Coexistence of Rotational Bands and \( \alpha \) Cluster Correlations in \( ^{42}\text{Ca} \)

Yasutaka TANIGUCHI

RIKEN Nishina Center for Accelerator-Based Science, RIKEN, Wako 351-0198, Japan

Coexistence of various low-lying rotational bands in \( N \neq Z \) nucleus \( ^{42}\text{Ca} \) and \( \alpha-^{38}\text{Ar} \) correlations in the deformed states have been investigated using the deformed-basis antisymmetrized molecular dynamics. Wave functions of low-lying states are obtained by the parity and angular momentum projections and the generator coordinate method (GCM). The GCM basis are obtained by energy variations with constraints on quadrupole deformation parameter \( \beta \) and intercluster distance between \( \alpha \) and \( ^{38}\text{Ar} \) clusters. The rotational band built on the \( J^\pi = 0^+_2 \) (1.84 MeV) state is reproduced, and coexistence of two more \( K^\pi = 0^+ \) rotational bands is predicted. Members of the ground state band and the rotational band built on the \( J^\pi = 0^+_3 \) (3.30 MeV) state contain \( \alpha-^{38}\text{Ar} \) cluster structure components. Variety of combinations of particle-hole configurations for protons and neutrons develops rich structures in \( ^{42}\text{Ca} \).

\( \S 1. \) Introduction

Drastic structure changes by low-excitation energies are important phenomena. Most ground states form spherical or slightly deformed shapes in nuclei, but recent experiments show largely deformed states exist in low-lying states. Recently, various excited rotational bands have been observed in \( A = 40 \) region.\(^1\)–\(^7\) Clustering is also a drastic structure change.

Excited states in \( ^{42}\text{Ca} \) has been studied via \( \gamma \)-spectroscopy experiments.\(^5\),\(^6\) A rotational band built on the \( J^\pi = 0^+_2 \) state has been observed up to the \( (12^+) \) state.\(^6\) Particle-hole configurations of low-lying states have been studied via an \( \alpha \) transfer experiment. \( \alpha \) transfer reactions populate the \( J^\pi = 0^+_1 \) and \( 0^+_3 \) states strongly, and the \( J^\pi = 0^+_2 \) state weakly. It is concluded that the \( J^\pi = 0^+_2 \) and \( 0^+_3 \) states have \( 6p4h \) and \( 4p2h \) configurations, respectively, relative to the \( sd \)-shell double closed structure. Theoretically, an \( \alpha + ^{38}\text{Ar} \) orthogonal condition model shows that the rotational band built on the \( J^\pi = 0^+_2 \) state has an \( \alpha-^{38}\text{Ar} \) cluster structure. And its dominant components have \( 4p2h \) configurations, which is inconsistent with experimental results.

Aims of the present study are to clarify structures of low-lying positive-parity states in \( N \neq Z \) nucleus \( ^{42}\text{Ca} \) focusing on coexistence of rotational bands. Effects of different proton and neutron numbers for the coexistence are discussed. \( \alpha-^{38}\text{Ar} \) cluster correlations in low-lying deformed states are also discussed.

\( \S 2. \) Framework

To obtain the wave functions in low-lying states, the parity and angular momentum projection (AMP) and the generator coordinate method (GCM) with deformed-basis AMD wave functions are performed. A deformed-basis AMD wave function \( |\Phi\rangle \)
is a Slater determinant of Gaussian wave packets that can deform triaxially, such that

\[ |\Phi\rangle = \hat{A} |\varphi_1, \varphi_2, \cdots, \varphi_A\rangle, \quad (2.1) \]

\[ |\varphi_i\rangle = |\phi_i\rangle \otimes |\chi_i\rangle \otimes |\tau_i\rangle, \quad (2.2) \]

\[ \langle r|\phi_i\rangle = \pi^{-3/4} (\det K)^{1/2} \exp \left[ -\frac{1}{2} (K r - Z_i)^2 \right], \quad (2.3) \]

\[ |\chi_i\rangle = \chi_i^{\uparrow}\langle \uparrow | + \chi_i^{\downarrow}\langle \downarrow |, \quad |\tau_i\rangle = |\tau\rangle \text{ or } |\nu\rangle, \quad (2.4) \]

where \( \hat{A} \) is the antisymmetrization operator and \( |\varphi_i\rangle \) are single-particle wave functions. The variables \( |\phi_i\rangle, |\chi_i\rangle, \) and \( |\tau_i\rangle \) are a spatial part, a spin part, and an isospin part, respectively, of every single-particle wave function \( |\varphi_i\rangle \). The real \( 3 \times 3 \) matrix \( K \) indicates the width of the Gaussian single-particle wave function that can deform triaxially, which is common to all nucleons. The \( Z_i = (Z_{ix}, Z_{iy}, Z_{iz}) \) are complex parameters that indicate a centroid of an each single-particle wave function. The complex parameters \( \chi_i^{\uparrow} \) and \( \chi_i^{\downarrow} \) represent spin directions. Time reversal or axial symmetries is not assumed. Basis wave functions of the GCM are obtained by the energy variation with two types of constraints, the quadrupole deformation parameter \( \beta \) of a total system and the intercluster distance, which is defined by positions of centroids of wave packets, between \( \alpha \) and \( ^{38}\text{Ar} \) clusters after the projection to positive parity states. In the energy variation with quadrupole deformation parameter \( \beta \), harmonic oscillator quanta for protons and neutrons relative to the lowest-allowed state are also constrained. Variational parameters are the \( K, Z_i, \) and \( \chi_i^{\uparrow\uparrow} (i = 1, \cdots, A) \). The isospin part of each single-particle wave function is fixed as a proton or a neutron. The Gogny D1S force is used as an effective interaction. Details of the framework are provided in Refs. 8)–10).

\section*{§3. Results}

Figure 1 shows \( \beta \)-energy curves obtained by energy variations with a \( \beta \) constraint. \( 2p, 4p2h, 6p4h, \) and \( 8p6h \) configurations are obtained from small to large \( \beta \). Details of particle-hole configurations are \( (2p)_{\nu}, (2p2h)^{\pi}(2p)_{\nu}, (2p2h)^{\pi}(4p2h)_{\nu}, \) and \( (4p4h)^{\pi}(4p2h)_{\nu} \) configurations for total \( 2p, 4p2h, 6p4h, \) and \( 8p6h \) states, respectively. After angular momentum projection, well deformed states gain much binding energies, and \( J^\pi = 0^+ \) energies of the \( 4p2h, 6p4h, \) and \( 8p6h \) states become similar.

Figure 2 shows energies of \( \alpha-^{38}\text{Ar} \) cluster structures as functions of intercluster-distances obtained by intercluster distance constrained energy variations. Energies of two configurations, \( A \) and \( B \), are adopted as GCM basis. Differences of these configurations are in positions of holes in the \( sd \)-shell for proton part of \( ^{38}\text{Ar} \), which represent rotation of the \( ^{38}\text{Ar} \) cluster partially. \( ^{38}\text{Ar} \) clusters in \( A \) and \( B \) types have two proton holes in parallel and vertical directions to an \( \alpha \) cluster, respectively.

Figure 3 shows level scheme of \( ^{42}\text{Ca} \) obtained by the GCM. Three \( K^\pi = 0^+ \) bands coexist in low-lying states, labeled ND1, ND2, and SD bands. Dominant components of the ND1, ND2, and SD states have \( 6p4h, 4p2h \) and \( 8p6h \) configurations, respectively. Their particle-hole configurations shows that the ND1 band
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Fig. 1. Solid and dashed lines show energies projected onto positive-parity and $J^\pi = 0^+$ states, respectively. Left and right numbers in parentheses show the numbers of protons and neutrons in $pf$-shell.

Table I. Theoretical and experimental $B(E2)$ values are listed in the Weisskopf unit. $I_i$ and $I_f$ indicate initial and final states, respectively. Experimental values are taken from Refs. 5) and 11).

<table>
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<tr>
<th>$I_i$</th>
<th>$I_f$</th>
<th>$B(E2)_{\text{theor}}$</th>
<th>$B(E2)_{\text{exp}}$</th>
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<tr>
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<td>33.12</td>
<td>57 ± 42</td>
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<td>50$^{+35}_{-16}$</td>
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corresponds to the rotational band built on the $J^\pi = 0^+_2$ state identified experimentally. And the $J^\pi = 0^+_4$ state in the ND2 band corresponds to the experimental $J^\pi = 0^+_3$ state. The ND2 and SD bands have never been observed experimentally. The ground state band and the ND2 band contain the A- and B-type $\alpha^{38}$Ar cluster structure components, respectively.
**Fig. 2.** Solid and dashed curves show energies of $\alpha$–$^{38}$Ar cluster structures for A and B types (see text), respectively, as functions of intercluster distance.

Table I shows $B(E2)$ values of in-band transitions of ND1, ND2 and SD bands in the Weisskopf unit. Theoretical in-band transition strengths of ND1 band are consistent with experimental data.

§4. Discussion

It is predicted that three $K^\pi = 0^+$ rotational bands coexist in low-lying states of $^{42}$Ca that has $(2p2h)_\pi(4p2h)_\nu$, $(2p2h)_\pi(2p)_\nu$, and $(4p4h)_\pi(4p2h)_\nu$ configurations (Figs 1 and 3), which has richer structures than an $N = Z$ isotope $^{40}$Ca that has two low-lying $K^\pi = 0^+$ rotational bands. Difference of proton and neutron numbers ($Z = 20, N = 22$) of $^{42}$Ca develops rich structures in low-lying states. In the case of low-lying positive parity states in $N = Z$ nucleus, protons and neutrons favor same configurations due to the isospin symmetry. In fact, both of protons and neutrons have $2p2h$ and $4p4h$ configurations in ND and SD states, respectively, in $N = Z$ nucleus $^{40}$Ca. Whereas, particle-hole configurations of protons and neutrons in $N \neq Z$ nuclei have several options for combinations of particle-hole configurations even in low-lying states. Neutron parts of ND1 and ND2 states in $^{42}$Ca have $4p2h$ and $2p$ configurations, respectively, although proton parts of both states have $2p2h$ configurations. Proton and neutron parts favor different deformations originally, but strong proton-neutron interactions stabilize intermediate deformations to enlarge
overlap between protons and neutrons.

A rotational band built on the $J^\pi = 0^+_3$ state has never been observed experimentally, though existence of the band are predicted (the ND2 band). The ND2 band are probably observed by an $\alpha$ transfer reactions to $^{38}$Ar and $\gamma$ spectroscopy experiments. The ND2 band contain $\alpha-^{38}$Ar cluster structure components as shown in the present study. Therefore, $\alpha$ transfer reactions populate the ND2 band selectively because of mixing of the $\alpha-^{38}$Ar cluster structure components. Observation of $\gamma$-ray from those populated states can confirm the ND2 band.

§5. Conclusions

Structures in deformed states in $^{42}$Ca are discussed using the AMD and the GCM. The rotational band built on the $J^\pi = 0^+_3$ state is reproduced, and coexistence of two more $K^\pi = 0^+$ bands is predicted. Theoretical $B(E2)$ values of in-band transitions of the ND1 band are consistent with experimental data. Since the ND2 band, which has never been confirmed, contains much $\alpha-^{38}$Ar cluster structure component, $\alpha$ transfer to $^{38}$Ar probably populate the ND2 band.

This different proton and neutron numbers develop rich structures in $^{42}$Ca.
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References