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Dynamic mathematical model of high rate algal ponds (HRAP)

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Abstract This article presents a mathematical model to describe High-Rate Algal Ponds (HRAPs). The hydrodynamic behavior of the reactor is described as completely mixed tanks in series with recirculation. The hydrodynamic pattern is combined with a subset of River Water Quality Model 1 (RWQM1), including the main processes in liquid phase. Our aim is to develop models for WSPs and aerated lagoons, too, but we focused on HRAPs first for several reasons:

- Sediments are usually less abundant in HRAP and can be neglected,
- Stratification is not observed and state variables are constant in a reactor cross section,
- Due to the system's geometry, the reactor is quite similar to a plugflow type reactor with recirculation, with a simple advection term.

The model is based on mass balances and includes the following processes:

- Phytoplankton growth with $NO3^-$, $NO2^-$ and death,
- Aerobic growth of heterotrophs with $NO3^-$, $NH4^+$ and respiration,
- Anoxic growth of heterotrophs with $NO3^-$, $NO2^-$ and anoxic respiration,
- Growth of nitrifiers (two stages) and respiration.

The differences with regard to RWQM1 are that we included a limiting term associated with inorganic carbon on the growth rate of algae and nitrifiers, gas transfers are taken into account by the familiar Adeney equation, and a subroutine calculates light intensity at the water surface. This article presents our first simulations.

Keywords Algal pond; biochemical processes; HRAP; hydrodynamic; mathematical model; RWQM1

Introduction

High-Rate Algal Ponds (HRAPs) for domestic sewage treatment are an attractive technology in hot climates, which are propitious for the growth of algal biomass combined with bacterial biomass. As for any other bioreactor, it is important to know the flow pattern in the tank. Obviously the HRAP design is conducive to plug flow, but with a large recirculation flowrate. Moreover, the pond's organic compound treatment capacity will depend on the net oxygen balance and thus photosynthesis and respiration.

Various models have been suggested to describe the HRAP. The first deterministic model appears to have been suggested by Buhr and Miller (1983), who described an already very sophisticated model that took the system's hydrodynamic behavior into account. Buhr and Miller (1983) simulated variations in the process parameters (pH, DO and substrate concentration) according to channel length and time of day, but did not describe the changes in bacterial and algal biomass. Moreover, examination of their equations shows that the model accounted for the algae's contribution to the oxygen balance through photosynthesis but neglected algal respiration. Despite this omission, their approach to modeling the system was good. Grobbelaar *et al.* (1988, 1990) modelled algal productivity in relation to temperature and incident light intensity as well. This model has been used by Martin and Fallowfield (1989) to estimate optimum pond area in Australia. More recently Portielje *et al.* (1996) employed a model, likewise based on the dissolved oxygen balance, in which the respiration rate was temperature dependent and photosynthetic activity was a P-I function. At the last IWA meeting on stabilization ponds some models were submitted on WSPs (Kayombo *et al.*, 1999; Soler *et al.*, 2000; Dochain *et al.*, 1999) but none on HRAPs.

In this paper we propose a model for HRAPs that is compatible with the ASM1 model for activated sludge and RWQM1 for rivers. As in the latter, the biochemical processes are based on elemental mass balances. The hydrodynamics of the system are modeled by a series of completely mixed reactors with recirculation. The biochemical processes parameters are taken from the RWQM1 model and the hydrodynamic parameters will depend on the geometry of the reactor and the mixing equipment (paddle wheel, air-lift, etc.).

Methods

The reactor

The HRAP we are taking as an example for model development and its main characteristics are presented in Figure 1. More details may be found in El Ouarghi *et al.* (2000), where the dye tracer experiments and gas transfer coefficient determination are described.

Hydrodynamics

As shown in Figure 3, the flow pattern of such a reactor is far from that of a completely mixed tank. Moreover, measurements of state variables such as oxygen, pH, and SS indicate that state variables can be considered constant in a cross-section of the reactor. Mixing in the reactor is due mainly to the fact that most of the flow is recycled. In our case the recirculated flowrate (430 m^3/h) is much higher than the inlet flowrate (8 m^3/h) (see Table 1).

For those reasons we chose to model the HRAP as shown in Figure 2. We also decided to calibrate the hydrodynamic model on the tracer dye experiment presented in Figures 3 and 4, corresponding to a pulse injection of the tracer, so as to describe the advection term in the system.

Figure 3 shows the simulated fluorescent dye concentration versus time after the pulse injection. As expected, we see that the C-diagram has multiple peaks (zoom in Figure 4). This phenomenon is due to the recirculation flowrate.

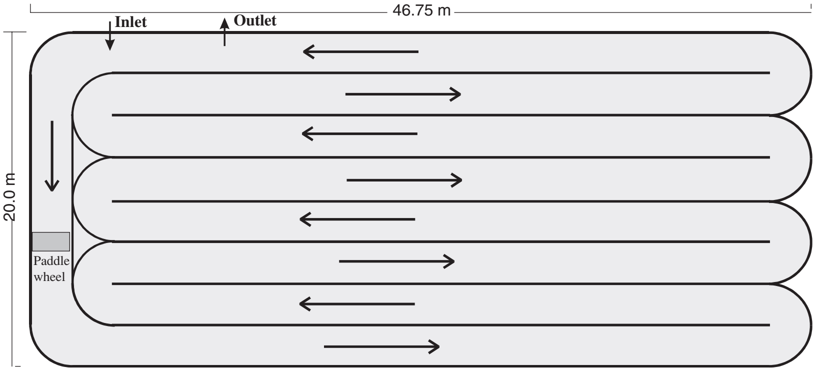

Figure 1 Diagram of the HRAP located in Rabat

Table 1 Main results of the hydrodynamic model calibration

Parameter	Value
Q	430 m^3/h
q	8 m^3/h
Number of reactors	25

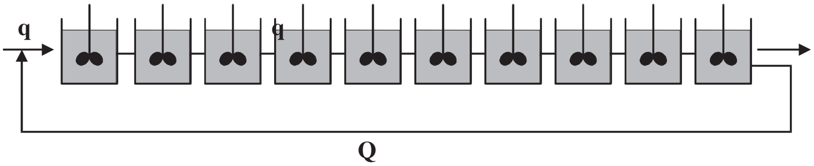

Figure 2 Schematic presentation of the HRAP model

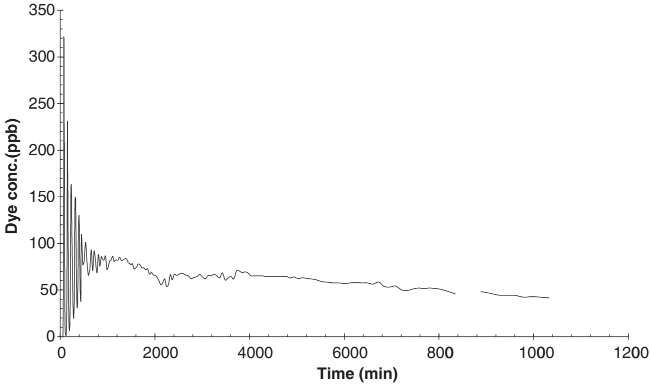

Figure 3 Tracer response curve after pulse injection (Rabat)

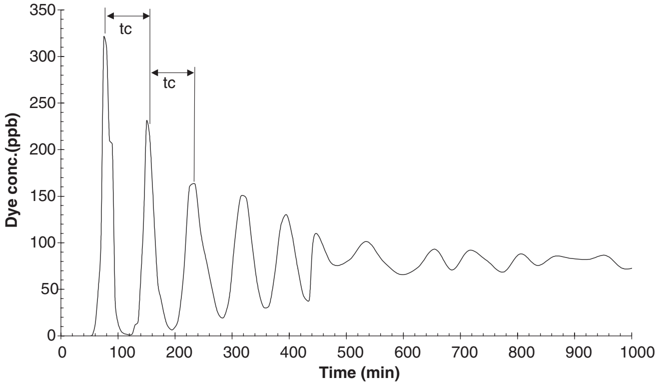

Figure 4 Tracer response curve after pulse injection (zoom on the first peaks)

Macroscopically and over a longer time the reactor continues to respond like a perfectly mixed model due to the large number of recirculations of an element of fluid before leaving the system.

Biochemical processes

Just as for the RWQM1 model, the main biochemical processes are described through mass balances, which yield the following stoichiometric matrix (Table 2). In our model all the biomass concentrations are expressed as dry weight.

Specific features of the model

Some processes are modeled in a way that is specific to our model.

SOD (sediment oxygen demand)

We are seeking to develop a model that will describe sediment activity more precisely. So far this activity is taken into account by an equation that is dependent on the substrate concentration and temperature (Chabir *et al.*, 2000), namely

Table 2 Stoichiometric matrix of the model

Components	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Processes	**SS**	**SI**	**SNH4**	**SNH3**	**SNO2**	**SNO3**	**SHPO4**	**SH2PO4**	**SO2**	**SCO2**	**SHCO3**	**SCO3**	**SH**	**SOH**	**SCA**	**XH**	**XN1**	**XN2**	**XALG**	**XS**	**XI**
1 Aerobic growth of heterotrophs with NH_4	−1.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	−1.4	0.4	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
2 Aerobic growth of heterotrophs with NO_3	−1.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	−1.3	0.4	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
3 Aerobic endogenous respiration of heterotrophs	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	−1.2	0.4	0.0	0.0	0.0	0.0	0.0	−1.0	0.0	0.0	0.0	0.0	0.2
4 Anoxic growth of heterotrophs with NO_3	−2.0	0.0	0.0	0.0	1.7	−1.7	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
5 Anoxic growth of heterotrophs with NO_2	−3.3	0.0	0.0	0.0	−2.6	0.0	0.0	0.0	0.0	1.4	0.0	0.0	−0.2	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
6 Anoxic endogenous respiration of heterotrophs	0.0	0.0	0.1	0.0	0.0	−0.4	0.0	0.0	0.0	0.4	0.0	0.0	0.0	0.0	0.0	−1.0	0.0	0.0	0.0	0.0	0.2
7 Growth of 1st stage nitrifiers	0.0	0.0	−7.7	0.0	7.6	0.0	0.0	0.0	−24.4	−0.5	0.0	0.0	1.1	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
8 Aerobic endogenous respiration of 1st stage nitrifiers	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	−1.2	0.4	0.0	0.0	0.0	0.0	0.0	0.0	−1.0	0.0	0.0	0.0	0.2
9 Growth of 2nd stage nitrifiers	0.0	0.0	0.0	0.0	−33.3	33.2	0.0	0.0	−35.9	−0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
10 Aerobic endogenous respiration of 2nd stage nitrifiers	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	−1.2	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	−1.0	0.0	0.0	0.2
11 Growth of algae with NH_4	0.0	0.0	−0.1	0.0	0.0	0.0	0.0	0.0	0.9	−0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0
12 Growth of algae with NO_3	0.0	0.0	0.0	0.0	0.0	−0.1	0.0	0.0	1.2	−0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0
13 Aerobic endogenous respiration of algae	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	−0.6	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	−1.0	0.0	0.2
14 Death of algae	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	−1.0	0.5	0.1
15 Hydrolysis	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	−1.0	0.0
16 Equilibrium $SCO_2/SHCO_3^-$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	−1.0	1.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17 Equilibrium $SHCO_3^-/SCO_3^{--}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	−1.0	1.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18 Equilibrium SH_2O/SH^{++} SOH^-	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19 Equilibrium SNH_4/SNH_3	0.0	0.0	−1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20 Equilibrium $SH_2PO_4/SHPO_4^-$	0.0	0.0	0.0	0.0	0.0	0.0	1.0	−1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21 Equilibrium SCA $SCO_3/SCACO_3$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.3	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0

$$R = \left[R_0 + R_{\max} \frac{S}{K_S + S} \right] \theta^{t-20}$$

with R: sediment respiration rate (g O_2 m^{-2} day^{-1})
R_0: endogenous respiration rate (g O_2 m^{-2} day^{-1})
$R_{\max}$: maximum respiration rate related to substrate (g O_2 m^{-2} day^{-1})
S: substrate concentration (mg COD l^{-1})
K_S: Michaelis constant (mg COD l^{-1})

Light intensity

For many years WSPs and HRAPs have been designed with empirical equations that usually take into account the load and water temperature. As the main biochemical processes such as photosynthesis are highly dependent on weather conditions, we feel that HRAP performance variables should be related to local conditions. For this reason the light intensity is calculated by a subroutine developed in the TRNSYS package (TRNSYS 15, 2000). This subroutine allows one to compute the average light intensity at the ground level anywhere on the earth. The inputs needed for this are:

h:	hour in the year (1st of Jan at 0:00 AM is 0)
step:	time step (in our case 1 hour)
alt:	location's elevation above sea level (m)
lat:	latitude in degrees (0 = Equator, >0 North)
long:	longitude in degrees (0 = Greenwich, >0 West)
lsm:	longitude for standard time meridian, in degrees

The light intensity used for the calculation in the case of Rabat HRAP is presented in Figure 5. It must be said that the calculation process does not take into account the meteorological conditions: the weather is assumed to be fine and the sky cloudless. Of course the model could easily be improved if meteorological data were monitored directly on site. Nevertheless from this model potential sites for HRAP installations can be designed.

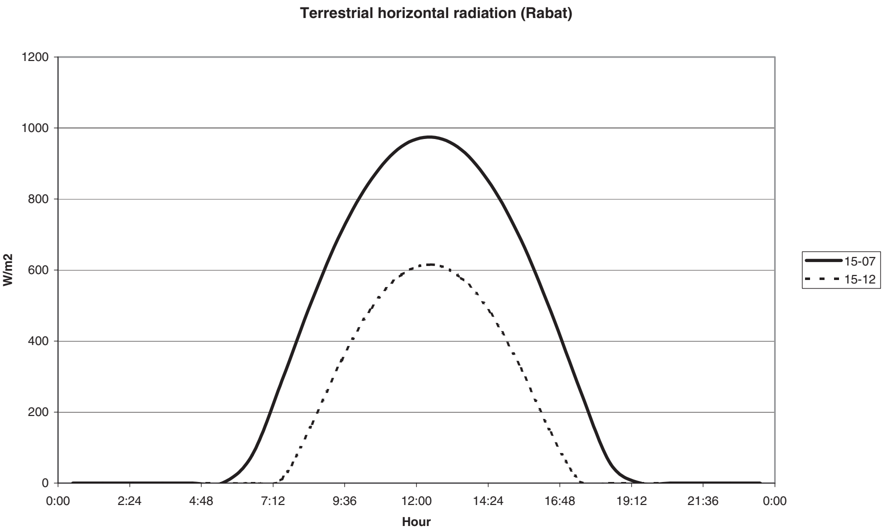

Figure 5 Terrestrial horizontal radiation in Rabat, Morocco, on December 15th and July 15th. Long.: 6.46° Lat.: 30.03° Alt. = 73.3 m Hour: GMT

Finally, we want to allow for the self-shading of the algae and biomass:

$$\eta = 0.32 + 0.03 * [\text{Suspended} \quad \text{Solids}]$$

$$I_{\text{eff}} = \sum_{0}^{\text{depth}} I_{\text{surf}} * \mathrm{e}^{-\eta * Z} * \mathrm{d}z \, / \, \text{depth}$$

with

depth: total depth of the pond (m)
I_{surf}: light intensity at the pond's surface (W/m^2)
I_{eff}: total light intensity in a vertical slice of the pond (W/m^2)
z: distance from the surface (m)
dz: calculation step (m)

Software and results

The model was developed using Matlab version 6 release 12, using the ODE solver provided in this package to resolve the differential equations. The following Figures (7–10) show a simulation of heterotrophic biomass, algae biomass, pH, carbon dioxide and oxygen

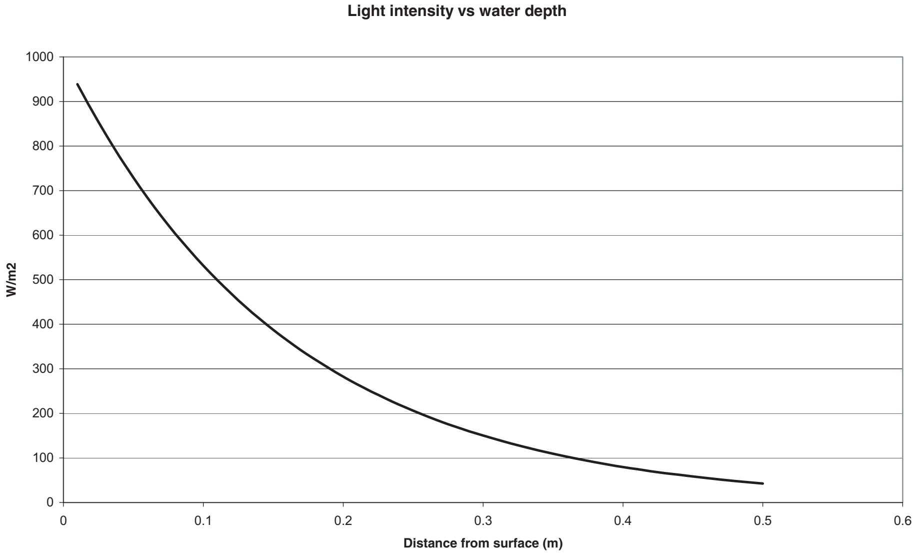

Figure 6 Example of light intensity versus water depth. (The surface intensity is assumed to be 1,000 W/m^2 and the suspended solids to have a uniform concentration of 200 g/m^3)

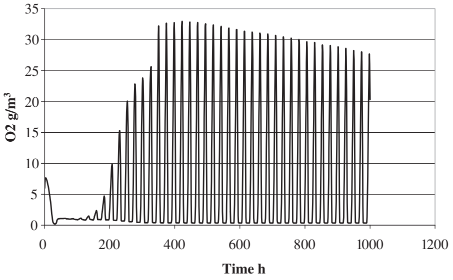

Figure 7 Example of oxygen outlet concentration

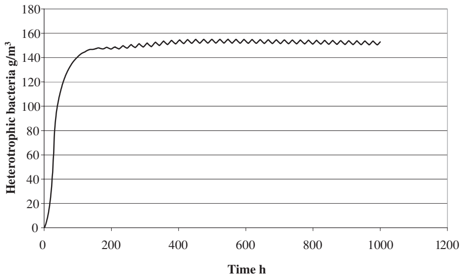

Figure 8 Example of heterotrophic bacteria outlet concentration

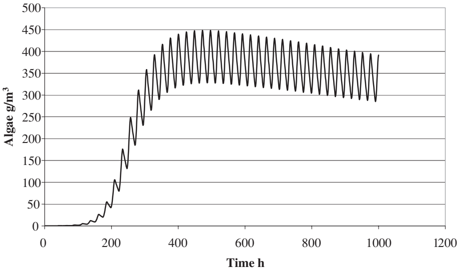

Figure 9 Example of algae outlet concentration

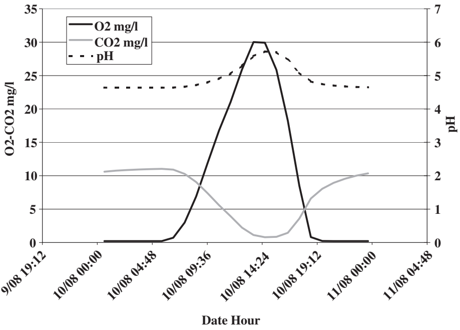

Figure 10 Example of O_2, CO_2 and pH daily variations

concentrations versus time in the HRAP with the classical cycles due to day/night conditions; Figure 10 shows the O_2, CO_2 and pH daily cycles. As we can see, the light influence is clearly marked on the oxygen and algae concentrations (slow decrease of the maximum daily concentration). Future papers will give more information on simulation parameters and their sensivity analysis.

Conclusion

The HRAP model presented in this paper is based on River Water Quality Model 1, with simplifications and extensions of the various biochemical process equations presented in Reichert *et al.* (2000). The system's hydrodynamics are particularly simple and the reactor is modeled by a series of completely mixed reactors with recirculation. Some specific features have been added, such as the calculation of light intensity as a function of location (long., lat.), date and hour, taking the sediment's oxygen demand into account, and the calculation of auto-shading by the suspended solids.

The model that we have developed is able to simulate the HRAP's operating cycles, several examples of which have been provided. The next step in this modeling work is to develop the sediment submodel to simulate lagoon and WSP operating conditions as well as development of calibration procedures for the model.

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