We suggest that in future work concerned with hyperporous media, one should distinguish between: (i) K_D, the usual "Darcy permeability" obtained experimentally using Darcy's law only; and (ii) K_h, the "hyperporous permeability," i.e., the parameter that appears in the momentum equation and represents a coefficient for viscous drag caused by the solid matrix only.

Vafai and Kim (1995) have also overlooked the fact that their Fig. 2, which presents plots of Nusselt number and centerline velocity versus Darcy number Da, for an unstated value of their inertia parameter Λ_l, shows a small but significant systematic discrepancy between their new numerical results and the results of Vafai and Kim (1989) as soon as Da is greater than about 0.01. Furthermore, the velocity profile displayed in their Fig. 1 reveals that for $\Lambda_l = 10$ and Da = 1 there is a significant discrepancy between the two sets of results. If they had performed calculations for smaller values of Λ_l they would have found a larger discrepancy. Although Vafai and Kim (1995) describe the solution of Vafai and Kim (1989) as an "exact solution," it in fact involves a boundary-layer approximation, and they have now merely shown that their approximate solution is accurate in those cases in which boundary layers occur (which is true for most, but not all, practical situations).

We note that Vafai and Kim (1995) have corrected a "typo" in Eq. (9) of their 1989 paper and that this conforms with the correct solution of the differential equation for the case $\Lambda_l = 0$, a solution originally obtained (essentially) by Kaviany (1985) and presented in rearranged form by Lauriat and Vafai (1991). However, Eq. (8) of Vafai and Kim (1989) actually leads to the asymptotic expression

$$u = 1 - \exp[\mathrm{Da}^{-1/2}(y - 1)]. \quad (1)$$

This result has been supplied to us by Professor Vafai (1996), and we have confirmed its correctness. The discrepancy in the predicted centerline velocity is thus of magnitude $\exp(-\mathrm{Da}^{-1/2})$, and this becomes significant as soon as Da is of order unity. For example, when $\Lambda_l = 0.1$, values of the discrepancy are 11 percent for Da = 0.2 and 37 percent for Da = 1.

In order to deal with a hyperporous medium an analytical solution of the momentum equation of Vafai and Kim (1989) valid for all values of Da is essential. Such a solution, including the effect of the Forchheimer term, has been reported by Nield, Junqueira, and Lage (1996). When the Forchhheimer term is negligible, the simple formula given by Kaviany (1985) is appropriate.

References

Nield, D. A., Junqueira, S. L. M., and Lage, J. L., 1996, "Forced Convection in a Fluid Saturated Porous Medium Channel With Isothermal or Isoflux Boundaries," *Journal of Fluid Mechanics,* Vol. 322, pp. 201–214.

Kaviany, M., 1985, "Laminar Flow Through a Porous Channel Bounded by Isothermal Parallel Plates," *International Journal of Heat Mass Transfer,* Vol. 28, pp. 851–858.

Lauriat, G., and Vafai, K., 1991, "Forced Convection Flow and Heat Transfer Through a Porous Medium Exposed to a Flat Plate or Channel," *Convective Heat and Mass Transfer in Porous Media,* S. Kakac et al., eds., Kluwer Academic, Dordrecht, pp. 289–327.

Vafai, K., 1996, personal communication.

Vafai, K., and Kim, S. J., 1989, "Forced Convection in a Channel Filled With a Porous Medium: An Exact Solution," ASME JOURNAL OF HEAT TRANSFER, Vol. 111, pp. 1103–1106.

Vafai, K., and Kim, S. J., 1995, discussion, ASME JOURNAL OF HEAT TRANSFER, Vol. 117, pp. 1097–1098.

Weinert, A., and Lage, J. L., 1994, "Porous Aluminum-Alloy Based Cooling Devices for Electronics," SMU-MED-CPMA Inter. Rep., 1.01/94.

Closure[4]

K. Vafai[5] and S. J. Kim[6]. We appreciate the comments by Nield and Lage on our discussion with Hadim (Vafai and Kim, 1995). We are thankful to the attention they have given to our original work (Vafai and Kim, 1989). Nield et al. (1996) have come up with a different perspective of the exact solution given by Vafai and Kim (1989). They have obtained an interesting mathematical representation of a numerical integration procedure which is a useful counterpart to the full numerical solution of the momentum equation.

The interesting porous medium (a layer which is one mm thick as cited by the authors) which resides in Professor Lage's laboratory, while being novel and obviously quite useful, is similar to a thin screen. Furthermore, the types of porous media Nield and Lage are considering do not satisfy the basic characteristics of what constitutes a porous medium. For example, their porous medium does not have a persistent solid phase, nor does it satisfy the Representative Elementary Volume (REV) requirement mentioned in various places, including Nield and Bejan (1992). In reality, what the authors should mention is that they like to extend the use of the porous medium formulation for situations other than those represented by a real porous medium. In fact, an approach using the porous medium formulation for situations in which there is, in essence, no real porous media has been used by our group in the past. We prefer to refer to all these cases as "pseudo porous medium." This, in our opinion, is a more accurate and representative term as it covers an entire class of materials which are not really porous media but for which the porous medium formulation is utilized to represent the transport processes. Therefore, we support the authors to follow up on the utilization of the porous medium formulation for the "pseudo porous medium" as we and a few other researchers have done in the past.

Vafai and Kim's (1989) solution is based on the free stream velocity, u_∞, which is equal to the center line velocity after the flow is fully developed, as long as the two boundary layers along the walls (top and bottom) don't interact with each other. This is because: (1) the thickness of the momentum boundary layer does not grow as the streamwise coordinate increases; and (2) the thickness of the momentum boundary layer is of the order of $\sqrt{K/\delta}/H$ or $\mathrm{Da}^{1/2}$. These two facts were shown by Vafai and Tien (1981) and later further substantiated by various other researchers (e.g., Kaviany, 1985) and were well addressed in Vafai and Kim (1989). The main difference between the interesting numerical solution presented by Nield et al. (1996) and the exact closed form solution presented by Vafai and Kim (1989) is in the use of the second derivative of the velocity with respect to y. Vafai and Kim (1989) assumed that outside the momentum boundary layer, in the core region,

$$\frac{d^2u}{dy^2} = 0.$$

The validity of this assumption, which can be shown by scaling analysis, was also rigorously proven and established by comparing the exact solution from Vafai and Kim (1989) with the "Full Numerical Solution" of the momentum Eq. (4) and boundary conditions (5a) and (5b) (using no slip boundary conditions on both walls of the channel) of Vafai and Kim (1989). As established by Hadim, the exact solution obtained by Vafai and Kim (1989) precisely matches (the curves corresponding to the numerical solution and the exact solution are inseparable for a vast range of parameters which covers, to the best of our knowledge, the entire range of known bona fide porous media) the numerical solution of the momentum equation given by Eq. (4) and boundary conditions (5a) and (5b) of Vafai and Kim (1989). The exact solution starts deviating from the numerical solution for Da > 1.

Nield et al. (1996) used Romberg's numerical integration to solve the integrals in their Eqs. (10) and (11). This is an interesting counterpart to the full numerical solution of the momentum equation which is an ODE. Even though they have presented a different numerical procedure, their solution cannot

[4] Only references which are not given, if any, in the discussion are cited.

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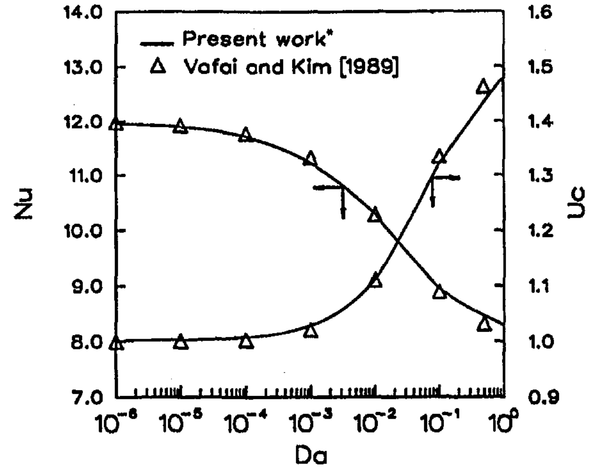

Fig. 2 Comparison of results with Vafai and Kim (1989) *Hadim, A, 1995, "Closure of 'Forced Convection in a Porous Channel with Localized Sources'," ASME Journal of Heat Transfer, Vol. 117, p. 1098.

be easily used for benchmarking the numerical solutions. However, as always, a full numerical solution can be compared with the numerical solution obtained by their Romberg's integration method and this can be a useful addition.

Vafai and Kim (1989) use $du/dy = 0$ when $u = 1$ (instead of $y = 0$) to satisfy $du/dy = 0$ as well as $d^2u/dy^2 = 0$ at $y = 0$. This strategy, which is based on the physics of the problem, as explained in Vafai and Kim (1989), is exact for known porous media. Essentially, this is exact for all practical porous media that we know of other than the interesting and unique set residing in Professor Lage's laboratory. It is important to note that Nield et al. (1996) satisfy the boundary condition $du/dy = 0$, when $y = 0$, implicitly, as was done (i.e., implicitly) in Vafai and Kim (1989). It should also be noted that the left hand side of their Eq. (8) is zero when $u = b_2$. It can then be seen that their numerical solution does not explicitly satisfy the boundary condition $du/dy = 0$ when $y = 0$, either. Their solution satisfies the boundary condition $du/dy = 0$ when $y = 0$, implicitly, which is the same way (i.e., implicitly) that Vafai and Kim (1989) arrived at their solution.

Another point that needs to be noted in Nield et al.'s (1996) work is with respect to recovering a previously obtained analytical solution from their numerical approach for the case $F = 0$. This recovery does not occur as their solution does not approach the known analytical solution as $F = 0$. They had solved the equation analytically for this new case. They did not use Eq. (11) to asymptotically get Eq. (21). When $F = 0$ their Eq. (11) takes the following form:

$$\frac{1}{\sqrt{MDa}}\, y = \int_u^{b_2} \frac{dt}{\sqrt{[t - (2\mathrm{Da} - b_2)](t - b_2)}},$$

which can be integrated to give

$$u = \mathrm{Da} - \Delta \cosh(\lambda y).$$

In our opinion, their Eq. (11) is not the final closed form solution but a mathematical representation of a numerical integration to solve an ordinary differential equation. In essence, this is equivalent to presenting

$$\int \frac{dt}{f(t)} = \int dy$$

as a solution to

$$\frac{du}{dy} = f(u)$$

which is a good representation for a numerical solution of the problem but in our opinion does not constitute an analytical solution.

Comparisons between the full numerical solution based on the momentum equation given by Eq. (4) and boundary conditions (5a) and (5b) of Vafai and Kim (1989) and the exact solution given in the same paper were shown in Figs. 1 and 2 of Vafai and Kim (1995). The exact solution starts deviating from the numerical solution for Da ~ 1. For a reasonably sized porous medium this translates to a permeability, K, of about 10^{-4} or 10^{-3} m^2 at most, and probably smaller. It should be noted that real porous media have permeabilities of at least 10^{-5} m^2 and smaller. Even for the extreme nonrealistic case of K ~ 10^{-2} m^2 and $\Lambda_l = 30$, the agreement is still within 0.7 percent. It should also be noted that Fig. 2 of the discussion given in Vafai and Kim (1995) was not presented by us (as Nield and Lage have incorrectly attributed to us) but rather it was produced independently using a full numerical solution by Professor Hadim. That figure, indeed, does show an excellent agreement up to Da = 1. For the benefit of the readers, Fig. 2 of that discussion, which was obtained by Professor Hadim, is reproduced here. We believe that the readers can easily see the differences between the numerical results of Hadim and the exact solution of Vafai and Kim (1989) in that uncomplicated figure and that there is no need for a guided tour. Furthermore, the cited numbers by Nield and Lage do not correspond to a real porous medium and as such do not relate to our exact solution which was for real porous media. However, we agree that the novel and interesting porous medium (a thin screen which is one mm thick) which resides in Professor Lage's laboratory falls under a different category which we refer to as a pseudo porous medium. Even though we appreciate the opportunity for the additional discussion on this subject with the authors, we believe any further discussion on what had already been presented at length would not serve any technical need.

Reference

Nield, D. A., and Bejan, A., 1992, *Convection in Porous Media*, Springer-Verlag.

Editorial Correction. The authors of the closure that appeared in the ASME Journal of Heat Transfer, Vol. 118, pp. 267–268 were K. Vafai and S. J. Kim. Our apologies for this inadvertent omission.
