in Vafai and Tien (1981). In the work presented in Vafai and Huang (1994), once again we have found that the convective term has essentially no effect on the results. We had found early that it makes no difference in our runs whether the epsilon-related coefficient in front of the convective term is present or not. This is because this term itself has no impact on the results. Therefore, in lieu of the fact that it has literally no impact, we had decided to just leave it out. This has no effect on any of the results. It is good, however, to see that Nield is moving ever more closely to the position that we have invariably held from the very beginning of proposing the generalized formulation. With respect to the thermal conductivity, we are thankful to Professor Nield for raising the question. However, it should be noted that the porosity in Eq. (6) of Vafai and Huang (1994) is embedded in the effective thermal conductivity. It should be noted that the effective thermal conductivity is already a function of the porosity. Therefore, the effective thermal conductivity is just presented in a different form. In essence, this is like representing $f_1(x)/x$ by $f_2(x)$. Conceptually, there is no difference between the two formulations. We are also thankful for Professor Nield's attention to this paper which is quite rich in physical and phenomenological descriptions that set a very innovative approach for heat transfer regulation and modification.

With respect to the comments on the vorticity-stream function formulation, here, once again, it appears that there is misunderstanding of the numerical procedure for handling a porous/fluid composite layer. As pointed out in Vafai and Kim (1990), one can use separate calculation schemes for the porous and fluid regions which would require an involved iterative procedure for matching the interface conditions. A more efficient approach is to combine the two sets of governing equations for the fluid region and the porous region into one set of conservation equations. In other words, the porous substrate and the fluid region can be modeled as a single domain governed by one set of equations, the solution of which satisfies the continuity of the velocities, stresses, temperature, and the heat fluxes across the porous/fluid interface. Hence, the inclusion of derivatives of the Darcy number and inertial coefficient in Eq. (17) of the first paper plays a role only across the interface. The same approach has been used by other investigators. It is good to hear about Professor Lage's personal feelings about this approach and we are thankful for his extra attention to our works. However, it should be noted that the one domain approach used by us is based on physical principles and it is not at all dependent on how our code is structured, as long as it is written correctly.

As for our statements regarding Chen and Chen's failure to set these coefficients equal, our statements and position stand as they are. Unfortunately, the statements by Professor Nield regarding the work of Brinkman (1947) stem from a confusion of the very issue that he is trying to discuss. Therefore, to clarify this issue once again we will briefly go through the explanation of the factors that must be considered and the clarifications regarding the effective viscosity and the fluid viscosity coefficients. For these coefficients to be equal, the coefficient for the Darcy term v/K and the coefficient for the Brinkman term $\nabla^2 v$ need to be set equal. This means that the effective viscosity, which refers to the coefficient in front of the Laplacian of the velocity of the fluid-saturated porous medium, should be equal to the fluid-viscosity. This point is directly and exactly consistent with the original formulation by Brinkman (1947) as displayed in his Eq. (5), Neale and Nader (1974) as displayed in their Eq. (1.7), as well as various other investigators. This is precisely what Vafai and Huang (1994) and Huang and Vafai (1994) have done. This is not a matter of style.

Finally, it is quite interesting to read after all these discussions that Nield is stating, "For the case of media of small or moderate permeability, it probably does not matter much, from a purely empirical as distinct from a scientific viewpoint, whether or not one takes the effective viscosity to be equal to the fluid viscosity, or that divided by the porosity, in the differential equation or the boundary conditions." We cannot help but think why we had to spend so much time on this issue.

In summary, we would like to thank Professor Nield for an extension of an extension of this discussion. However, we have already discussed these issues at length and with clarity with respect to several inapplicable points. As such, we believe any further discussion on what has already been presented at length would not serve any technical need.

Additional Reference

Nield, D. A., Vafai, K., and Kim, S. J., 1996, "Closure Statements on the Brinkman-Forchheimer Extended Darcy Model," *International Journal of Heat and Fluid Flow*, Vol. 17, pp. 34–35.

Discussion of a Discussion by K. Vafai and S. J. Kim[1]

D. A. Nield[2] and J. L. Lage.[3] In their discussion of a recent paper, Vafai and Kim (1995, p. 1097) stated that they had shown that, "the solution of Vafai and Kim (1989) . . . is valid for Da < 1, which covers the entire range of all porous media." In making this statement they have overlooked the fact that there is a class of materials of practical importance which are appropriately modeled by the Brinkman-Forchhiemer equation and for which the Darcy number can exceed unity. For example, Weinert and Lage (1994) reported permeabilities of compressed aluminum-alloy foams as high as 8×10^{-6} m^2, and for a 1.0 mm thick layer of such material the Darcy number is equal to about eight. Materials in this class have a connected solid matrix and a connected void space.

For this type of porous medium that we propose be called a "hyperporous medium," the Brinkman term has order of magnitude comparable with that of the Darcy term throughout the medium (rather than just in boundary layers near solid walls). The "permeability" K which appears in the Darcy term of the general momentum equation is no longer determined, in a Darcy type experiment, as simply the fluid viscosity times the mean Darcy velocity divided by the pressure gradient. Rather, one has to allow for the contribution of the Brinkman term to the pressure gradient. As the porosity becomes closer to unity, the Brinkman term dominates over the Darcy term, and the value of K (and hence the Darcy number) can increase without limit (other than that imposed by the strength of the solid material from which the matrix is constructed).

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We suggest that in future work concerned with hyperporous media, one should distinguish between: (i) K_D, the usual "Darcy permeability" obtained experimentally using Darcy's law only; and (ii) K_h, the "hyperporous permeability," i.e., the parameter that appears in the momentum equation and represents a coefficient for viscous drag caused by the solid matrix only.

Vafai and Kim (1995) have also overlooked the fact that their Fig. 2, which presents plots of Nusselt number and centerline velocity versus Darcy number Da, for an unstated value of their inertia parameter Λ_I, shows a small but significant systematic discrepancy between their new numerical results and the results of Vafai and Kim (1989) as soon as Da is greater than about 0.01. Furthermore, the velocity profile displayed in their Fig. 1 reveals that for $\Lambda_I = 10$ and Da = 1 there is a significant discrepancy between the two sets of results. If they had performed calculations for smaller values of Λ_I they would have found a larger discrepancy. Although Vafai and Kim (1995) describe the solution of Vafai and Kim (1989) as an "exact solution," it in fact involves a boundary-layer approximation, and they have now merely shown that their approximate solution is accurate in those cases in which boundary layers occur (which is true for most, but not all, practical situations).

We note that Vafai and Kim (1995) have corrected a "typo" in Eq. (9) of their 1989 paper and that this conforms with the correct solution of the differential equation for the case $\Lambda_I = 0$, a solution originally obtained (essentially) by Kaviany (1985) and presented in rearranged form by Lauriat and Vafai (1991). However, Eq. (8) of Vafai and Kim (1989) actually leads to the asymptotic expression

$$u = 1 - \exp[\mathrm{Da}^{-1/2}(y - 1)]. \qquad (1)$$

This result has been supplied to us by Professor Vafai (1996), and we have confirmed its correctness. The discrepancy in the predicted centerline velocity is thus of magnitude $\exp(-\mathrm{Da}^{-1/2})$, and this becomes significant as soon as Da is of order unity. For example, when $\Lambda_I = 0.1$, values of the discrepancy are 11 percent for Da = 0.2 and 37 percent for Da = 1.

In order to deal with a hyperporous medium an analytical solution of the momentum equation of Vafai and Kim (1989) valid for all values of Da is essential. Such a solution, including the effect of the Forchheimer term, has been reported by Nield, Junqueira, and Lage (1996). When the Forchhheimer term is negligible, the simple formula given by Kaviany (1985) is appropriate.

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Closure[4]

K. Vafai[5] and S. J. Kim[6]. We appreciate the comments by Nield and Lage on our discussion with Hadim (Vafai and Kim, 1995). We are thankful to the attention they have given to our original work (Vafai and Kim, 1989). Nield et al. (1996) have come up with a different perspective of the exact solution given by Vafai and Kim (1989). They have obtained an interesting mathematical representation of a numerical integration procedure which is a useful counterpart to the full numerical solution of the momentum equation.

The interesting porous medium (a layer which is one mm thick as cited by the authors) which resides in Professor Lage's laboratory, while being novel and obviously quite useful, is similar to a thin screen. Furthermore, the types of porous media Nield and Lage are considering do not satisfy the basic characteristics of what constitutes a porous medium. For example, their porous medium does not have a persistent solid phase, nor does it satisfy the Representative Elementary Volume (REV) requirement mentioned in various places, including Nield and Bejan (1992). In reality, what the authors should mention is that they like to extend the use of the porous medium formulation for situations other than those represented by a real porous medium. In fact, an approach using the porous medium formulation for situations in which there is, in essence, no real porous media has been used by our group in the past. We prefer to refer to all these cases as "pseudo porous medium." This, in our opinion, is a more accurate and representative term as it covers an entire class of materials which are not really porous media but for which the porous medium formulation is utilized to represent the transport processes. Therefore, we support the authors to follow up on the utilization of the porous medium formulation for the "pseudo porous medium" as we and a few other researchers have done in the past.

Vafai and Kim's (1989) solution is based on the free stream velocity, u_∞, which is equal to the center line velocity after the flow is fully developed, as long as the two boundary layers along the walls (top and bottom) don't interact with each other. This is because: (1) the thickness of the momentum boundary layer does not grow as the streamwise coordinate increases; and (2) the thickness of the momentum boundary layer is of the order of $\sqrt{K/\delta}/H$ or $\mathrm{Da}^{1/2}$. These two facts were shown by Vafai and Tien (1981) and later further substantiated by various other researchers (e.g., Kaviany, 1985) and were well addressed in Vafai and Kim (1989). The main difference between the interesting numerical solution presented by Nield et al. (1996) and the exact closed form solution presented by Vafai and Kim (1989) is in the use of the second derivative of the velocity with respect to y. Vafai and Kim (1989) assumed that outside the momentum boundary layer, in the core region,

$$\frac{d^2u}{dy^2} = 0.$$

The validity of this assumption, which can be shown by scaling analysis, was also rigorously proven and established by comparing the exact solution from Vafai and Kim (1989) with the "Full Numerical Solution" of the momentum Eq. (4) and boundary conditions (5a) and (5b) (using no slip boundary conditions on both walls of the channel) of Vafai and Kim (1989). As established by Hadim, the exact solution obtained by Vafai and Kim (1989) precisely matches (the curves corresponding to the numerical solution and the exact solution are inseparable for a vast range of parameters which covers, to the best of our knowledge, the entire range of known bona fide porous media) the numerical solution of the momentum equation given by Eq. (4) and boundary conditions (5a) and (5b) of Vafai and Kim (1989). The exact solution starts deviating from the numerical solution for Da > 1.

Nield et al. (1996) used Romberg's numerical integration to solve the integrals in their Eqs. (10) and (11). This is an interesting counterpart to the full numerical solution of the momentum equation which is an ODE. Even though they have presented a different numerical procedure, their solution cannot

[4] Only references which are not given, if any, in the discussion are cited.

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