Table 1 Cross-sectional properties

Ele. No.	l (mm)	X (mm)	O.D. Mass (mm)	O.D. Stiffness (mm)	I.D. Mass (mm)	I.D. Stiffness (mm)	Unbalance (mm)
1	118	118	32.6	30.5	0.0	0.0	0.0
2	106	224	72.3	48.3	0.0	0.0	0.0
3	265	489	71.8	71.8	0.0	0.0	0.0
4	50	539	270.0	72.0	110.6	0.0	0.01
5	400	9394	270.0	165.1	0.0	0.0	0.0
6	50	989	270.0	72.0	110.6	0.0	0.01
7	265	1254	70.8	70.5	0.0	0.0	0.0
8	105	1359	71.5	71.4	0.0	0.0	0.0

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23 Nataraj, C., "Analysis of the Dynamics of Cracked Rotors," M.S. Thesis, Mechanical and Aerospace Engineering Department, Arizona State University, May 1984.

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APPENDIX

Grabowski Rotor Model

The parameters used in this analysis for Grabowski's model [6] are as listed below. The two bearings, located at stations 3 and 8, are assumed to be identical, purely translational, and independent of speed.

$$k_{YY} = 0.17658\ 10^9 \text{ N/m} \quad c_{YY} = 0.24\ 10^6 \text{ N}-\text{s/m}$$
$$k_{YZ} = 0.26000\ 10^8 \text{ N/m} \quad c_{YZ} = 0.42\ 10^6 \text{ N}-\text{s/m}$$
$$k_{ZY} = -0.68000\ 10^7 \text{ N/m} \quad c_{ZY} = 0.12\ 10^4 \text{ N}-\text{s/m}$$
$$k_{ZZ} = 0.10300\ 10^9 \text{ N/m} \quad c_{ZZ} = 0.14\ 10^6 \text{ N}-\text{s/m}$$

The cross-sectional properties are listed in Table 1 and the associated stiffness and inertia properties for $E = 20.601\ 106$ N/m^2 and $\rho = 7{,}850$ kg/m^3 are listed in Table 2.

Table 2 Rotor stiffness and inertial properties

Ele. No.	EI_0 $(N-mm^2)$	μ (kg/mm)	$J = \rho I_0$ (kg − mm)
1	$0.8751\ 10^{10}$	$0.6542\ 10^{-2}$	$3.3346\ 10^{-4}$
2	$0.5496\ 10^{11}$	$0.3236\ 10^{-1}$	$2.0942\ 10^{-3}$
3	$0.2683\ 10^{12}$	$0.3177\ 10^{-1}$	$1.0223\ 10^{-2}$
4	$0.2718\ 10^{12}$	0.3740	$1.0357\ 10^{-2}$
5	$0.7509\ 10^{13}$	0.4500	$2.8613\ 10^{-1}$
6	$0.2718\ 10^{12}$	0.3740	$1.0357\ 10^{-2}$
7	$0.2497\ 10^{12}$	$0.3091\ 10^{-1}$	$9.5148\ 10^{-3}$
8	$0.2632\ 10^{12}$	$0.3152\ 10^{-1}$	$1.0029\ 10^{-2}$

DISCUSSION

L. S. Jenkins.[1] This paper, although quite informative in developing a good mathematical model based on the familiar concepts of shaft bending moments of inertia, introduces some confusion with use of the terms "subcritical" and "subharmonic." New criticals below the normal first, second, third, etc., are not observed due to shaft cracks, as experiments will show. Rather a peak response occurs at 1/2, 1/3, etc., of critical speed. Figures 7 and 9 in the paper imply that there are "subcriticals" although the text describes the new peaks as 2× and 3× peaks.

Therefore, the terminology in the paper needs a little rework to maintain internal consistency. The graphical presentations showing amplitudes versus speed should also be modified to rescale the locations of the 2× and 3× peaks (new scales on the abscissa will help).

Incidentally, this confusion was not mine only; I discussed this subject with several other people in attendance at the conference as well as one of the authors (Prof. Nelson) of the paper. Professor Nelson did agree that the terminology and graphical presentation could be improved.

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Authors' Closure

The terms "subcritical resonance" and "subcritical speed" are commonly used in the literature of dynamics of cracked shafts. Nevertheless, their meaning can be confusing, as Mr. Jenkins has pointed out, and both terms should have been more clearly defined in the paper. For single-shaft systems, synchronous critical speeds are defined as those shaft speeds where peak (or resonant) responses occur. For lightly damped systems, these speeds essentially correspond to the system natural whirl frequencies. In this work, the interaction of the crack, gravity, and unbalance results in an n-per-revolution resonance at shaft speeds of nominally $1/n$ times the synchronous critical speeds. Each of these speeds is called a "subcritical speed" and the corresponding response phenomenon is called a "subcritical resonance."

The abscissa in Figs. 7 and 8 is the shaft spin speed and it is identical to the first harmonic of the response frequency. Referring to Fig. 7, it is seen that a synchronous critical speed (1× response) exists at a shaft spin speed of approximately 4000 rpm, a subcritical speed of order 1/2 (2× response) exists at a shaft spin speed of approximately 2000 rpm, and a subcritical speed of order 1/3 (3× response) exists at a shaft spin speed of approximately 1667 rpm. A 3-D frequency spectrum, as used in reference [6], clarifies the dynamic character of the response and should have been included in this paper.