Possible Experimental Tests on the Decay Interactions of Hyperons

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We examine what information could be obtained by the measurement of the characteristic quantities of the \((N+\pi)\)-decays of \(\Sigma\) and \(\Lambda\)-hyperons. In particular, the possibility of experimental tests for the \(|\Delta I|=1/2\) rule and the postulate of time-reversal invariance is discussed. We also discuss that the relative magnitudes of the effective strengths of the \(|\Delta I|=1/2\) and other (if they exist) interactions could be known under suitable assumptions on the dynamical nature of the interactions. Some physically interesting examples of such assumptions are presented and it is shown that the validity of those assumptions can be tested experimentally. For obtaining such considerably definite information, it is very much desired to measure, at least, the ratio of the asymmetry parameters for various modes of decays.

§ 1. Introduction

Stimulated by the success of the Nishijima and Gell-Mann scheme, many authors have investigated the characteristic isotopic spin changes in the decay processes of the strange particles, and pointed out the importance of the selection rule \(|\Delta I|=1/2\). In fact, the experimental value of the branching ratio between the two modes of the \(\Lambda^0\)-decay is consistent with the prediction of the \(|\Delta I|=1/2\) rule. Moreover, the predictions of this rule are not inconsistent also with the data for the branching ratio of the \(\Sigma^+\)-decay and the ratio of the lifetimes of \(\Sigma^-\) and \(\Sigma^0\), provided that the parity does not conserve in these decays. The validity of the latter assumption is evident nowadays. However, it should be noted that these facts do not necessarily mean the non-existence of the other types of the decay interactions (e.g. \(|\Delta I|=3/2\)). For example, even under the postulate of time-reversal invariance (\(T\)-invariance), the theory basing on the \(|\Delta I|=1/2\) rule involves three adjustable parameters for the case of \(\Sigma\)-decay (see later). Therefore, for the more definite test on this rule, it is necessary also to use the data for the other quantities. The situation is similar also for the case of \(\Lambda^0\)-decay. We shall mention in § 3. B on the possible method for such a detailed test on the \(|\Delta I|=1/2\) theory.

Although the theoretical investigations which have thus far been carried out on the hyperon decays are almost confined within the test for the \(|\Delta I|=1/2\) rule, we cannot be nowadays satisfied by only such a "test", itself. The reason is as follows: For the decays of \(K\)-mesons, it has already been evident that the selection rule \(|\Delta I|=1/2\) is never absolute, and the contribution of the interactions other than the one of the type \(|\Delta I|=1/2\) may amount to about 10\% (in the decay amplitude) of the latter. This fact suggests the possibility that the situation might be similar also in the hyperon decays.
Therefore, what we want to know is not only whether the \( |d| = 1/2 \) rule is consistent with the experimental data, but also how much contribution is given by each of the \( |d| = 1/2 \) and the other interactions (if they exist). Also, it is desired to obtain the quantitative information for other properties of the decay interactions, e.g., the ratio of the contributions from the parity-conserving and -reversing interactions.

One way to clarify the structure of the particles and their interactions is the "dynamical" method, with which one starts from the suitable models for the dynamical nature of the interactions and compares the computational results with the experimental data. For example, the attempt to extrapolate the "one-to-one" law\(^{13,14}\) also for the case of the hyperon decay belongs to this method. However, here we want to emphasize also the importance of the kinematical attack. The latter needs only a few assumptions that are strongly confirmed by many experiences, and therefore, the conclusions deduced by this method are highly reliable. For example, it does not generally need the perturbational treatment of the effects of the strong interactions, in contrast with the "dynamical" method. Therefore, it would be very important to investigate the various possibilities involved in the kinematical analyses of the decay processes, and, if possible, to test the dynamical models thus far proposed for the nature of the interactions. This would also be helpful for the construction of a more reliable model. Of course, the kinematical method can generally give only limited information. However, at the present stage of the research on the strange particle decays, it may be said that even those limited possibilities are not yet investigated enough.

From such a viewpoint, we shall investigate in the present paper what information could be obtained by the kinematical analysis of the \((N + \pi)\) decays of hyperons, and what data are desired for obtaining such information. We shall also show the possibility of the experimental test for some dynamical models or assumptions thus far proposed on the nature of the interactions. Throughout the present paper, we postulate the following three assumptions:

1) Conservation of the total angular momentum in all reactions.
2) Isotopic spin assignments of the Nishijima and Gell-Mann scheme.
3) Conservation of parity and charge-independence in the strong reactions.

In addition to these, taking into account many experimental information, we treat here \(\Lambda\) and \(\Sigma\) as the spin 1/2 particles.\(^{15}\) However, the discussions in the present paper can be easily extended also to the cases of the spins of the hyperons not to be 1/2.

§ 2. Kinematics of the decays of charged \(\Sigma\)

Mainly for the illustration of the meanings of the notations, we first begin with a brief summary of the kinematics of the decays of the charged \(\Sigma\)-particles.

Observed decays of the \(\Sigma\)-particles into \((N + \pi)\) systems are

\* This terminology is used here to mean the assumption that the form of the ordinary space part of the decay interaction Hamiltonian is (symbolically) of the type \((1 \pm \gamma_\delta) \cdot Q_i\). See also § 3. D.
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$\begin{align*}
\Sigma^+ &\rightarrow p+\pi^0 \quad (0) \\
\Sigma^+ &\rightarrow n+\pi^+ \quad (+) \\
\Sigma^- &\rightarrow n+\pi^- \quad (-)
\end{align*}$

(2.1)

In the following, we call these three modes $0$, $+$, and $-$, respectively, and distinguish the quantities related to each of these modes by the suffix $i$ $(i=0, +, -)$. The decay matrix element for each mode of (2.1) is given by

$$M_i = A_i + B_i \quad (i=0, +, -),$$

(2.2)

where $A$ and $B$ correspond to the two possible orbital angular momentum states $l=0$ ($\Sigma$) and $l=1$ ($P$), respectively, of the final $(N+\pi)$ system.

Since the isotopic spin of $\Sigma$-particle is 1, the only decay interactions that can contribute to the processes (2.1) are of the types $|\Delta I|=1/2$, $3/2$, and $5/2$. The interaction of the type $|\Delta I|=1/2$, for example, means that the interaction Hamiltonian representing it transforms like one component of a spinor under the rotation in the isotopic spin space. With the use of the Clebsch-Gordan coefficients, $A's$ of (2.2) can be written as the sums of the contributions from the $|\Delta I|=1/2$, $3/2$, and $5/2$ interactions:

$$A_i = \frac{1}{3} (\sqrt{5} \left( \sqrt{10} a_i - 4b_i + \sqrt{6} c_i \right) - (1/3 \sqrt{2}) (2a_i + b_i),$$

where $a$, $b$, and $c$ represent the parts caused by the $|\Delta I|=1/2$, $3/2$, and $5/2$ interactions, respectively, and the suffices 1 and 3 indicate the isotopic spin state of the final $(N+\pi)$ system.

In the processes (2.1), the $|\Delta I|=5/2$ interaction can give rise to the final $I=3/2$ state only, and thus $c_i$ and $c_i'$ are absent in the expressions (2.3) and (2.4).

By the unitarity of the $S$-matrix, the following relations hold between the phases of the “amplitudes” and the $\pi-N$ scattering phase shifts $\delta's$ at the $\Sigma$-decay energy:

$$a_i = \epsilon_i^{(\pi)} |a_i| \cdot \exp i (\delta_i + \eta_i),$$

$$b_i = \epsilon_i^{(\pi)} |b_i| \cdot \exp i (\delta_i + \eta_i),$$

$$c_i = \epsilon_i^{(\pi)} |c_i| \cdot \exp i (\delta_i + \eta_i),$$

$$a_i' = \epsilon_i^{(\pi)} |a_i'| \cdot \exp i (\delta_i + \eta_i)$$

where $\epsilon_i^{(\pi)}$ is the unitarity of the $S$-matrix, the following relations hold between the phases of the $|\Delta I|=0$, $+$, and $-$ amplitudes.
\[
\begin{align*}
\{ b'_j &= \mathcal{E}^{(IV)} \mid b'_j \mid \exp(i(\delta_m + \eta_m)) \\
\{ c'_j &= \mathcal{E}^{(III)} \mid c'_j \mid \exp(i(\delta_m + \eta_m)), \quad (j = 1, 3)
\end{align*}
\] (2.5)

where \(\mathcal{E}'s\) are the undetermined sign factors, and the upper suffices (1, 3, 5) of \(\mathcal{E}'s\) and the second suffices (1, 3, 5) of \(\eta\)'s correspond to the \(|\Delta I| = 1/2, 3/2, \text{ and } 5/2\) interactions, respectively. Not \(\eta\)'s themselves, but the differences between them are physically significant, which indicate the degree of violation of the time-reversal invariance in the decay processes in question. If the processes are \(T\)-invariant, then all the differences of \(\eta\)'s vanish.

Now, let us pay notice to the following characteristic quantities of the decay processes:

1) The branching ratio \(\xi\) between the two modes \((i = 0 \text{ and } +)\) of \(\Sigma^+\)-decay, and the ratio \(\zeta\) of the lifetimes of \(\Sigma^-\) and \(\Sigma^+\),

\[
\begin{align*}
\xi &= w(\Sigma^+ \rightarrow p + \pi^+)/w(\Sigma^+ \rightarrow n + \pi^+) = (\mid A_0 \mid^2 + \mid B_0 \mid^2) / (\mid A_+ \mid^2 + \mid B_+ \mid^2), \\
\zeta &= \tau(\Sigma^-) / \tau(\Sigma^+) = (\mid A_0 \mid^2 + \mid B_0 \mid^2 + \mid A_+ \mid^2 + \mid B_+ \mid^2) / (\mid A_- \mid^2 + \mid B_- \mid^2).
\end{align*}
\] (2.6)

2) The quantities \(\alpha, \beta\) and \(\gamma\) (or \(\alpha'\) and \(\phi\)), which are defined for each mode by

\[
\begin{align*}
\alpha &= 2 \text{Re} (A^* B) / (\mid A_+ \mid^2 + \mid B_+ \mid^2), \\
\beta &= 2 \text{Im} (A^* B) / (\mid A_+ \mid^2 + \mid B_+ \mid^2) \equiv (1 - \alpha^2)^{1/2} \cos \phi, \\
\gamma &= (\mid A_+ \mid^2 - \mid B_+ \mid^2) / (\mid A_0 \mid^2 + \mid B_0 \mid^2) \equiv (1 - \alpha^2)^{1/2} \sin \phi,
\end{align*}
\] (2.7)

\[(\alpha^2 + \beta^2 + \gamma^2 = 1).\]

The quantities (2.7) are the ones introduced by Lee and Yang,\(^{10}\) and especially \(\alpha\)'s are the so-called asymmetry parameters.

Taking into account (2.3) \sim (2.5), all of the quantities defined by (2.6) and (2.7) can be easily expressed in terms of \(\alpha\)'s, etc. We shall first give those expressions for a simple case of the \(|\Delta I| = 1/2\) interaction only (Subsection \(A\)), and next consider more general cases.

\(A.\) The case of only the \(|\Delta I| = 1/2\) interaction

We first introduce the following parameters:\ *

\[
\begin{align*}
x_1 &= \mathcal{E}^{(IV)} \mathcal{E}^{(IV)} [\mid a_1 \mid^2 + \mid a'_1 \mid^2] / (\mid a_0 \mid^2 + \mid a'_0 \mid^2)], \\
\tan \theta_1 &= \mathcal{E}^{(IV)} \mathcal{E}^{(IV)} \mid a_1 \mid / \mid a'_1 \mid, \quad \tan \phi_1 = \mathcal{E}^{(IV)} \mathcal{E}^{(IV)} \mid a_0 \mid / \mid a'_0 \mid, \\
\eta_1 &= \eta_m = \eta_n.
\end{align*}
\] (2.8)

\(x_1\) gives a measure for the ratio of the effective "amplitudes" of the final \(I = 1/2\) and \(3/2\) states caused by the \(|\Delta I| = 1/2\) interaction. Similarly, \(\theta_1\) and \(\phi_1\) give that of the final \(S\)- and \(P\)-states.

In terms of these four parameters, the quantities defined by (2.6) and (2.7) are expressed as follows:

\[* These are quite similar to the parameters introduced by Kawaguchi.\(^{39}\) \(\theta_1\) and \(\phi_1\) are equal to his \(\theta\) and \(\phi\), respectively. However, \(x_1\) is \((1/\sqrt{2})\) times of his \(x\), and \(\eta_1\) is absent in ref. 3.\]
Similarly, in the case of co-existence of the interactions which give rise to the final to the phase difference \( y \) in addition to the eight parameters defined by (2.8) and (2.11). Roughly speaking, because of the absence of 1/2 and 3/2, we need two more parameters defined by (2.5) and (2.7) can be expressed in terms of only four adjustable parameters corresponding to (2.8), i.e.

\[
\begin{align*}
A & = - \sin \theta_1 \sin \Phi_1 \cos (\delta_1 - \delta_3) \pm \cos \theta_1 \cos \Phi_1 \cos (\delta_{11} - \delta_{31}) \\
B & = \sin \theta_1 \cos \Phi_1 \cos (\delta_{11} - \delta_1 + \gamma_1) \\
C & = \sin \theta_1 \cos \Phi_1 \cos (\delta_{31} - \delta_1 + \gamma_1) + \cos \theta_1 \sin \Phi_1 \cos (\delta_{11} - \delta_3 + \gamma_3) \\
D & = \sin \Phi_1 \cos \Phi_1 \cos (\delta_{31} - \delta_3 + \gamma_3). 
\end{align*}
\( (2.10) \)

The expressions for \( \beta \)'s are obtained from those of \( \alpha \)'s only by replacing the cos-functions of \( \beta \)'s and \( \gamma \)'s by the corresponding sin-functions. What is important here is that all of these expressions involve only four adjustable parameters defined by (2.8).

B. General case including all the types of interactions

We note first that, also in the case of the \(|\mathcal{M}| = 3/2 \) interaction only, the quantities (2.6) and (2.7) can be expressed in terms of only four adjustable parameters corresponding to (2.8), i.e.

\[
\begin{align*}
x_0 & = \varepsilon_3^{(3)} \varepsilon_3^{(3)} [ (|b_1|^2 + |b_1'|^2) / (|b_3|^2 + |b_3'|^2) ]^{1/2}, \\
\tan \theta_3 & = \varepsilon_3^{(3)} \varepsilon_3^{(3)} |b_1| / |b_1'|, \quad \tan \Phi_3 = \varepsilon_3^{(3)} \varepsilon_3^{(3)} |b_3| / |b_3'|, \\
\gamma_3 & = \gamma_{a3} - \gamma_{a5}. 
\end{align*}
\( (2.11) \)

In the case of the \(|\mathcal{M}| = 5/2 \) interaction only, we need only two parameters \( \Phi_3 \) and \( \gamma_3 \), because of the absence of \( c_1 \) and \( c_1' \) in our formulas:

\[
\begin{align*}
\tan \Phi_3 & = \varepsilon_3^{(3)} \varepsilon_3^{(3)} |c_1| / |c_1'|, \quad \gamma_3 = \gamma_{a3} - \gamma_{a5}. 
\end{align*}
\( (2.12) \)

Finally, when there coexist the two types of the interactions, for example, \(|\mathcal{M}| = 1/2 \) and \( 3/2 \), we need two more parameters defined by

\[
\begin{align*}
\gamma & = \varepsilon_3^{(3)} \varepsilon_3^{(3)} [ (|b_3|^2 + |b_3'|^2) / (|a_3|^2 + |a_3'|^2) ]^{1/2}, \quad \gamma_{a3} = \gamma_{a3} - \gamma_{a3}, 
\end{align*}
\( (2.13) \)

in addition to the eight parameters defined by (2.8) and (2.11). Roughly speaking, \( \gamma \) represents the ratio of the effective strengths of the parts of the \(|\mathcal{M}| = 1/2 \) and \( 3/2 \) interactions which give rise to the final \( I = 3/2 \) state, and \( \gamma_{a3} \) is the parameter relating to the phase difference between the \( S \)-state amplitudes caused by these two interactions. Similarly, in the case of co-existence of the \(|\mathcal{M}| = 1/2 \) and \( 5/2 \) interactions, we need
two more parameters $\zeta$ and $\gamma_{31}$, in addition to the six parameters defined by (2·8) and (2·12):

$$\zeta = E^{(n)}_3 E^{(m)}_8 \left[ (|c_8|^2 + |c_9|^2) / (|d_8|^2 + |d_9|^2) \right]^{1/2}, \quad \gamma_{31} = \gamma_{35} - \gamma_{41}. \quad (2·14)$$

Thus, in the most general case including all of the above three types of interactions, we need altogether 14 independently adjustable parameters:

$$x_1, \theta_1, \phi_1, \gamma_1; x_3, \theta_3, \phi_3, \gamma_3; \theta_5, \gamma_5; \gamma, \zeta, \gamma_{31}, \text{ and } \gamma_{51}.$$ 

In any case, the expressions for $\xi$, $\zeta$, etc., in terms of the relevant adjustable parameters can be obtained by straightforward (but rather cumbersome) calculations. However, since those expressions are generally very complicated, we shall not here give them explicitly. The details of those expressions for the most general case (with and without the postulate of $T$-invariance) will be given in Appendix.

§ 3. Possible experimental information on the $\Sigma$-decay interactions

A. General considerations

Among the quantities defined by (2·6) and (2·7), the number of the independently variable ones is only eight. For example, we can take

$$\begin{cases} \alpha \text{ and } \phi \text{ for each of three modes } (i=0, +, -), \\ \xi \text{ and } \zeta \end{cases} \quad (3·1)$$

as those independent quantities. This number, 8, indicates the upper limit of the possible experimental knowledge on the structure of the $\Sigma$-decay interaction. This is because that the physical properties of each mode $(i=0, +, -)$ of the decay processes are completely determined by the totally three quantities, i.e. the magnitudes of the two parts $A_i$ and $B_i$ of the decay matrix element $M_{is}$, and their relative phase. Among these totally 9 quantities, the absolute value of $|A_i|^2 + |B_i|^2$ is arbitrarily adjustable within the numerical factor common to all of three modes by a suitable choice of the average strength of coupling. Thus, not the absolute values of $A$'s and $B$'s, but their relative magnitudes can serve for our purpose.

The above number, 8, is much smaller than the number, 14, of the independent parameters in the most general theory.* Therefore, even if all of the eight quantities (3·1) were measured with sufficient accuracy, it is impossible to determine the values of all these parameters and obtain the definite information on the details of the $\Sigma$-decay interactions. To obtain some information on these interactions from the experimental data, it is necessary to reduce the number of the independent parameters by the suitable assumptions or models on the nature of the interactions. For example, the postulate of

* This is mainly owing to that the possible $(N+\pi)$-decay of the $\Sigma^0$-particle can be hardly observed by the occurrence of the faster reaction, $\Sigma^0 \to A_0 + \gamma$. If the decay $\Sigma^0 \to N+\pi$ were observable, then we had possessed six more data, i.e. $\tau(\Sigma^-)/\tau(\Sigma^0)$, the branching ratio between the two modes $\Sigma^0 \to p+\pi^-$ and $\Sigma^0 \to n+\pi^0$, and the quantities $\alpha$ and $\phi$ for each of these two modes.
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T-invariance imposes that all $\eta$'s should vanish. Thus, there still remain the possibilities to determine the values of the parameters under such suitable assumptions, and in some cases, even to test those underlying assumptions by examining the consistency with the experimental data. There would exist many varieties of such attempts. In the following subsections, however, we shall consider only several examples for such possibilities that seem to be particularly interesting.

Before entering into the discussions of those examples, let us consider here which of the quantities (2·6) and (2·7) seem to be practically measurable. We have already known the following data\(^9,17\) :

$$\xi = 0.88 \pm 0.12, \quad \zeta = 2.2 \pm 0.5, \quad (3·2)$$

and the "up-down" asymmetries,

$$P^+ \cdot \alpha_0 = -0.37 \pm 0.19, \quad P^+ \cdot \alpha_+ = -0.36 \pm 0.21, \quad P^- \cdot \alpha_- = -0.13 \pm 0.26, \quad (3·3)$$

where $P^+$ and $P^-$ are the average polarization degrees of the parent $\Sigma^+$ and $\Sigma^-$, respectively. Since the signs and the magnitudes of $P'$s are unknown, the data which are available at present are only those for

$$\xi, \zeta, \text{ and } \alpha_0/\alpha_+.$$  

(3·4)

However, some of the other quantities are also measurable in principle. For example, the ratio $\alpha_0 : \alpha_+ : \alpha_-$ would be obtained by the measurement of the ratio $P \cdot \alpha_0 : P \cdot \alpha_+ : P \cdot \alpha_-$ of the "up-down" asymmetries in the decays of $\Sigma^{\pm}$ which are produced by the reactions\(^{17}\)

$$\pi^+ + p \rightarrow \Sigma^+ + K^+ \quad \text{and} \quad \pi^- + n \rightarrow \Sigma^- + K^0, \quad (3·5)$$

because these $\Sigma^{\pm}$ have the same degrees of polarization, provided that they are produced under the same experimental conditions (energy of the incident pion beam and the production angle). (Note that we have assumed the charge-independence of the strong reactions.) Moreover, Lee and Yang\(^{16}\) have recently suggested the method for directly obtaining the values of $\alpha$ and $\phi$.* This is, in principle, not impossible at least for the

* The value of $\alpha$ can be obtained by the measurement of the polarization (longitudinal) of the decay nucleon from the unpolarized $\Sigma$. The values of $\beta$ and $\gamma$, or $\phi$, are to be known from the transverse polarization of the decay nucleon from the completely polarized hyperon. However, the value of $\phi$ can be known (in principle) also in the practical case where the parent hyperon is partially polarized. In such a case, the polarization vector of the decay nucleon is given by

$$(1 + P \cdot \alpha \cos \chi)^{-1} \left[ (\alpha - P \cos \chi) p + P \beta \rho \times s + p (p \times s) \times p \right],$$

instead of the formula given in ref. 16). Here, $s$ and $p$ are the unit vectors along the directions of the hyperon polarization and the nucleon emission, respectively. $P$ is the polarization degree of the parent hyperon and $\chi$ is the angle between the directions of the pion emission and $s$ ($\cos \chi = -p \cdot s$). In addition to this, note that $P$ can be known from the values of $\alpha$ and the "up-down" asymmetry $P \cdot \alpha$. The value of $P$ would be useful for the analysis of the production process of the hyperon. The direct measurement of $\alpha$ is thus very much desired, since it serves for the analysis not only of the decay but also of the production processes.
case of the decay nucleon being a proton (mode \( i = 0 \)). If \( \alpha_0 \) is known, then also \( \alpha_+ \) and \( \alpha_- \) could be known through the knowledge on the ratio \( \alpha_0 : \alpha_+ / \alpha_- \) obtained by the above-mentioned method. After all, we can conclude as follows: Under the favorable circumstances, it would be possible to measure the six quantities,

\[
\hat{\xi}, \zeta, \alpha_0, \alpha_+, \alpha_- \text{, and } \phi_0.
\]

(3.6)

Even if we take into account that the direct measurement of \( \alpha \) is considerably difficult owing to the very low energy of the decay nucleon, it would not be difficult to measure the ratios \( \alpha_0 / \alpha_+ \) and \( \alpha_0 / \alpha_- \). Thus, the data at least for the four quantities,

\[
\hat{\xi}, \zeta, \alpha_0 / \alpha_+, \text{ and } \alpha_0 / \alpha_-
\]

(3.7)

would be expected to become available enough.

B. The \( |D| = 1/2 \) theory

We shall first investigate the selection rule \( |D| = 1/2 \), as an example of the possibilities mentioned in \( A \). As has already been pointed out, a rather large possibility is expected that the \( |D| = 1/2 \) rule might be violated to some extent also in the case of the hyperon decay. However, on the other hand, another possibility has not yet been excluded that this rule is absolutely true for the case of the hyperon decay, and this might really be the characteristic difference between the decays of the hyperons and of the K-mesons. Whichever it may be, the important fact is that the evidence for the violation of the \( |D| = 1/2 \) rule in the hyperon decay has not yet been found.\(^{39-41}\) Therefore, among all, the detailed test on this rule would be the first problem which we must investigate.

The test on the \( T \)-invariant \( |D| = 1/2 \) theory seems to be relatively simple, since this theory contains only three adjustable parameters \( x_1, \theta_1 \), and \( \phi_1 \). The experimental knowledge on the values of the four quantities, e.g. \( \hat{\xi}, \zeta, \alpha_0 / \alpha_+ \) and \( \alpha_0 / \alpha_- \), or \( \hat{\xi}, \zeta, \alpha_0 \) and \( \alpha_+ \), would overdetermine the values of those three parameters, and thus provide a detailed test for the theory.\(^{39-41}\) Thus, the measurement of \( \alpha_0 \), itself, or at least the ratio \( \alpha_0 : \alpha_+ : \alpha_- \) by the method explained in \( A \) is very much desired.

In connection with this, note that it is sufficient for the determination of the values of those adjustable parameters to make use of the three known data for \( \hat{\xi}, \zeta \), and \( \alpha_0 / \alpha_+ \). The parameter values thus determined give the individual values of \( \alpha \)'s. Or otherwise, in this case, we can express \( \alpha \)'s in terms of the three known quantities \( \hat{\xi}, \zeta \), and \( \alpha_0 / \alpha_+ \) by eliminating the parameters \( x_1, \theta_1 \), and \( \phi_1 \) from the \( (T \)-invariant \( |D| = 1/2 \) theoretical expressions for \( \hat{\xi}, \zeta \) and \( \alpha_0 / \alpha_+ \).\(^{39-41}\) By such a method, Eguchi and Nagata\(^{41}\) have obtained\(^*\)

\[
\begin{align*}
\alpha_0 &= \pm (0.68 \pm 0.29), \\
\alpha_+ &= \pm (0.70 \pm 0.28), \\
\alpha_- &= \pm (0.84 \pm 0.23).
\end{align*}
\]

\(^*\) As a means of testing the theory, they have considered the criterion that the magnitudes \( |P^\pm| \) of the \( \Sigma^\pm \)-polarization deduced from these values of \( \alpha \)'s and the experimental data (3.3) should not be larger than unity. The above values of \( \alpha \)'s do not contradict this criterion.
Moreover, we want here to add the following remarks:

1) Also the general $|\mathbf{J}| = 1/2$ theory without $T$-invariance postulate can be tested, if at least the five quantities $\xi$, $\zeta$, $\alpha_0$, $\alpha_+$, and $\alpha_-$ are experimentally known by the method explained in A. This is owing to that the general $|\mathbf{J}| = 1/2$ theory contains only four adjustable parameters, $x_1$, $\theta_1$, $\Phi_1$, and $\gamma_1$.

2) Of course, to obtain the decisive conclusion on the validity of the $|\mathbf{J}| = 1/2$ rule, it is best to start from the more general theory including all of the $|\mathbf{J}| = 1/2$, $3/2$, and $5/2$ interactions, and examine whether the magnitudes $\gamma$, $\gamma_\alpha$, and $\zeta$ of the contributions of the $|\mathbf{J}| = 3/2$ and $5/2$ interactions vanish or not. This is possible under the suitable assumptions on the nature of the interactions (see Subsection D).

This is important not only because of the above-mentioned reason but also for that, when the $|\mathbf{J}| = 1/2$ rule breaks down, the values of the above parameters are necessary as the important information on the $\Sigma$-decay interactions.

C. The Postulate of $T$-invariance

As has been pointed out in refs. 10 and 16), when $T$-invariance holds, the parameters $\gamma$'s vanish exactly and thus all of $\beta$'s become very small. This is seen from the formulas (A·5) given in Appendix. Thus, if the measurements yield the large value for any one of $\beta$'s, then we can conclude that the postulate of $T$-invariance is surely violated. Note that this conclusion does not depend on any other assumptions (e.g. the $|\mathbf{J}| = 1/2$ rule, etc.).

In addition to this, we want to point out the following facts: According to the formulas given in Appendix, the only changes, caused by the violation of $T$-invariance, in the expressions for $\xi$, $\zeta$, $\alpha$'s, and $\gamma$'s are the replacements,

$$\cos (\beta's) \rightarrow \cos (\beta's + \gamma's). \quad (3.8)$$

In practice, when $T$-invariance holds, the cos-functions of the left-hand side of (3.8) can be approximated by unity. This is owing to the very small values of the $\pi-N$ scattering phase shifts $\beta$'s at the $\Sigma$-decay energy:

$$\begin{align*}
\delta_1 &\approx 13^\circ, \quad \delta_2 \approx -8^\circ, \quad \delta_{11} \approx \delta_{12} \approx -2.5^\circ, \quad \text{and thus} \\
\cos (\delta_1 - \delta_2) &\approx 0.93, \quad \cos (\delta_{11} - \delta_{12}) \approx 1.00, \\
\cos (\delta_{11} - \delta_2) &\approx \cos (\delta_{11} - \delta_1) \approx 0.99, \\
\cos (\delta_{11} - \delta_1) &\approx \cos (\delta_{12} - \delta_1) \approx 0.96. \quad (3.9)
\end{align*}$$

For such small angles, the changes of cos-function caused by the relatively small changes of the angle are almost negligible, while the changes of the corresponding sin-function are proportional to those of the angle. Therefore, we can say roughly that the relatively small deviation from $T$-invariance can affect only the values of $\beta$'s. Unless the definite evidence for the violation of $T$-invariance is found, there would be a considerably high reliability in the values of the parameters which are determined consistently from the data for $\xi$, $\zeta$, $\alpha$'s and $\gamma$'s, with the postulate of $T$-invariance. Particularly, if the main decay interaction is the $|\mathbf{J}| = 1/2$ type, then the expressions for $\xi$, $\zeta$, and $\gamma$'s are practically
independent of the $T$-invariance postulate. On the basis of the above considerations and because of the fact that any evidence against $T$-invariance has not yet been found, we shall assume this in the following.

Strictly speaking, even if the experimental data for all of $\beta$'s are quite small, we cannot conclude that $T$-invariance holds, unless the values of all $\gamma$'s are found to be exactly zero. However, as has already been seen, the number of the parameters in the most general theory without $T$-invariance is too large to determine their values by the experiments. The situation is not much altered even under the postulate of $T$-invariance. (The number of the parameters in this case is 9.) Thus, as far as it concerns only with the decays (2·1), we can conclude as follows: Even if all of the quantities (3·1) were measured with sufficient accuracy, the best we can do is to examine the consistency of the $T$-invariant theory* with the suitable assumptions on the dynamical nature of the interactions.

D. Assumption of the decay Hamiltonian and the “one-to-one” law

The quantities $|a|'$s, etc., can generally involve not only the effects of the elementary decay interactions but also those of the strong (e.g. the final state $\pi N$) interactions, and accordingly the relation between our parameters and the form of the decay Hamiltonian is very complicated. However, if the effects of the strong interactions can be neglected, then there holds some direct and simple relation between them. According to the results of the perturbational computation, those effects of the strong interactions are, in fact, very small. Of course, the perturbational treatment of the strong interactions is very doubtful in many respects. However, here we shall tentatively adopt this assumption.**

In addition to this, let us assume that both parts of the $|\Delta I|=1/2$ interaction which give rise respectively to the final $I=1/2$ and $3/2$ states have the same types of coupling. Here the “type of coupling” means the form of the spin- and momentum-dependence of the interaction Hamiltonian. For example, if the interaction is of the Yukawa type, then the Hamiltonian can be written symbolically as, say,

$$g \bar{\psi}_N(1+r\gamma_5)\psi_2 \cdot \phi_n,$$

except for the isotopic spin factors. The “type of coupling” means the form of the operator $(1+r\gamma_5)\psi_2$. Within a general frame of the terminology “$|\Delta I|=1/2$ interaction”, the above two parts can have different types of coupling. However, in the usual dynamical approach, one would tentatively assume such a relatively simple Hamiltonian (e.g. see (3·23) or refs. 7 and 18). Similarly, let us assume the corresponding two parts of the $|\Delta I|=3/2$ interaction to have the same types of coupling to each other. Under such assumptions, we have

$$\theta_1 = \phi_1 \quad \text{and} \quad \theta_2 = \phi_2. \quad (3·10)$$

* The only exception is the $|\Delta I|=1/2$ theory. We have seen in B that the test for this theory does not necessarily need $T$-invariance.

** Umezawa et al. have insisted on the validity of this assumption, basing on their speculations on the difference of the effective domains of the strong and weak interactions.
As an alternative assumption, we can set also as
\[ \theta_1 = \theta_3 = \theta, \quad \phi_1 = \phi_3 = \phi = 0, \quad (3 \cdot 11) \]
instead of \(3 \cdot 10\). This corresponds to the assumption that the types of coupling of the parts, responsible for the final \(I=3/2\) state, of the \(|dI|=1/2, 3/2\) and \(5/2\) interactions are of the same to each other, and the similar assumption for the parts responsible for the final \(I=1/2\) state. \((3 \cdot 10)\) with the postulate of \(T\)-invariance reduces the number of parameters to only 7. \((As for the case of \(3 \cdot 11\), see later.)\)

Moreover, for example, if the part of the \(|dI|=1/2\) interaction which gives rise to the final \(I=1/2\) state has the \((V-A)\) coupling of the Yukawa type described symbolically as
\[ \bar{\psi}_N(g-g')g'' g' \bar{\psi} \eta \partial_\mu \phi_\eta, \quad (3 \cdot 12) \]
then \(^7\)
\[ a_i/a_i' = (g/g') \left[ (M-m)/(M+m) \right] \left[ (M+m)^2 - \mu^2 \right] / \left[ (M-m)^2 - \mu^2 \right]^{1/2}, \quad (3 \cdot 13) \]
where \(M, m\) and \(\mu\) are the masses of \(\Sigma\), nucleon and pion, respectively. Under the "one-to-one" law \((g = \pm g')\), \((3 \cdot 13)\) yields
\[ \theta_1 \approx \pm 50^\circ. \quad (3 \cdot 14) \]

Similarly, if it is the \((S-P)\) coupling, then
\[ a_i/a_i' = (g/g') \left[ (M+m)^2 - \mu^2 \right] / \left[ (M-m)^2 - \mu^2 \right]^{1/2}, \quad (3 \cdot 15) \]
and, under the "one-to-one" law,
\[ \theta_1 \approx \pm 85^\circ. \quad (3 \cdot 16) \]

Thus, corresponding to the particular assumption for the coupling types of the various interactions, the values of \(\theta_1's\) and \(\phi_1's\) are determined uniquely, and the number of the free parameters becomes only 4, at most, \((\text{i.e.} \ x_1, \ x_3, \ \gamma, \ \zeta)\). Therefore, if four quantities, at most, say \((3 \cdot 7)\), be measured, we could know the strengths of the \(|dI|=3/2\) and \(5/2\) interactions required by the particular assumption on the types of coupling. \((\text{For example, see \((3 \cdot 21)\).})\) Of course, it is possible also to test such an assumption on the coupling type, when the five quantities \(\zeta, \ \zeta', \ \alpha_2, \ \alpha_1, \ \text{and} \ \alpha_-\) can be experimentally known. \((\text{See also the remarks towards the end of this Subsection.})\)

With a similar assumption as above, Eguchi-Nagata\(^7\) and Nakagawa-Umezawa\(^6\) have investigated the compatibility of the \(|dI|=1/2\) and the "one-to-one" laws, and obtained the negative answer. However, their result can deny neither one of these two laws, since both of them are never the ones beyond the merely tentative assumptions. In view of the success of the "one-to-one" law in the lepton processes, it would be very interesting to investigate this law in the more general frame including also the interactions other than the \(|dI|=1/2\) one. The above consideration show that such an investigation is, in fact, possible.

Finally, we want to add the following remarks:
1) Assumption (3·11) seems to reduce the number of parameters to 6. However, actually, it reduces this number to only 4. Under this assumption, the expressions for \( \xi, \zeta, \) etc., become as

\[
\xi = (1/2) [(2A-B)^2 + 4(1-q)AB] / [(A+B)^2 - 2(1-q)AB]
\]

\[
\zeta = (1/(18C^2)) [(2A-B)^2 + 2(A+B)^2]
\]

\[
\alpha_0 = (4A^2s - 4ABp + B^2t) / [(2A-B)^2 + 4(1-q)AB]
\]

\[
\alpha_+ = (A^2s + 2ABp + B^2t) / [(A+B)^2 - 2(1-q)AB]
\]

\[
\alpha_- = s,
\]

with

\[
A = 1 - 2\sqrt{2/5}y + \sqrt{3/5}z, \quad B = 2x_1 + yx_2, \quad C = 1 + \sqrt{2/5}y + \sqrt{1/15}z,
\]

and

\[
p = \sin(\Theta + \Phi), \quad q = \cos(\Theta - \Phi), \quad s = \sin(2\Theta), \quad t = \sin(2\Theta).
\]

\( \gamma \)'s are obtained from the corresponding \( \alpha \)'s by replacing the sin-functions of \( \Theta \) and \( \Phi \) by the minus of the corresponding cos-functions. Thus, in this case, the essential parameters are eventually only \( B/A \), \( C/A \), \( \Theta \) and \( \Phi \). Under the particular assumption for the types of coupling, we have only two adjustable parameters \( B/A \) and \( C/A \), which can be determined by the known data (3·2) (e.g. see (3·21)). It is remarkable that, in such a case, we cannot know the individual values of \( x_1, x_2, y, \) and \( z \). The meanings of the parameters \( B/A \) and \( C/A \) are obvious: \( C/A \) gives a measure for the degree of violation of the \( |\Delta I| = 1/2 \) rule, since the condition \( y = z = 0 \) imposes that \( C/A = 1 \). Similarly, \( B/A \) gives a measure for the ratio of the effective strengths of the two parts of the total decay interaction which give rise respectively to the final \( I = 1/2 \) and \( 3/2 \) states.

2) Under the combined assumption of (3·10) and (3·11),

\[
\xi = (1/2) \left[ \frac{(2A-B)/(A+B)} \right], \quad \zeta = (1/(18C^2)) \left[ \frac{(2A-B)^2 + 2(A+B)^2} \right],
\]

\[
\alpha_0 = \alpha_+ = \alpha_- = \sin(2\Theta), \quad \gamma_0 = \gamma_+ = \gamma_- = -\cos(2\Theta).
\]

Note that the relations \( \alpha_0 = \alpha_+ = \alpha_- \) and \( \gamma_0 = \gamma_+ = \gamma_- \) hold independently of the coupling type, and thus, this model can be directly tested, if the radio \( \alpha_0, \alpha_+, \alpha_- \) be measured. The data (3·3) seem to indicate that \( \alpha_0 = \alpha_+ \). If we extrapolate this also to \( \alpha_- \) and assume (3·10) and (3·11) to be true, then the data (3·2) are quite inconsistent with the \( |\Delta I| = 1/2 \) rule. Namely, the expressions (3·20) and the data (3·2) lead to

\[
\begin{align*}
|B/A| & \approx 0.29 \pm 0.04 & |B/A| & \approx -10.2 \pm 2.8 \\
|C/A| & \approx 0.40 \pm 0.05 & |C/A| & \approx 2.8 \pm 0.6.
\end{align*}
\]

Moreover, if the coupling is the \((V-A)\) or \((S-P)\) one of the Yukawa type with \( g = \pm g' \) (the "one-to-one" law), then we get by (3·14) and (3·16)

\[
\alpha_0 = \alpha_+ = \alpha_- = \pm 1, \quad \alpha_0 = \alpha_+ = \alpha_- = \pm 0.2,
\]

(3·22)
respectively. Both of these two cases are not inconsistent with the data (3·3). It is obvious that the previously quoted works of refs. 7) and 18) are the special cases of the present consideration.

E. Conclusion

We have investigated what information could be obtained, at least in principle, by the measurements of the decay processes (2·1). The essential limitation of such information (owing to the insufficiency of the number of the observable quantities) is clarified, and within this limitation, the various possibilities are discussed. Especially, the measurement of the individual values of \( \alpha' \)'s, or at least, that of the ratio \( \alpha_0 : \alpha_+ : \alpha_- \), is very much desired for the more definite tests on the \( |\Delta I|=1/2 \) rule and the analysis discussed in \( D \).

We have seen that, as long as the effects of the strong interactions are neglected, the values of our parameters \( \theta' \)'s and \( \phi' \)'s have certain correspondences with the "dynamical" assumption for the form of the decay Hamiltonian. In other words, the "dynamical" approach, in which only the lowest order effects of the decay interactions are taken into account, is never the one beyond our "phenomenological" analysis. For example, to assume tentatively the decay Hamiltonian

\[
H = [g_0 \bar{p}(1 + \gamma_5)\gamma^\mu \partial_\mu I^0 + g_+ \bar{N}(1 + \gamma_5)\gamma^\mu \partial_\mu I^+ \nonumber \\
+ g_- \bar{N}(1 + \gamma_5)\gamma^\mu \partial_\mu I^- ] + \text{h.c.} \quad (3·23)
\]

is equivalent to our combined assumption (3·10), (3·11), and (3·14). In fact, except for the absolute values of \( g' \)'s which relate to the absolute probabilities of decays, the essentially free parameters involved in (3·23) are \( g_0/g_+ \), and \( g_0/g_- \), whose number is only two in agreement with that of the parameters appearing in (3·20).

However, the analysis of the present paper is advantageous in that, in general, it does not necessarily need the neglect or the perturbational treatment of the strong interactions. For example, even if the full effect of the final state interactions are taken into account, the relation (3·11), and accordingly (3·17) would be expected to hold, provided that the types of coupling are the same for all of the \( |\Delta I|=1/2, 3/2, \) and \( 5/2 \) interactions. Similarly, the discussion in B on the \( |\Delta I|=1/2 \) theory is completely independent of the assumption for the dynamical nature of the interactions. Thus, we feel that the analysis of the present paper covers almost all that we can do at present on the \( \Sigma \)'-decays.

§ 4. Decay of the \( \Lambda' \)-particle

A similar analysis to the one for the \( \Sigma \)'-decays is applicable also to the case of the \( \Lambda' \)-decay. In the following, we shall briefly mention on this case.

A. Kinematics

Since the isotopic spin of \( \Lambda' \) is zero, the types of the interactions which can contribute to the decay \( \Lambda' \to N^+ + \pi \) are only \( |\Delta I|=1/2 \) and \( 3/2 \). The final states produced by these two types of the interactions are purely \( I=1/2 \) and \( 3/2 \), respectively, and thus...
it is unnecessary to add the suffix (1 or 3) to the quantities \( a \) and \( b \).* Thus, even in

the most general theory, the free parameters are only

\[
\begin{align*}
\gamma &= \mathcal{E}^{(1)} \mathcal{E}^{(1)'} \left[ (|b|^2 + |b'|^2) / (|a|^2 + |a'|^2) \right]^{1/2}, \\
\tan \theta &= \mathcal{E}^{(1)} \mathcal{E}^{(1)'} \cdot |a| / |a'|, \\
\tan \phi &= \mathcal{E}^{(1)} \mathcal{E}^{(1)'} \cdot |b| / |b'|,
\end{align*}
\]

where

\[
\begin{align*}
\gamma &= \gamma_{\text{obs}} - \gamma_{\text{th}}, \\
\gamma_p &= \gamma_{\text{ps}} - \gamma_{\text{ps1}}, \\
\eta &= \gamma_{\text{ps1}} - \eta_{\text{ps1}}, \quad (4.1)
\end{align*}
\]

\( \theta, \phi, \) and \( \gamma \) in (4.1) correspond to \( \theta_1, \Phi_3, \) and \( \gamma/\chi \) for the case of \( \Sigma \)-decays, respectively.

We can express the branching ratio \( \xi = \omega(A^0 \rightarrow p + \pi^-) / \omega(A^0 \rightarrow n + \pi^0) \), the asymmetry parameters \( \alpha \)'s and so on, in terms of the above six free parameters. Those expressions basing on the most general theory will be given in Appendix, (A·10). Under the postulate of \( T \)-invariance, they become as follows:

\[
\begin{align*}
\xi &= [1 - 2 \sqrt{2} \gamma \cos (\theta - \Phi) + \gamma^2] / [1 + 2 \sqrt{2} \gamma \cos (\theta - \Phi) + 2\gamma^2], \\
\alpha_a &= [2 \sin (2\theta) - 2 \sqrt{2} \gamma \sin (\theta + \Phi) + \gamma^2 \sin (2\theta)] / \text{(numerator of } \xi_a), \\
\alpha_s &= [\sin (2\theta) + 2 \sqrt{2} \gamma \sin (\theta + \Phi) + 2\gamma^2 \sin (2\theta)] / \text{(denominator of } \xi_a), \\
\gamma_a &= -[2 \cos (2\theta) - 2 \sqrt{2} \gamma \cos (\theta + \Phi) + \gamma^2 \cos (2\theta)] / \text{(numerator of } \xi_a), \\
\gamma_b &= -[\cos (2\theta) + 2 \sqrt{2} \gamma \cos (\theta + \Phi) + 2\gamma^2 \cos (2\theta)] / \text{(denominator of } \xi_a), \\
\text{and} \quad \beta_a = \beta_s = 0, \quad (4.3)
\end{align*}
\]

where the suffices \( a \) and \( b \) denote that the quantities are respectively corresponding to the decay modes,

\( A^0 \rightarrow p + \pi^- (a), \) and \( A^0 \rightarrow n + \pi^0 (b). \) \( (4.4) \)

In (4.3), we have approximated the \( \cos \)-functions of the differences of \( \delta \)'s by unity, since the \( \pi -N \) scattering phase shifts are very small at the \( A^0 \)-decay energy.

Similarly, under the \( |dI| = 1/2 \) theory,

\[
\begin{align*}
\xi_A &= 2, \\
\gamma_a = \gamma_b &= -\cos (2\theta), \\
\alpha_a &= \alpha_b = \sin (2\theta) \cos (\delta_{11} - \delta_1 + \gamma) \approx \sin (2\theta) \cos \gamma, \\
\beta_a &= \beta_b = \sin (2\theta) \sin (\delta_{11} - \delta_1 + \gamma) \approx \sin (2\theta) \sin \gamma. \quad (4.5)
\end{align*}
\]

In (4.5), we have not postulated \( T \)-invariance.

* The meanings of the notations are similar as those of § 2 and 3. Although the interactions for the \( A^0 \)-decay are not necessarily same as those for the \( \Sigma \)-decay, we use here the same letters \( a \) and \( b \) to avoid unnecessary complications.
B. Possible experimental information

Through similar discussions to those in the previous sections, what can help to determine the nature of the decay interactions of this case are at most five quantities, e.g. $\xi_\Lambda$, $\alpha_a$, $\alpha_b$, $\phi_a$ and $\phi_b$. The data obtained thus far are

\[
\begin{aligned}
&\frac{w(A^0 \rightarrow n + \pi^0)}{w(A^0, \text{ total})} = 0.32 \pm 0.05 \quad (\xi_\Lambda \approx 2.1) \\
&p \cdot \alpha_a = +0.44 \pm 0.11,
\end{aligned}
\]

(4.6)

where $p$ is the average degree of polarization of the parent $A^0$.

Under such a circumstance and with the formulas (4.3) $\sim$ (4.5) and (A.10), we can draw several conclusions on the possible information obtainable from the experimental analysis of the $A^0$-decays. Since the details of the discussions are quite similar to those of § 3, we shall not here repeat them. However, we want here to emphasize the importance of the measurement of $\alpha_a$ and $\alpha_b$, or at least their ratio* $\alpha_a/\alpha_b$. This is owing to the following reasons:

1) Analogously to the discussion in § 3, let us assume T-invariance and $\theta = \phi$. In this case, (4.3) reduces to

\[
\begin{aligned}
&\xi_\Lambda = \left[ \frac{(\sqrt{2} - \gamma)}{(1 + \sqrt{2} \gamma)} \right]^2 \\
&\alpha_a = \alpha_b = \sin(2\theta) \quad \gamma_a = \gamma_b = -\cos(2\theta).
\end{aligned}
\]

On the other hand, we have seen that, if the $|\mathcal{D}|=1/2$ theory be true, the relations $\alpha_a = \alpha_b$ and $\phi_a = \phi_b$ hold independently of any other assumptions. Thus, the measurement of the ratio $\alpha_a : \alpha_b$ provides a means of test for the above two models. However, the following should be noted: From the conditions $\alpha_a = \alpha_b$ and $\phi_a = \phi_b$ alone, we cannot judge which of the above two models is true. When the measurement shows $\alpha_a = \alpha_b$, the test of the $|\mathcal{D}|=1/2$ rule requires a more accurate measurement of $\xi_\Lambda$. If we assume (4.7) to be true, then the data (4.6) confine the permissible region of the value of $\gamma$ within

\[-2.5 \geq \gamma \geq -2.8 \quad \text{or} \quad +0.03 \geq \gamma \geq -0.03.\]

(4.8)

2) If the individual values of $\alpha_a$ and $\alpha_b$ were known, then we could determine the parameters of the general T-invariant theory including both of the $|\mathcal{D}|=1/2$ and $3/2$ interactions. Moreover, if we neglect the effects of the strong interactions, then, from the values of $\theta$ and $\phi$ thus determined, we would be able to know the coupling type of the decay interactions. For example, if we assume the coupling type of both of the $|\mathcal{D}|=1/2$ and $3/2$ interactions to be $(V - A)$ or $(S - P)$ of the Yukawa type with the "one-to-one" law, then we get (similarly to (3.22))

\[
\alpha_a = \alpha_b \approx \pm 0.9 \quad \text{or} \quad \pm 0.1,
\]

(4.9)

* The measurement of at least the ratio $\alpha_a/\alpha_b$ would certainly be possible, although it would be more difficult than the measurement of the corresponding ratio $\alpha_0 : \alpha_\pm$ in the $\Sigma$-decays, owing to that both of the two decay products, $n$ and $\pi^0$, in the mode $b$ are neutral.
respectively. It is remarkable that the \( (V-A) \) coupling is not inconsistent with the data \((4.6)\), but the \( (S-P) \) coupling is completely ruled out in this case. (Note that the magnitude of \( \bar{p} \) in \((4.6)\) cannot be larger than unity.)

### Appendix. General formulas for \( \xi, \zeta, \) etc.

#### A. The case of the \( \Sigma \)-decays

1) \( T \)-invariant theory including all the possible interactions

The expressions for \( \xi \) and \( \zeta \) are given by

\[
\xi = \frac{N_0}{N_+} \quad \text{and} \quad \zeta = \frac{(N_0 + N_+) / (9N_-)}{, (A\cdot 1)}
\]

where

\[
N_0 = 2 + 2x_1^2 - 4x_1 \cos (\Theta_1 - \Phi_1) + (8/5) \gamma^2 (2 + (5/16) x_3^2 + (5/\sqrt{10}) x_3 \cos (\Theta_1 - \Phi_3) - 2y (V_2/S) \cos (\Theta_1 - \Phi_2) + x_1 y \sin (\Theta_1 - \Phi_3) + x_3 y \sin (\Theta_3 - \Phi_3) - 2x_1 \cos (\Theta_1 - \Phi_3) - x_3 y \cos (\Theta_3 - \Phi_3) - 4V_2/S \gamma \sin (\Theta_1 - \Phi_3) - (4V_6/S) y \sin (\Theta_1 - \Phi_3),
\]

\[
N_+ = 1 + 4x_1^2 + 4x_1 \cos (\Theta_1 - \Phi_1) + (8/5) \gamma^2 (1 + (5/8) x_3^2 - (5/\sqrt{10}) x_3 \cos (\Theta_1 - \Phi_3) - 2y (V_2/S) \cos (\Theta_1 - \Phi_2) + x_1 y \sin (\Theta_1 - \Phi_3) + x_3 y \sin (\Theta_3 - \Phi_3) - 2x_1 \cos (\Theta_1 - \Phi_3) - x_3 y \cos (\Theta_3 - \Phi_3) - 4V_2/S \gamma \sin (\Theta_1 - \Phi_3) - (4V_6/S) y \sin (\Theta_1 - \Phi_3),
\]

\[
N_- = 1 + (2/5) \gamma^2 + 2V_2/S \gamma \cos (\Theta_1 - \Phi_3) + (1/15) \gamma^2
\]

\[
+ (2/\sqrt{15}) \gamma \cos (\Theta_1 - \Phi_3) + (2/5) \sqrt{\gamma} \gamma \cos (\Theta_3 - \Phi_3), \quad (A\cdot 2)
\]

and thus the numerator of \( \zeta \) is

\[
N_0 + N_+ = 3 \left[ 1 + 2x_1^2 + (8/5) \gamma^2 (1 + (5/16) x_3^2) - 4V_2/S \gamma \cos (\Theta_1 - \Phi_3) + 2x_1 y \sin (\Theta_1 - \Phi_3) + (3/5) \gamma^2 + 2V_2/S \gamma \cos (\Theta_1 - \Phi_3) - (4V_6/S) y \sin (\Theta_1 - \Phi_3) \right]. \quad (A\cdot 3)
\]

The expressions for \( \alpha's \) are as follows:

\[
\alpha_0 = (2/N_0) \left[ \sin (2\Theta_1) - 2x_1 \sin (\Theta_1 + \Phi_1) + x_3 \sin (2\Theta_3)
\right.
\]

\[
- \gamma (4V_2/S \gamma \sin (\Theta_1 + \Phi_3) - x_1 \sin (\Theta_1 + \Phi_3) + x_3 \sin (\Theta_3 + \Phi_3))
\]\n
\[
+ x_1 y \sin (\Theta_1 + \Phi_3) - x_3 y \sin (\Theta_3 + \Phi_3)
\]

\[
+ (4/5) \gamma^2 \left( 2 \sin (2\Phi_3) + (5/\sqrt{10}) x_3 \sin (\Theta_3 + \Phi_3) + (5/16) x_3 \sin (2\Theta_3) \right)
\]

\[
+ \sqrt{3/5} \gamma \left( 2 \sin (\Theta_3 + \Phi_3) + 2x_1 \sin (\Theta_3 + \Phi_3) - 4V_2/S \gamma \sin (\Theta_3 + \Phi_3)
\right.
\]

\[
- 4V_6/S y \sin (\Theta_3 + \Phi_3) + (3/5) \gamma^2 \sin (2\Phi_3),
\]

\]

\[
(2/N_0) \left[ \sin (2\Theta_1) - 2x_1 \sin (\Theta_1 + \Phi_1) + x_3 \sin (2\Theta_3)
\right.
\]

\[
- \gamma (4V_2/S \gamma \sin (\Theta_1 + \Phi_3) - x_1 \sin (\Theta_1 + \Phi_3) + x_3 \sin (\Theta_3 + \Phi_3))
\]\n
\[
+ x_1 y \sin (\Theta_1 + \Phi_3) - x_3 y \sin (\Theta_3 + \Phi_3)
\]

\[
+ (4/5) \gamma^2 \left( 2 \sin (2\Phi_3) + (5/\sqrt{10}) x_3 \sin (\Theta_3 + \Phi_3) + (5/16) x_3 \sin (2\Theta_3) \right)
\]

\[
+ \sqrt{3/5} \gamma \left( 2 \sin (\Theta_3 + \Phi_3) + 2x_1 \sin (\Theta_3 + \Phi_3) - 4V_2/S \gamma \sin (\Theta_3 + \Phi_3)
\right.
\]

\[
- 4V_6/S y \sin (\Theta_3 + \Phi_3) + (3/5) \gamma^2 \sin (2\Phi_3),
\]

\]
Possible Experimental Tests on the Decay Interactions of Hyperons

\( \alpha_+ = (2/N_+)[(1/2) \sin (2 \Phi_i) + 2x_1 \sin (\theta_1 + \Phi_i) + 2x_1^2 \sin (2 \Phi_i) 
- \gamma [2\sqrt{2/5} [\sin (\Phi_1 + \Phi_3) + 2x_1 \sin (\theta_1 + \Phi_3)]
- x_2 \sin (\theta_2 + \Phi_i) - 2x_1 x_3 \sin (\theta_1 + \theta_3) \]
+ (4/5) x_3^2 \sin (2 \Phi_2) \]
+ \sqrt{3/5} \gamma [\sin (\Phi_1 + \Phi_3) + 2x_1 \sin (\theta_1 + \Phi_3) - 2\sqrt{2/5} \gamma \sin (\Phi_3 + \Phi_3)
+ yx_3 \sin (\theta_2 + \Phi_i) \] + (3/10) x_3^2 \sin (2 \Phi_2) \],
\( \alpha_- = (2/N_-)[(1/2) \sin (2 \Phi_i) + \sqrt{2/5} \gamma \sin (\Phi_1 + \Phi_3) + (1/5) \gamma \sin (2 \Phi_3)
+ \gamma [(1/\sqrt{15}) \sin (\Phi_1 + \Phi_3) + (1/5) \sqrt{2/3} \gamma \sin (\Phi_3 + \Phi_3)
+ (1/30) x^2 \sin (2 \Phi_2) \], \quad (A\cdot4)

The expressions for \( \gamma \)'s are obtained from those for \( \alpha \)'s by replacing the \sin\-functions of \( \theta \)'s and \( \Phi \)'s by the minus of the corresponding \cos\-functions. Finally, the expressions for \( \beta \)'s are obtained from those for the corresponding \( \alpha \)'s by the following replacements:

\[
\begin{align*}
\sin (\Phi_i + \Phi_j) &\to 0.1 \sin (\Phi_i + \Phi_j), \\
\sin (\theta_i + \theta_j) &\to -0.27 \sin (\theta_i + \theta_j), \\
\sin (\Phi_i + \theta_j) &\to 0.1 \sin (\Phi_i + \theta_i), \\
&= - (1/2) [0.17 \sin (\Phi_i + \theta_i) - 0.37 \sin (\Phi_i + \theta_j)] , \\
(i, j=1, 3, 5). \quad (A\cdot5)
\end{align*}
\]

In (A\cdot1) \sim (A\cdot4), we have approximated the \cos\-functions of the differences of \( \delta \)'s [e.g. \cos (\delta_1 - \delta_3), etc.] by unity. This gives rise to the errors of only a few percent. The exact expressions (without this approximation) can be easily obtained from the expressions in the most general case only by setting all \( \gamma \)'s as zero. [See (A\cdot7) \sim (A\cdot9).]

In (A\cdot5), we have taken into account that, by (3\cdot7),
\[
\begin{align*}
\sin (\delta_{11} - \delta_2) &\approx \sin (\delta_{11} - \delta_2) \approx 0.1 \\
\sin (\delta_{11} - \delta_2) &\approx \sin (\delta_{11} - \delta_2) \approx -0.27. \quad (A\cdot6)
\end{align*}
\]

2) The most general theory without the postulate of \( T \)-invariance.

In this case, the required expressions are obtained from the formulas (A\cdot1) \sim (A\cdot4) of the \( T \)-invariant theory by the following replacements:

a. The expression for \( \xi \) and \( \zeta \)

\[
N_i \to N'_i \quad (i=0, +, -), \quad (A\cdot7)
\]

where \( N'_i \) is the quantity obtained from \( N_i \) by the replacements,
\[
\begin{align*}
\cos (\theta_1 - \Phi_j) &\to \sin \theta_1 \sin \Phi_j \cos (\delta_1 - \delta_3 + \gamma_{ij}) \\
&+ \gamma \cos \theta_1 \cos \Phi_j \cos (\delta_{11} - \theta_3 + \gamma_i - \gamma_j + \gamma_{ij}) , \\
\cos (\Phi_i - \Phi_j) &\to \sin \Phi_i \sin \Phi_j \cos \gamma_{ij} + \cos \Phi_i \cos \Phi_j \cos (\gamma_i - \gamma_j + \gamma_{ij}) ,
\end{align*}
\]
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\[\cos (\theta_i - \theta_j) \rightarrow \sin \theta_i \sin \theta_j \cos \gamma_{ij} + \cos \theta_i \cos \theta_j \cos (\gamma_{ij} - \gamma_j + \gamma_i),\]

with \[\gamma_{ij} = -\gamma_{ji}, \quad \gamma_{ii} = 0, \quad \text{and} \quad \gamma_{i} = \gamma_{i-1} - \gamma_{i+1}. \quad (i, j = 1, 3, 5) \quad (A \cdot 8)\]

b. The expressions for \(\alpha's\) and \(\gamma's\)

These require, in addition to (A \cdot 7), the following replacements for the \(\sin\)-functions of \(\theta's\) and \(\Phi's\) in the expressions (A \cdot 4) of \(\alpha's\), and the corresponding \(\cos\)-functions in the expressions of \(\gamma's\):

\[\sin (\theta_i + \Phi_j) \rightarrow \sin \theta_i \cos \Phi_j \cos (\delta_{i1} - \delta_{i1} + \gamma_{i1} + \gamma_{j})\]

\[+ \cos \Phi_i \sin \theta_j \cos (\delta_{j1} - \delta_{j1} + \gamma_{j} - \gamma_{i})],\]

\[\sin (\theta_i + \Phi_j) \rightarrow \sin \theta_i \cos \Phi_j \cos (\delta_{i1} - \delta_{i1} + \gamma_{i} + \gamma_{j})\]

\[+ \cos \Phi_i \sin \theta_j \cos (\delta_{j1} - \delta_{j1} + \gamma_{j} - \gamma_{i})],\]

\[\sin (\theta_i + \Phi_j) \rightarrow \sin \theta_i \cos \Phi_j \cos (\delta_{i1} - \delta_{i1} + \gamma_{i} + \gamma_{j})\]

\[+ \cos \Phi_i \sin \theta_j \cos (\delta_{j1} - \delta_{j1} + \gamma_{j} - \gamma_{i})],\]

\[\cos (\theta_i + \Phi_j) \rightarrow \cos \phi_i \cos \Phi_j \cos (\gamma_{i} - \gamma_{i} + \gamma_{j}) - \sin \phi_i \sin \Phi_j \cos \gamma_{j} + \cos \theta_i \cos \Phi_j \cos (\gamma_{i} - \gamma_{i} + \gamma_{j})\]

\[+ \cos \phi_i \sin \Phi_j \cos (\gamma_{i} - \gamma_{i} - \gamma_{j})\]

\[\cos (\theta_i + \Phi_j) \rightarrow \cos \phi_i \cos \Phi_j \cos (\gamma_{i} - \gamma_{i} + \gamma_{j}) - \sin \phi_i \sin \Phi_j \cos \gamma_{j} + \cos \theta_i \cos \Phi_j \cos (\gamma_{i} - \gamma_{i} + \gamma_{j})\]

\[+ \cos \phi_i \sin \Phi_j \cos (\gamma_{i} - \gamma_{i} - \gamma_{j})\]

\[\cos (\theta_i + \Phi_j) \rightarrow \cos \phi_i \cos \Phi_j \cos (\gamma_{i} - \gamma_{i} + \gamma_{j}) - \sin \phi_i \sin \Phi_j \cos \gamma_{j} + \cos \theta_i \cos \Phi_j \cos (\gamma_{i} - \gamma_{i} + \gamma_{j})\]

\[- \sin \phi_i \sin \Phi_j \cos (\gamma_{i} - \gamma_{i} + \gamma_{j}). \quad (A \cdot 9)\]

Note that the replacement (A \cdot 9) keeps \(\cos (2\theta_i)\) and \(\cos (2\Phi_i)\) unchanged.

c. The expressions for \(\beta's\)

These are obtained from the expressions for \(\alpha's\) (in the present case) by replacing the \(\cos\)-functions of \(\delta's\) and \(\gamma's\) by the corresponding \(\sin\)-functions.

B. The case of the \(\Lambda^0\)-decays

\[\xi_A = (2 + \gamma^2 - 2 \sqrt{2} \Lambda^0 \cdot \gamma) / (1 + 2 \gamma^2 + 2 \sqrt{2} \Lambda^0 \cdot \gamma),\]

\[\alpha_a = 2 (2B + Dy^2 - \sqrt{2} Cy) / (2 + \gamma^2 - 2 \sqrt{2} \Lambda^0 \cdot \gamma),\]

\[\alpha_b = 2 (B + 2 Dy^2 + \sqrt{2} Cy) / (1 + 2 \gamma^2 + 2 \sqrt{2} \Lambda^0 \cdot \gamma),\]

\[\gamma_a = -[2 \cos (2\theta') + \gamma^2 \cos (2\Phi) + 2 \gamma \Lambda^0 \cdot \gamma] / (1 + \gamma^2 + 2 \sqrt{2} \Lambda^0 \cdot \gamma),\]

\[\gamma_b = -[2 \cos (2\theta') + \gamma^2 \cos (2\Phi) - 2 \gamma \Lambda^0 \cdot \gamma] / (1 + \gamma^2 + 2 \sqrt{2} \Lambda^0 \cdot \gamma),\]

\[\quad (A \cdot 10)\]

where

\[A^\pm = \sin \theta \sin \phi \cos (\delta - \delta_1 + \gamma) \pm \cos \theta \cos \phi \cos (\delta_{j1} - \delta_{j1} + \gamma),\]

\[B = \sin \theta \cos \phi \cos (\delta_{j1} - \delta_1 + \gamma),\]

\[C = \sin \theta \cos \phi \cos (\delta_{j1} - \delta_1 + \gamma_0 + \gamma) + \cos \theta \sin \phi \cos (\delta_{j1} - \delta_3 - \gamma_0 + \gamma),\]

\[D = \sin \phi \cos \phi \cos (\delta_{j1} - \delta_3 + \gamma_0 - \gamma_0 + \gamma). \quad (A \cdot 11)\]
\( \beta \)'s are obtained from \( \alpha \)'s by replacing the cos-functions of \( \delta \)'s and \( \gamma \)'s by the corresponding sin-functions.

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