Remarks on the Ferromagnetism of the Semi-conductors

A possible explanation of the ferromagnetism of the semi-conductors was proposed by S. Miyahara on the basis of Wilson's model of semi-conductors and of Slater's theory of the ferromagnetism. We try to treat the same problem by the statistical methods starting from Heitler-London model. We assume that each atom has two energy levels, ground level and excited level, and that at absolute zero temperature ground levels are fully occupied by two electrons with antiparallel spins, while excited levels are all unoccupied. In the suitable temperature range one or two electrons in each ground level may be excited, and the remaining electrons of the singly excited atoms contribute to the ferromagnetism while the excited electrons do not.

Basing on this model, the statistical treatments are performed wholly analogous to the well-known two treatments for the order-and disorder-problem of alloys, namely, Bragg-Williams' theory and Bethe's theory.

1) In the treatment of Bragg-Williams' type, we assume the ordering force $\gamma M$ which make the spins of the remaining electrons of the singly excited atoms parallel and which we suppose, as in Weiss' theory of the ferromagnetism, to be proportional to the present magnetization $M$. From the simple consideration about the free energy of the system it can be shown that $M$ is given by the root of following a multaneous equations similar to those of Weiss' theory:

$$y = \frac{kT}{\mu N}, \quad \frac{2(e_1 - e_2)}{e_1 + e_2 + 2(e_1 + e_2)}$$

where $y = M/N\mu$ and $\lambda = U/kT$, and $\mu$ is the magnetic moment of an electron, $N$ being the number of the total atoms.

2) In the treatment of Bethe's type, we consider the interaction of the atoms only between nearest neighbors and suppose that they are exchange type.

Therefore, we put the potential energies between neighboring atoms as follows:

$$V_{++} = +J, \quad V_{--} = -J, \quad V_{-+} = -\lambda (\lambda > 0), \quad V_{+-} = +\lambda (\lambda > 0).$$
where $V$ means the potential energy between both singly excited atoms, one has a remaining electron with $+$ spin, the other with $-$ spin, and so forth.

The potential energies of all other types of neighboring atoms are supposed to be zero. Following Bethe, we take up a typical atom which we shall call the central atom, and its $Z$-nearest neighbors which shall be called as the boundary atoms. We shall treat the interactions between them precisely and the influence of the other exterior atoms on the boundary atoms are taken into account in average such as it causes a tendency which makes the singly excited boundary atom to take the spin of the remaining electron $+$. Then the relative probability for the circumstance that the central atom is singly excited, the spin of the remaining electron being $+$, and that among the boundary atoms $l$ atoms are doubly excited, $m$ atoms are singly excited of which $n$ atoms have the remaining electrons of $+$ spin, is

$$P(+, l, m, n) = 2\varepsilon \left( \frac{Z}{l} \right)^{l} \left( \frac{Z-l}{m} \right)^{m} \left( \frac{n}{n} \right)^{n} \left( \omega_{0}^{n}(\omega_{0}^{n})^{n} - (m-n) \right)$$

where $\varepsilon = e^{-U/kT}$, $\omega = e^{-J/kT}$ and $\varepsilon$ is the factor which express the above denoted tendency due to the exterior atoms.

The relative probability for the circumstance that the central atom is singly excited and the remaining electron has $+$ spin regardless of the situation of the boundary atoms is, thus

$$P(+) = \sum_{l=0}^{Z} \sum_{m=0}^{Z-l} \sum_{n=0}^{m} P(+; l, m, n)$$

$$= 2\varepsilon [1 + \varepsilon^{2} + 2\varepsilon (\omega_{0} + \varepsilon^{-1} \omega_{0}^{-1})^{2}]$$

All the other probabilities $P(-)$, $P(0)$, $P(\pm)$, the meaning of the notation being clear, are found easily in similar manner.

Now, in equilibrium, $Z[P(+) - P(-)]$ should be the difference of the average number of singly excited boundary atoms with remaining $+$ spin and those with $-$ spin. Hence we have

$$P(+) - P(-) = \frac{e}{Z} \frac{\partial}{\partial \varepsilon} \left[ P(+) + P(0) + P(\pm) \right]$$

from which we can determine $\varepsilon$. Finally $M/N_{H}$ can be given by

$$\frac{M}{N_{H}} = \frac{P(+) - P(-)}{P(+) + P(-) + P(0) + P(\pm)}$$

The relation between $M/N_{H}$ and the temperature in both treatments are plotted for $Z=12$, $U=10^{-1}$ and $J=10^{-10}$. The value of the constant $\gamma$ is estimated by the relation $N_{H}R^{2} = ZI$. We hope that the reason of the discrepancy of the results of two treatments will be understood by the more elaborate consideration.

![Graph](image_url)

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