Conflict between Matter and Field.*
—An Analysis of the Difficulties of the Theory of Elementary Particles—

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The present status of theoretical physics is confronted with difficulties of extremely ambiguous nature. These difficulties can be glossed over in various ways, but no one believes that a definite solution has been attained. The reason for this is that on one hand present theoretical physics itself has logical difficulties, while on the other, there is no decisive experiment whereby to determine the theory uniquely. In other words, the present difficulties manifest themselves in that the theory itself involves certain diverging results, as well as deviating a little from corresponding experiments; and they are as yet not adequately sorted out. But it is generally accepted that these difficulties cannot be overcome by anything in the way of a trivial modification. In short, the state of affairs is just like that on the eve of the advent of quantum mechanics. For in that case too, the discrepancies with experiment were, quantitatively speaking not so great. But unlike those on the eve of quantum mechanics, the present difficulties are not yet sorted out into a radical form such as the incompatibility and inconsistency between fundamental concepts, so that there seems to be some uncertainty as to the point at which to commence attack of the problem. We must set these in order, run them down into an inconsistency between basic concepts, and ascertain the shape of the inconsistency, thus paving the way to a new theory. Recent developments (up to the early stages of the Pacific War) have furnished much material for this purpose. The present inconsistencies and difficulties are believed to touch the internal structure of this theory.

* This is a synopsis of what I made public at the "Nuclear Physics Colloquium" in May 1943, and also what was discussed with Prof. S. Sakata and others at the Physical Department of the Nagoya University in November 1945. Although it is by no means perfect yet, I have decided to publish it, hoping it might furnish some information or other, and also that it might lead to another series of discussions. A concrete analysis, forming the latter half in the MS, has been omitted, be ingerserved for another opportunity. I extend my sincere thanks to Prof. Tomonaga and his coworkers as well as to Prof. Sakata and others for having discussed the matter over with me, and also to Mr. M. Tatsuoka who has helped to prepare present paper in this form.
of elementary particles themselves. I believe that the clue to an analysis at this juncture is provided by identifying this fundamental inconsistency between concepts with the conflict between matter and field.* The two fundamental concepts of matter and field are those most essentially standing against each other at present, embodying the most essential problem of contemporary theory—just as the two incompatible concepts of wave and particle gave birth to quantum mechanics. Much beating about the bush having been made in connection with this problem. Now it becomes the central problem and its definite solution is an urgent matter of the day.

It is a well-known fact that, in the formulation of electron theory, the energy of the field becomes infinitely large unless the electron in classical theory is taken to possess a finite radius, so Lorentz considered a model in which the charge was distributed over a small sphere. As a result, however, forces interact between each charge density, so in order that the electron be kept rigid notwithstanding, a sort of cohesive force must be introduced. This is no longer an electro-magnetic force. Therefore, the necessity arises of considering a point model of the electron. Later on, in quantum mechanics the superposition of material waves was considered in order to denote the existence in space of an electron, in which case no "size" could be attributed to it. So here again was a point-model required. In a point electron, firstly, Lorentz's equation of motion cannot be applied at the place where the charge exists, as the field strength becomes infinitely large there. Secondly, as mentioned above, we obtain an infinitely large electro-magnetic energy at the point of charge.

These difficulties are not confined to classical theory alone, but in quantum theory too, in quantum electro-dynamics, they are present in the fact that a point electron possesses an infinitely large interaction with itself. The above difficulties in classical electron theory and quantum electro-dynamics were, however, rather merely theoretical, and had the character of an "academic problem" as it were. For these difficulties offer no

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* This is, of course, a conjecture which is to be tested in the process of actual analysis by its success or failure, and gradually the correct key will be found.

This much is, however, certain: that this suffices to summarize the many attempts of various physicists up to now, to clarify their results, and to furnish some clue to future analysis.

** Matter, here is no philosophical concept. That is, it does not imply materialism, but is a common term among physicists, meaning particles which act as sources of fields, for example electrons, protons, neutrons, charged mesons and so on.
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hindrance in explaining theoretically all the experimental facts we obtain concerning an electron, and they had significance only as a problem in themselves, independent of experimental facts. The only experimental fact in this connection is the mass $m$ of an electron.

But there have been much meandering concerning this question. That is to say, until mesons were discovered within cosmic-rays, the large penetrating power of the latter believed the fact that, according to Dirac's relativistic equation for an electron, the energy loss should be great when high-energy electrons penetrate through matter. Also, the experimental fact that there was a phenomenon in cosmic-rays called shower, in which many particles are produced explosively, it seemed, in a single process,* could not be explained. Therefore, these difficulties came to be ascribed to the same cause as the above-mentioned difficulties in electron theory and quantum electrodynamics. To these were further associated the difficulty of the nuclear electron, which was considered in relation with $\beta$-decay. However, when the meson was discovered and the large penetrating power of cosmic-rays attributed to it, and the theory of cascade shower was established at about the same time (1937) doing away with the necessity of considering showers as single processes explosively producing many negative and positive electrons, thus making Dirac's equation acceptable even for extremely high energies, once more did the fundamental difficulty of electron theory and quantum electrodynamics come to lose its realistic foothold.

But as meson theory made further progress, the quantum electrodynamical difficulty made its second appearance—in a new light: the field theory was developed in a manner similar to electron theory, and, evidently in compliance with the correspondence principle, the meson field was quantized in the same way as in quantum electrodynamics. Thus, it was natural that the same difficulty presented itself here as in the quantum electrodynamical case.

In this case, however, the difficulty turned out to be more realistic and abundant in experimental evidence—as I frequently emphasized at the time we** were performing the quantization of the meson field (1937–8). One of the experimental evidences is the anomalous magnetic moment of

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* It was later ascertained experimentally that this is not a single process.
** Drs. Yukawa, Sakata and I.
heavy particles (protons and neutrons), which is obtained, in accordance with perturbation theory, by assuming intermediate states and considering mesons to dissociate around heavy particles and interact with the magnetic field. This involves a calculation similar to that for self-energies, so that the result diverged just as the self-energies did. Thus, unlike in the case of the electron, the difficulty in the case of mesons had a close connection with experimental fact. It was for this reason that I stated in those days: "This time for certain, it will be impossible to develop meson theory leaving this difficulty as it is."

To cope with this difficulty, the method of "cut-off", in which the momentum integration of the intermediate state is cut off at a certain value in order to prevent its diverging, has been employed (It is the same as inserting a suitable converging factor).

Later on as the meson theory made further development, it was found that when vector and pseudoscalar mesons were scattered by heavy particles, and also when charged vector mesons interacted with an electromagnetic field, the scattering cross-section became larger as the energy increased, which contradicts experiment. Further, while the interaction due to the Coulomb field about an electron is \(-\frac{e^2}{r}\) (where \(e\) is the charge and \(r\) the distance), an interaction term \(-g^2\frac{\alpha^2}{r}\) (where \(g\) is the constant of interaction between a meson and a heavy particle, and \(\alpha = \frac{m_e c}{\hbar}\) \([m_e: \text{meson mass, } c: \text{light velocity, } \hbar: \text{planck's constant } \div 2\pi]\)) presents itself in the meson field surrounding a heavy particle, and such a singularity brings about a difficulty in the solution of the equation of motion. Hence the method of "cut-off", in which the interaction is cut off for momentam above a certain value, becomes necessary in various cases. And since momentum implies wave length, the cutting off at a certain momentum value involves the necessity of introducing some sort of an universal length, as Heisenberg emphasize.

Later, the problem developed in a rather different direction, and assumed a more concrete form. In the case of strong interactions like that of the meson field, it is natural that the applicability of perturbation theory be doubted; and even if perturbation theory can be used, it is naturally to be expected that a damping term must be introduced, as in the case of deducing a dispersion formula. And I have thought that some sort of a dispersion
formula would be given by the correct theory. A similar line of thought was taken by Landau.\(^\text{(4)}\)

In this way, various trials were made to depart from the perturbation method of radiation fields. For instance, a classical solution of the equations was carried out, and it was made clear that certain important parts of the difficulty had their origin in an inappropriate use of perturbation theory.

This was performed by Heisenberg,\(^\text{(5)}\) Bhabha,\(^\text{(6)}\) Wentzel,\(^\text{(7)}\) and Tomonaga.\(^\text{(8)}\)

Also it was shown by Heither\(^\text{(9)}\) and Wilson\(^\text{(10)}\) that the divergence could be removed in the case of high energy scattering if radiation damping was taken into account in perturbation calculations of radiation theory.

Thus it was found that a part of the above-mentioned difficulty arose because radiation damping had not been taken into account, and the state of affairs assumed a rather different aspect. Many previous calculations now became meaningless. Perturbation theory could no longer be used without due precaution. For instance the method of Christy and Kusaka\(^\text{(11)}\) which attempts to determine the meson spin from the interaction between mesons and an electro-magnetic field would give according to Wilson,\(^\text{(10)}\) a value too much definite to be anticipated.

In the perturbation theory of radiation, the difficulty of field theory does not present itself in the first term. It is well known that this occurs in the second term. The afore-mentioned difficulty which was removed by considering radiation damping is one relating to this first term, and the second term which involves self-energies still remains divergent. Thus the introduction of radiation damping seemed to have brought about success in cornering the difficulty to a certain extent. But in meson theory, the interaction is strong, and much doubt is to be entertained in a perturbation method where phenomena involving a single meson alone come into the first term. So a solution which lays more stress upon phenomena involving many mesons at once comes to be sought for. This was what Heisenberg previously considered in connection with his universal length relating to the theory of bursts, and a brilliant solution was achieved by Tomonaga.\(^\text{(11)}\)

As frequently extorted by Tomonaga, it is not sufficient merely to introduce damping in the calculations, as done by Heither and Wilson. When the perturbation method becomes unavailable, this second and diverging term must, of necessity, be touched. Tomonaga used Fock's method and solved
the equation for a meson in configuration space. Making calculations by this method for the case of mesons scattered by heavy particles, it was found that self-energy played an important role in the case of scattering too. This takes the form, as anticipated, of a dispersion formula. *

In this formula, a diverging self-energy enters as resonance energy. Also the radiation damping enters as the width of resonance. This self-energy must be taken into consideration, as it becomes infinitely large. It is impossible in this calculation, to apply the subtraction method as in Dirac's theory. On the other hand, if it is made finite by the method of "cut-off", it acts as the energy deviation in the scattering process, and as the inertia of the proper field advocated by Heisenberg. This is directly connected with the scattering cross-section for low-energy mesons, and improves the agreement with experiment. In this way, divergency problems related to self-energy called for solution, coming to the foreground en bloc in an internally rearranged form. That is to say, it become a theme in hand to solve this problem of "field and matter".

Various standpoints have hitherto been taken concerning the problem of matter and field. The usual method adopted in quantum mechanics is to treat matter and field equivalently, consider their respective equations of motion, and introduce a term expressing their interaction. This has been done by Heisenberg and Pauli, and leads to the well-known divergence of self-energy. In this standpoint the interaction between two fields is considered, and there is no differentiating which is matter and which is field. Only when one associates the rest-mass of one of the fields with its self-energy, does one regard that field as matter, and the other, that causes self-energy from the interaction with the former, is regarded as field. This involves the divergence of self-energy, to cope with which a cutting-off or the artificial convergence factor is introduced, as done by Watagin. 

* The scattering cross-section for charged vector mesons is

\[ Q = \pi \frac{4(2\pi)^4 \hbar^8}{k_0^3} \left( K_0 - \frac{\hbar^2}{L^2} \right) + (2\pi)^4 \hbar^8 = \frac{\pi}{L^2} \frac{4(2\pi)^4 \hbar^8}{(K_0 - K(0))^2 + (T/2)^2} \]

where \( K \) is the momentum \( (k, h, h) \), \( k_0 \) the momentum of the incident meson, \( \varepsilon = mc^2/\hbar \)

\[ L = \tan \frac{\varepsilon}{2\pi} \frac{1}{\sqrt{2\varepsilon - \varepsilon^2}} \]

This equation is a dispersion formula with resonance energy \( K(0) = \rho \int \frac{\hbar^2}{K(K_0 - K)} \) and \( T/2 = (2\pi)^4 \hbar^8 \) its amplitude.
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and Scherzer\(^{(10)}\) in early stages. In either case the result is that something of the dimension of length is introduced in the interaction term, which leads us to anticipate a contact with the question of the structure of elementary particles. Of course it is evident that this length is not of such a purely substantialistic nature as Lorentz' electron radius, but it is also certain that it touches, in some sense or other, the question of structure.

Now, as the logical structure of quantum mechanics deals persistently with the existence probability of matter regarded as a point, the logical structure of observation is so built up that it is impossible to take up separately into question the "left or right half" of matter regarded as having some sort of magnitude or other. In other words, quantization leads to the existence probability of particles regarded as geometrical points. Unless this framework is altered, it would be impossible to deal with the structure of elementary particles. Of course "structure" in this case is not merely a spatial, but a spatio-temporal structure, and is intimately connected with the laws of relativity. The above mentioned standpoint of the interaction of two fields cannot deal with such a spatio-temporal structure. In particular the magnitude of spin (angular momentum of rotation) has a quantum number \(\frac{1}{2}\) in objects regarded mainly as matter.

This value \(\frac{1}{2}\) follows as the natural consequence of the group theory of spatial rotations, and though the operator representing angular momentum is incommutable with space co-ordinates when its quantum number is 1, it is commutable with space-time co-ordinates in this case. Although spin does not, of course, signify any directly perceptible rotation of a body with extension, the spin \(\frac{1}{2}\) is, in this sense, to be associated only with the existence probability of an elementary particle, and not with its spatio-temporal structure.

Markov,\(^{(16)}\) in the theory of field quantization, postulated characteristic commutation-relations between the four-dimensional co-ordinates of the test-body employed to measure the field strength and the field quantities, introducing in them a length corresponding to the radius of the test-body, with some success over the divergence difficulty. Markov's theory shows that the mutual interaction type of treatment current in quantum mechanics is inapplicable to observations concerning the structure of elementary particles unless due modifications be made; and also that when such observations are in question, the space-time co-ordinates no longer merely specify the
point of existence of matter, but possess organic inter-relations with field quantities, suggesting that the observation should be associated with the structure of elementary particles. But his theory does not penetrate into the question of spin, and also the mode of observation is not radically re-established but simply inherited from quantum mechanics.

Applying the method of Dirac, Fork and Podolsky's \cite{NP1949} "many-time theory", Wentzel\cite{Wen51} has obtained the difference between retarded and advanced fields by causing the test-body to approach the particle from a time-like direction, thus attaining some success in preventing the self-energy divergence. We learn, from papers which came to hand after the termination of the war, that Dirac has adopted this method in the form of the \( \lambda \)-limiting process, which is further developed by Pauli.\cite{Pau51} Using this, and at the same time introducing negative probability and photons with negative energy, Pauli has succeeded, at least formally, in removing all the divergence difficulties.

This is quite interesting from the consequential viewpoint, and especially noteworthy is the fact that certain divergences, which cannot be removed by either the \( \lambda \)-limiting process or the introduction of negative probability and energy alone, vanish on a simultaneous application of both these methods. However, there is a possibility that this might considerably affect hitherto satisfactory results too; and also, the comparison with experiment of the results cleared of divergencies is still incomplete. So nothing can be said, as yet, even of the results themselves. Furthermore, the structure of the theory is not logically consistent but extremely artificial and ambiguous, and contains many paradoxes, so that it must be glossed by introducing diverse conditions. It is evidently necessary to make further quests into the inconsistencies of this theory.

A radically new mode of measurement cannot be obtained by dealing with the existence probabilities of point-particles. It is the complete measurement of field quantities within a certain space-time domain that must be dealt with. This is connected with the many-time theory, originating in Dirac\cite{Dir38} and discussed by Yukawa,\cite{Yuk46} Tomonaga,\cite{Tom47} Watanabe\cite{Wat50} and Tanikawa,\cite{Tani51} and verges upon an attempt to re-establish the problem of measurement from a purely relativistic standpoint. But these trials stand little hope of further development as they possess no substantial content whatever.

Next comes the monistic field theory, which treats both field and matter
with a single field equation, considering the singular points of the field as matter, and regarding matter as being perfectly derivable from the field equation. This was first done by Mie, followed by Born and Infeld, and then by Bopp. These must be said to be of perfectly substantialistic character. They are logically consistent, but too much emphasis is laid upon the divergence difficulties, and though these may be solved, the theories are too far removed from other experimental facts and give an impression of awkwardness. It might be said in their defence that this is because they penetrate further into the intrinsic nature of things than do other theories. But there remains the next decisive contradiction that in monistic field theory the only field capable of admittance is a Bose field, and therefore no possible combination of this type of field avails in creating matter with spin $\frac{1}{2}$ and obeying Fermi statistics. Herein lies the most essential difficulty of monistic field theory. To conquer this, a considerably different method of attack would be required. Another difficulty is that an equation of motion for the singular points expressing matter does not always follow as a matter of course. Also, for instance, if some special significance is attached to the charge current density in Bopp's theory, it would mean making a great departure from the standpoint of monistic field theory. This difficulty is just the opposite of that arising in the quantum mechanical treatment by considering mutual interaction. In that case spins of the value $\frac{1}{2}$ and Fermi statistics could be introduced without any difficulty, but the structure of elementary particles did not come in, whereas in the case of monistic field theory, the structure of elementary particles is first introduced, but spin $\frac{1}{2}$ and Fermi statistics are shut out. It is here that we can perceive the fundamental inconsistency of concepts.

Another problem confronting monistic field theory is the way in which to treat matter interacting with many kinds of fields. Concerning this point studies are being made by the Nagoya University group under the direction of Dr. S. Sakata, and an interesting solution is already within sight in a different direction from monistic field theory; viz. the mixed field theory.

Nextly, quantization is a great difficulty in the case of monistic field theory. It is found that the ordinary procedure of quantization is alien and quite unfitting to this standpoint. To carry out quantization, it is believed that there is no alternative but to proceed from the above-mentioned standpoint of complete measurements of field quantities within a certain space-
The peculiar method of quantization employed in Born's first paper seems to be suggestive of pointing to the correct direction, though of course it is attended by serious mathematical difficulties.

To recapitulate, monistic field theory must solve the problems of: spin and statistics; equation of motion for the singular points; finding an appropriate method of quantization.

We believe that it can be said from the above dialectic logical analysis that the conflict between the two fundamental concepts, matter and field is the contradiction in hand, which points out to us the direction in which we should proceed.

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References

1. Dirac: Proc. Roy. Soc., London (A) 167 (1938), 148, where the problem of a point-electron is solved classically, and the divergence separated. In connection with this singularity, too, a cohesive force has been considered. (Pryce: ibid. 168 (1938), 389).

2. Thursday-lecture at the Department of Physics, Osaka Imperial University.


18. Wentzel: Zeit. f. Phys. 8; (1933), 479, 635; 87 (1934), 726

19. Pauli: Rev. of Mod. Phys. 15 (1943), 175. My sincere thanks are due to Drs. Tomonaga and Miyatima for discussing this paper with me.


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(29) Born: (26).