Similarly, the coefficients A_0, A_1, and A_2 for the rotor are given as follows:

$$A_0 = \frac{\sin(\alpha_r + \alpha_s)}{2\pi^2 R_r} \sum_{n=0}^{B-1} \int_{-c_s}^{c_s} \left(\frac{c_s - x_s}{c_s + x_s}\right)^{1/2} \times [(c_s - x_s)\sin(\alpha_r + \alpha_s) + nd_s \cos\alpha_r + b' \sin\alpha_r] \times \int_0^{\pi} \left\{\frac{R_2R_r - R_r^2 - l_n^2}{l_n^2[l_n^2 + (R_2 - R_r)^2]^{1/2}} - \frac{R_1R_r - R_r^2 - l_n^2}{l_n^2[l_n^2 + (R_1 - R_r)^2]^{1/2}}\right\} \times dx_s d\theta_r \quad (21)$$

$$A_1 = \frac{\sin(\alpha_r + \alpha_s)}{2\pi^2 R_r} \sum_{n=0}^{B-1} \int_{-c_s}^{c_s} \left(\frac{c_s - x_s}{c_s + x_s}\right)^{1/2} \times [(c_s - x_s)\sin(\alpha_r + \alpha_s) + nd_s \cos\alpha_r + b' \sin\alpha_r] \times \int_0^{\pi} \left\{\frac{R_2R_r - R_r^2 - l_n^2}{l_n^2[l_n^2 - (R_2 - R_r)^2]^{1/2}} - \frac{R_1R_r - R_r^2 - l_n^2}{l_n^2[l_n^2 + (R_1 - R_r)^2]^{1/2}}\right\} \cos\theta_s dx_s d\theta_r \quad (22)$$

$$A_2 = \frac{\sin(\alpha_r + \alpha_s)}{2\pi^2 R_r} \sum_{n=0}^{B-1} \int_{-c_s}^{c_s} \left(\frac{c_s - x_s}{c_s + x_s}\right)^{1/2} \times [(c_s + x_s)\sin(\alpha_r + \alpha_s) + nd_s \cos\alpha_r + b' \sin\alpha_r] \times \int_0^{\pi} \left\{\frac{R_1R_r - R_r^2 - l_n^2}{l_n^2[l_n^2 + (R_2 - R_r)^2]^{1/2}} - \frac{R_1R_r - R_r^2 - l_n^2}{l_n^2[l_n^2 + (R_1 - R_r)^2]^{1/2}}\right\} \times \frac{\cos 2\theta_r}{l_n^2} dx_s d\theta_r \quad (23)$$

and the lift force, L_r, is

$$L_r = 2\pi\rho U_\infty^2 \left[(A_0 + A_1)C(\omega) + (A_0 - A_2)\frac{i\omega}{2}\right]_{t=0} \quad (24)$$

Equations (1) through (6) have been programmed for the IBM Model 7090 digital computer and calculations have been compared with experimental data in Figs. 8 and 9.

DISCUSSION

B. Mugridge[3]

The paper presents a theoretical treatment which is an interesting addition to those already in existence. In particular, the use of Prandtl's lifting line equation to assimilate the steady circulation about a finite blade span suggests that the treatment could be extended to investigate some of the simpler three-dimensional effects of the problem.

For instance, the authors postulate a two-dimensional model and consequently neglect the downwash velocity that would occur due to the trailing vortex, whose circulation would be equal to that of the bound vortex sheet of the finite blade, and which would be shed downstream from some point near the blade tip. Although this velocity would be small in comparison with the blade's relative inlet velocity, it may be presumptuous to neglect this effect in relation to upwash induced by the bound vortex sheet on which the calculations for fluctuating stator lift are based.

Furthermore, it would be interesting to determine the spanwise variation of this resultant velocity upstream of the blade even for a constant circulation distribution. With regard to this aspect of the problem, the work which is in hand at this University in comparing measured upstream velocities with those computed from a two-dimensional model is hampered by the fact that the measured results of the velocity decay rate are somewhat dependent on the radial (spanwise) velocity variation at a plane tangential to the blade leading edge (used as the generating plane). This type of behavior has of course already been pointed out by Tyler and Sofrin during their investigation into propagating modes in ducts. It would therefore be interesting if Prandtl's lifting line theory for a finite blade could be used to predict some of the simpler upstream to spanwise velocity characteristics for a single rotor.

Taking the argument one step further, a nonuniform circulation distribution could be specified for the rotor and the resulting velocity compared with experimental values for the case where the chosen circulation is believed to be that existing on the actual rotor blading. This situation is of course covered by the theory of the present paper [equations (7), (8), (9)] although the authors appear to allow their radial load distribution to degenerate into a modified constant distribution, equation (9) (Appendix).

It is surprising that a more rigorous treatment was not adopted in developing equation (10). For instance, a modified elliptical load distribution of the form:

$$\Gamma_r(R) = \Gamma_0 \sqrt{1 - \left(\frac{R - R_h}{R_t - R_h}\right)^2} \left\{1 + a\left[\frac{R - R_h}{R_t - R_h}\right]^2\right\}$$

could have been adopted, where

$$\Gamma_0(\text{hub}) = 2V \sin\theta \int_{-c}^{c} \sqrt{\frac{c - x_r}{c + x_r}} dx_r$$

for $a > 0$, θ = angle of incidence, and subscripts t and h refer to tip and hub, respectively.

This would modify equation (10) to:

$$q_s = \frac{2VS_m\theta}{4\pi} \int_{R_h}^{R_t} \left\{\frac{\ln}{[\ln^2 + (R - R_s)^2]^{1/2}}\right\}^3 \frac{1}{\ln^2} \times \left[\sqrt{1 - \left(\frac{R - R_h}{R_t - R_h}\right)^2} \left\{1 + a\left(\frac{R - R_h}{R_t - R_h}\right)^2\right\}\right] \times \int_{-c}^{c} \sqrt{\frac{c - x_r}{c + x_r}} dx_r dR$$

This equation would be suitable for determining the velocity upstream of the single rotor stage having a low solidity (C/S). However, it would require careful thought and consideration before this upwash velocity could be directly used for the two-dimensional unsteady lift equation derived by Kemp and Sears [equation (2)], especially if the spanwise velocity variation were large.

Regarding the use of isolated blade data for cascade problems it would be necessary to determine, in particular, the maximum (C/S) ratio for a given blade form, that can be used before mutual interference between blades becomes an important factor. Would the authors care to comment on what they consider to be the maximum solidity ratio for their calculations to still be applicable? Part of our own investigation is aimed at comparing actual theoretical velocity distributions about two-dimensional cascades incorporating both finite blade widths and straight and cambered lines, with the results from the simple vortex model calculations in an attempt to determine the accuracies of the latter.

In this instance it should be comparatively straightforward to compare the upstream velocities parallel with the cascade yielded by an actual two-dimensional flow field with those of a modified form of equation (11) (Appendix), i.e., the velocity parallel to the cascade:

$$v_s = \frac{1}{2\pi} \sum_{n=-B/2}^{n=\frac{B-1}{2}} \int_{-c}^{c} \left(\frac{b + (c + x_r)\cos\alpha_r}{\ln^2}\right) \times \left\{2V\sin\theta\sqrt{\frac{c - x_r}{c + x_r}} + 2V_c \cdot \frac{y}{\sqrt{c^2 - x_r^2}(c - x_r)}\right\} dx_r$$

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for both straight or cambered thin aerofoils, where

$$\ln = (b - a) + (c + x_r)e^{i\alpha r} + nd_r e^{i\pi/2}$$

$$d_r = S = \text{blade pitch}$$

and b and a denote positions upstream in the x, y-directions relative to the leading edge of the center blade.

The experimental results of the paper in general agree with those reported by other workers in this field. However, it is surprising that the theoretical and measured values are in such close agreement, since on no account are the effects of viscous wakes considered.

Kemp and Sears show that, under certain conditions, the fluctuating lift on a rotor downstream of a stator caused by the viscous wake of that blade is of the same order of magnitude as the lift produced by the induced potential effects of steady stator circulation. Even if one assumes that the potential effects induced on the downstream rotor are less than those on the upstream stator, it still appears that viscous wake effects contribute substantially to discrete frequency noise generation. One therefore questions the accuracy of the theoretical results obtained in the paper when eventually wake effects are included, especially in the case of the higher harmonics.

R. Parker[4]

In the theories of noise generation quoted in the paper there appear to be no terms relating to the blade wakes, and the analyses, in fact, refer to inviscid flow. In choosing the test configuration, the author states that the choice was made because "the wake from a stator is assumed to be relatively stronger than that from a rotor." This seems to imply that some further effects, not covered by the theory, were expected but no further mention is made of them. The configuration shown in Fig. 2 will have considerably higher velocities relative to the rotor than the stator and, therefore, greater velocity variation between the center of the wakes and freestream; this also appears to be inconsistent with the stated assumption regarding the relative strength of the wakes.

In my own experience wake effects have always proved to be insignificant compared with the interference between the blade rows in the potential flow (see, for example, footnote 5) and the authors' results appear to be consistent with this.

Could the authors please explain how they separated "thickness noise" from "rotor-stator interaction noise" in the experimental results shown in Figs. 8 and 9, and can they give corresponding results for "blade loading," bearing in mind their conclusion that "the thickness noise level is comparatively lower than that from blade loading?"

When dealing with single-stage machines with relatively thick blades, my own experience is that the noise levels are practically independent of the operating point on the characteristic, suggesting that the noise does not vary with blade loading; I feel that the conclusion in the paper cannot be applied to machines with thick blades.

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[5] W. Risk and D. F. Seymour, *Proceedings of the Institution of Mechanical Engineers*, vol. 179, 1964–1965, p. 21.