On the Aliasing and Resolving Power of Sea Level Low-Pass Filtered onto a Regular Grid from Along-Track Altimeter Data of Uncoordinated Satellites: The Smoothing Strategy

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ABSTRACT
It is shown that smoothing (low-pass filtering) along-track altimeter data of uncoordinated satellites onto a regular space–time grid helps reduce the overall energy level of the aliasing from the aliasing levels of the individual satellites. The rough rule of thumb is that combining $N$ satellites reduces the energy of the overall aliasing to $1/N$ of the average aliasing level of the $N$ satellites. Assuming the aliasing levels of these satellites are roughly of the same order of magnitude (i.e., assuming that no special signal spectral content significantly favors one satellite over others at certain locations), combining data from uncoordinated satellites is clearly the right strategy. Moreover, contrary to the case of coordinated satellites, this reduction of aliasing is not achieved by the enhancement of the overall resolving power. In fact (by the strict definition of the resolving power as the largest bandwidths within which a band-limited signal remains free of aliasing), the resolving power is reduced to its smallest possible extent. If one characterizes the resolving power of each satellite as a spectral space within which all band-limited signals are resolved by the satellite, then the combined resolving power of the $N$ satellite is characterized by the spectral space that is the intersection of all $N$ spectral spaces (i.e., the spectral space that is common to all the resolved spectral spaces of the $N$ satellites, hence the smallest). It is also shown that the least squares approach is superior to the smoothing approach in reducing the aliasing and upholding the resolving power of the raw data. To remedy one of the shortcomings of the smoothing approach, the author recommends a multismoother smoothing strategy that tailors the smoother to the sampling characteristics of each satellite. Last, a strategy based on the least squares approach is also described for combining data from uncoordinated satellites.

1. Introduction
For a single altimetric satellite in exact-repeat sampling mode or for a coordinated constellation of satellites (wherein the coordination is such that the collective samplings appear as if they were carried out by a single exact-repeat satellite; see more elaborations in appendix A), Tai (2004, hereafter T04) has been able to determine the resolving power (i.e., the largest bandwidths within which a band-limited signal can remain aliasing-free; see more elaborations in appendix B). Moreover, Tai (2006, hereafter T06) has been able to derive the aliasing formulas (i.e., the aliasing from spectral components beyond these bandwidths into components within these bandwidths). However, when the satellites are not coordinated, the combined resolving power and aliasing have been somewhat obscure. Greenslade et al. (1997) have shown that when the data from uncoordinated satellites are combined, it does not appear to enhance the original resolving power of any of the individual satellites. Thus combining the data appears to offer few advantages. Yet, Le Traon and Dibarboure (1999) as well as Ducet et al. (2000) have found that the mapping errors are much reduced if data from uncoordinated missions are combined [as has the overall error level computed in Greenslade et al. (1997)]. The main purpose of the present investigation is to document a proof that they are both right and that there is no contradiction here, because even if the combination does not enhance the resolving power it may reduce the aliasing. In addition, this paper recommends the optimal strategies for combining data from uncoordinated satellites.

T06 has shown that the aliasing computed using the least squares (i.e., the best-fitting results to the raw
data) is different from the aliasing computed if the raw data are first low-pass filtered onto a regular space–time grid. The aliasing results favor the least squares approach, which incurs the least amount of aliasing. (Hereafter, the low-pass-filtered data are referred to as the smoothed data, the low-pass-filtering approach as the smoothing approach, and the least squares approach as the LS approach.) It is no surprise that how one processes the data can affect the aliasing. But the difference here extends to the resolving power. That is, the resolving power of the smoothed data is smaller than what the raw data are capable of resolving. With few exceptions, most studies utilizing altimeter data have used the smoothing approach, including the seemingly contradictory studies cited in the previous paragraph. And the proof that will be presented here is also for the smoothed data, whereas the proof for the LS approach is still elusive because of the extreme difficulty in trying to get the analytic solutions. But clearly a numerical LS solution in combining the data from uncoordinated satellites would incur the least amount of aliasing. However, even in the absence of such a solution, it is still possible to combine the LS results obtained from individual satellites in the most advantageous way.

Because the proof cannot be developed coherently without a brief description of how the aliasing occurs for the smoothed data of a single satellite, this is what will be outlined first but the details will be left to T06 (see section 8). Furthermore, this paper will provide more detailed explanations where T06 has failed to explain fully.

2. Aliasing for the smoothed data from one exact-repeat satellite

a. The resolving power and aliasing of altimetric exact-repeat sampling

The peculiar sampling patterns of the exact-repeat sampling are characterized by four parameters. Spatially, the sampling is carried out along two sets of parallel tracks (ascending and descending). The east–west (north–south) distance between adjacent parallel tracks is a constant called $X$ ($Y$). Temporally, the entire sampling pattern is repeated after a repeat period called $T$. Additionally, the sampling time difference between adjacent parallel tracks is also a constant called $\tau$. T04 has shown that the resolving power is characterized by a resolved spectral space $R_c$, where the wavenumbers $|k| < k_c = 2\pi/X$ and $|l| < l_c = 2\pi/Y$ and the frequencies $|\omega| < \omega_c = \pi/T$.

T06 has shown that the spatial aliasing occurs in the following manner: a spectral component outside $R_c$, called $\alpha$, may be indistinguishable from up to two spectral components inside $R_c$, called $\beta$ and $\gamma$, along the ascending or descending tracks. For this to occur, $\alpha$ has to reside in special regions in the spectral space. Section 5 (section 8) of T06 gives the LS (smoothing) solutions to the spatial aliasing. The temporal aliasing involves one extra step. First, pure temporal aliasing causes $\alpha$ to appear as $\alpha'$ (because of the constant sampling time difference $\tau$ between adjacent parallel tracks) along the ascending or descending tracks (see section 3 of T06) and $\alpha'$ may or may not be inside $R_c$ already. Then $\alpha'$ may be indistinguishable from up to two spectral components inside $R_c$ along the ascending or descending tracks as well. Again, for this to occur, $\alpha'$ has to reside in special regions in the spectral space. Section 6 (8) of T06 gives the LS (smoothing) solutions to the temporal aliasing.

b. The advantage of the LS approach over the smoothing approach

T06 has made the assertion that the LS approach produces the least aliasing, but the explanation leaves something to be desired. We will use this opportunity to offer a better explanation. In comparing the two approaches, one needs to distinguish two sources of aliasing, namely, the inherent aliasing and the artificial aliasing. By the inherent aliasing, we mean the aliasing that arises from the sampling’s inability to distinguish two spectral components from each other, such as mentioned in section 2a, an outside spectral component $\alpha$ is indistinguishable from up to two inside spectral components $\beta$ and $\gamma$ along the ascending or descending tracks. Thus the inherent aliasing is unavoidable no matter what method is used to process the data. The artificial aliasing, on the other hand, refers to the inefficiencies of the method deployed to treat the data. In the LS approach, all the aliasing arises from the sampling’s inability to distinguish two spectral components from each other. There is no artificial aliasing in the LS approach. However, the smoothing approach induces a lot of artificial aliasing in addition to the inherent aliasing (see elaborations below). Despite its superior quality, it is much harder to apply the LS methodology. In practice, almost all investigations have adopted the smoothing approach.

c. The artificial aliasing induced in the smoothing approach

One would get a better grasp of the situation if we contrast the altimetric sampling with the ideal textbook case in which data are available continuously, thus filtering can be regarded as a convolution operation. Then the Fourier transform of the filtered data is the
multiplication of the Fourier transforms of the data and the filter (e.g., the convolution theorem in Bracqell [1986, 108–112]). However, when a well-defined filter (i.e., one with a known Fourier transform) is applied to the altimetric sampling (i.e., along-track altimeter data repeating at the repeat period), the result is anything but clear-cut. A spectral component when filtered in the ideal textbook case would only have its magnitude changed but would remain at the same frequency and wavenumber. Yet, in the actual case, the filtering causes leakage into many other frequencies and wavenumbers. This leakage is in fact what causes the artificial aliasing even when the optimal smoother is adopted.

The amount of artificial aliasing incurred in the smoothing approach, of course, depends on the exact nature of the smoother that is employed. However, two sources of artificial aliasing can be readily identified. The first and more dominant source is the leakage from the unresolved high-frequency and/or high-wavenumber spectral components. An ideal smoother would smooth away all the high-frequency and/or high-wavenumber spectral components that are not resolved by the sampling (aside from those spectral components that induce the inherent aliasing, which no smoothing can remove), lest their residuals on the regular grid become the artificial aliasing. That is, the smoother should smooth away all of the outside spectral component \( \alpha \) that is distinguishable from all spectral components inside \( R_e \). The residual of \( \alpha \) becomes a combination of many low-frequency and low-wavenumber spectral components. Even though artificial aliasing is unavoidable in the smoothing approach, one should always try to reduce its severity. That is, one should always strive to use a smoother that would be capable of removing the unresolved spectral components if the data were continuous. Failing to do so will cause even more artificial aliasing. The second and lesser source is the leakage from the resolved spectral components into other components in the resolved spectral range.

d. The smoothing mechanism

Notwithstanding the wide variety of smoothers and regular grids being utilized, it is possible to set out the common framework for computing the aliasing when smoothing is adopted, from which one can deduce some common characteristics of the aliasing under smoothing. The smoothing operation (low-pass filtering) can be written as

\[
\hat{h}(x_i, y_j, t_j) = \sum_j A_{ij} h(x_i, y_j, t_j),
\]

where \( \hat{h}(x_i, y_j, t_j) \) are the smoothed data on a regular space–time grid of \((x_i, y_j, t_j); h(x_i, y_j, t_j) \) are the raw data along track at \((x_i, y_j, t_j); A_{ij} \) is the smoothing coefficient; and the summation is effectively only over the raw data inside some sphere of influence around \((x_i, y_j, t_j) \) appropriate for the search radius of the smoother. That is, if a raw data point \( j \) is outside the sphere of influence of the smoothed data point \( i \), then \( A_{ij} = 0 \). Moreover, one can separate the raw data according to ascending or descending data. And (1) can be rewritten as

\[
\hat{h}(x_i, y_j, t_j) = \sum_j (A_a)_{ij} h(x_i, y_j, t_j) + \sum_j (A_d)_{ij} h(x_i, y_j, t_j),
\]

where \((A_a)_{ij} = A_{ij} \) if \( j \) is an ascending point, otherwise \((A_a)_{ij} = 0 \); likewise, \((A_d)_{ij} = A_{ij} \) if \( j \) is a descending point, otherwise \((A_d)_{ij} = 0 \). Symbolically, we can rewrite (1) and (2) as

\[
\hat{h} = A_h = A_a h + A_d h,
\]

where \( \hat{h} \) and \( h \) are the smoothed and raw data vectors, respectively, and \( A, A_a, \) and \( A_d \) are matrices representing the smoothing operation as well as its ascending and descending parts, respectively.

Clearly, the smoothing operation is a linear operator (no matter how \( A \) is computed so long as it does not depend on \( h \) and \( h \)). Thus one is free to consider individual spectral components independently. To be a well-designed low-pass filter, it must pass without distortion the truly large-scale and long-period spectral components. Thus, substituting the spatial and temporal means of \( h \) and \( \hat{h} \), which must be the same for an unbiased smoother, in (1) and (2) leads to the following formula, which is true for all \( i \):

\[
1 = \sum_j A_{ij} = \sum_j (A_a)_{ij} + \sum_j (A_d)_{ij},
\]

(Note that this well-known and seemingly trivial property is crucial to the understanding of the reduction of aliasing when data from two or more uncoordinated satellites are combined.) If the sphere of influence is large enough (coupled with a well-chosen regular grid) to have roughly equivalent distributions of ascending and descending points, then (4) becomes

\[
1/2 \approx \sum_j (A_a)_{ij} \approx \sum_j (A_d)_{ij}.
\]

At the risk of being simplistic, we can characterize the smoother (low-pass filter) by two sets of spectral limits: first, let \( k_o, l_o, \) and \( \omega_o \) be the spectral limits such that higher wavenumber and/or frequency components
(if they are not aliased inside, i.e., if they cannot masquerade as low-wavenumber and low-frequency components along one set of parallel tracks) would be smoothed out by the smoother if data were continuous. Second, let \( k_1, l_1, \) and \( \omega_1 \) be the spectral limits such that the lower wavenumber and frequency spectral components would not be attenuated by the smoother if the data were continuous.

To put these statements into mathematical formulation, let us define two more three-dimensional spectral ranges in addition to \( \mathbf{R}_c \): first, \( \mathbf{R}_0 \) for spectral components with \(|k| < k_o, |l| < l_o, \) and \(|\omega| < \omega_o\); and second, \( \mathbf{R}_1 \) for spectral components with \(|k| < k_1, |l| < l_1, \) and \(|\omega| < \omega_1\). In the following, we will ignore the artificial aliasing for the purpose of deriving the formulas for the inherent aliasing. Let \( \alpha \) be a spectral component outside \( \mathbf{R}_0 \) and let \( \mathbf{h}_\alpha \) be the raw data vector representing \( \alpha \). If \( \alpha \) is distinguishable from all spectral components inside \( \mathbf{R}_0 \), along the ascending (and/or descending) tracks, then

\[
\mathbf{A}_\alpha \mathbf{h}_\alpha = \mathbf{0} \quad \text{(and/or} \quad \mathbf{A}_\alpha \mathbf{h}_\alpha = \mathbf{0}) \tag{6}
\]

where \( \mathbf{0} \) is the null vector on the regular grid. Also, let \( \beta \) denote a spectral component inside \( \mathbf{R}_0 \) and let \( \mathbf{h}_\beta \) and \( \mathbf{h'_\beta} \) denote the data vectors representing the spectral component \( \beta \) on the raw and regular grids, respectively, before smoothing. Then

\[
\mathbf{A}_\beta \mathbf{h}_\beta = \mathbf{h'_\beta}. \tag{7}
\]

Moreover, if (5) is true, then

\[
\mathbf{A}_\alpha \mathbf{h}_\beta \approx \mathbf{A}_\alpha \mathbf{h'_\beta} \approx (1/2)\mathbf{h'_\beta}. \tag{8}
\]

To the extent that (6) and (7) are not true, these are manifestations of the artificial aliasing.

It is clear that \( k_1 < k_o, l_1 < l_o, \) and \( \omega_1 < \omega_o \) (i.e., \( \mathbf{R}_1 \) lies inside \( \mathbf{R}_0 \)). The more one smooths, the smaller \( \mathbf{R}_0 \) and \( \mathbf{R}_1 \) become. It also is clearly desirable to have \( k_o \leq k_c, l_o \leq l_c, \) and \( \omega_o \leq \omega_c \) (i.e., \( \mathbf{R}_o \) should be no larger than \( \mathbf{R}_c \)), lest any remnants from unresolved spectral components linger after smoothing. Because \( \mathbf{R}_o \) also determines the density of the regular grid (note that it makes no sense to have a regular grid whose Nyquist frequency and wavenumber are greater than \( \omega_o, k_o, \) and \( l_o \) when the smoother tries to smooth out any high-frequency and/or wavenumber spectral terms outside \( \mathbf{R}_0 \) in the raw data), the regular grid should be no denser than the regular grid with \( \Delta x = X/2, \Delta y = Y/2, \) and \( \Delta t = T; \) to do otherwise is not only wasting resources but is also misleading by giving the impression that a higher resolution has been achieved.

To compute the inherent aliasing under the smoothing approach, it is simply a matter of finding where \( \alpha \) (or \( \alpha' \)) must reside in the spectral space for \( \alpha \) (or \( \alpha' \)) to be indistinguishable from \( \beta \) (and/or \( \gamma \)) along the ascending (or descending) tracks, which is tabulated in detail in T06. Then the inherent aliasing is described below.

If \( \alpha \) (or \( \alpha' \)) is indistinguishable from \( \beta \) (and/or \( \gamma \)) along the ascending (or descending) tracks, then

\[
\mathbf{A}_\alpha \mathbf{h}_\alpha = \mathbf{A}_\alpha \mathbf{h}_\beta \quad \text{(and/or} \quad \mathbf{A}_\alpha \mathbf{h}_\alpha = \mathbf{A}_\alpha \mathbf{h}_\gamma) \tag{9}
\]

or

\[
\mathbf{A}_\alpha \mathbf{h}_\alpha = \mathbf{A}_\alpha \mathbf{h}_\beta \quad \text{(and/or} \quad \mathbf{A}_\alpha \mathbf{h}_\alpha = \mathbf{A}_\alpha \mathbf{h}_\gamma). \tag{10}
\]

### 3. Smoothing formulas valid for combining two uncoordinated satellites

The formulation in section 2d has its natural and more complicated extension when dealing with two satellites. The satellites most often used for multisatellite studies are the Ocean Topography Experiment [TOPEX/Poseidon (T/P)] and the European Remote Sensing Satellite (ERS). Thus we will use the superscript \( p \) or \( e \) to denote symbols pertaining to T/P and ERS, respectively, even though it could be any two satellites as long as they are uncoordinated.

Now the smoothing operation can be written as (note that the usual practice is to employ a common smoother for both sets of data from the two uncoordinated satellites, but we will develop the formulas allowing for two different smoothers to accommodate arguments in favor of two different smoothers)

\[
\hat{h}(x_i, y_j, t_l) = \sum_j A_{ij}^p \hat{h}_p(x_j, y_j, t_l) + \sum_m A_{im}^e \hat{h}_e(x_m, y_m, t_m), \tag{11}
\]

where \( \hat{h}(x_i, y_j, t_l) \) are the smoothed data on a regular space–time grid of \((x_j, y_j, t_l); \hat{h}_p(x_j, y_j, t_l) \) are the raw T/P data along track at \((x_j, y_j, t_l); \hat{h}_e(x_m, y_m, t_m) \) is the raw ERS data along track at \((x_m, y_m, t_m); A_{ij}^p \) and \( A_{im}^e \) are the smoothing coefficients pertaining to the respective smoothers; and the summation is effectively only over the raw data inside some specified sphere of influence around \((x_i, y_j, t_l) \) for each smoother, respectively. That is, if a raw data point \( i \) or \( m \) is outside the respective sphere of influence around the smoothed data point \( i \), then \( A_{ij}^p = 0 \) or \( A_{im}^e = 0 \). Moreover, one can separate the raw data according to ascending or descending data. And (11) can be rewritten as
\[
\hat{h}(x_i, y_i, t_i) = \sum_j (A^p_{ij}) h(x_i, y_i, t_i) + \sum_j (A^e_{ij}) h(x_i, y_i, t_i) + \sum_m (A_{im}) h(x_i, y_i, t_i),
\]

(12)

where \((A^p_{ij})\) and \((A^e_{ij})\) are the respective ascending and descending parts for T/P and ERS, 
are matrices representing the smoothers as well as their respective ascending and descending parts for T/P and ERS. The necessity for two different smoothers is made clear by the need to eliminate the artificial aliasing as much as possible (discussed at length in section 2). If a common smoother is to be used (the usual practice), it dictates more smoothing than necessary on both datasets in order to smooth out unresolved spectral components in both T/P and ERS; the \(R_e\) for the common smoother has to be no larger than the intersection of \(R^e_p\) and \(R^e_c\) (i.e., the spectral space that is common to \(R^e_p\) and \(R^e_c\)). In other words, the regular grid is to be no denser than the midpoint (i.e., the along-track points that are midway between two crossover points) grid of T/P with a 35-day sampling period. In essence, this common smoother smooths away the higher spatial resolution attained by ERS and the higher temporal resolution attained by T/P. The need for this unwarranted excess smoothing argues against the adoption of a common smoother. Rather, one should use two different smoothers, each best suited for the sampling characteristics of the satellite data it is smoothing, as formulated here.

However, a common regular grid is absolutely necessary. This needs to be dense enough so all possibly resolved spectral components can be adequately represented without man-made aliasing, which occurs when the grid is not dense enough to represent the spectral component. In other words, the common grid for T/P and ERS should have the spatial density of the mid-point grid of ERS with a 10-day sampling period. Contrary to the situation for a single satellite, the adoption of such a dense regular grid is not a waste of resources but is necessary. However, this denser than warranted (in terms of resolving power) regular grid could be quite misleading indeed as we will prove next that the resolving power is not enhanced by combining data from uncoordinated satellites. The necessity for two different smoothers is made clear by the need to eliminate the artificial aliasing as much as possible (discussed at length in section 2). If a common smoother is to be used (the usual practice), it dictates more smoothing than necessary on both datasets in order to smooth out unresolved spectral components in both T/P and ERS; the \(R_e\) for the common smoother has to be no larger than the intersection of \(R^e_p\) and \(R^e_c\) (i.e., the spectral space that is common to \(R^e_p\) and \(R^e_c\)). In other words, the regular grid is to be no denser than the midpoint (i.e., the along-track points that are midway between two crossover points) grid of T/P with a 35-day sampling period. In essence, this common smoother smooths away the higher spatial resolution attained by ERS and the higher temporal resolution attained by T/P. The need for this unwarranted excess smoothing argues against the adoption of a common smoother. Rather, one should use two different smoothers, each best suited for the sampling characteristics of the satellite data it is smoothing, as formulated here.

4. Proof that the combination of datasets from two uncoordinated satellites does not improve the resolving power over either dataset but reduces the overall aliasing

Consistent with the textbook description of Nyquist frequency and wavenumber as well as aliasing for regular grids (see appendix B), the resolving power is defined here as the largest spectral range, within which all spectral components can be distinguished (i.e., re-
solved) from each other by the sampling; in other words, it can be defined as the largest bandwidths within which a band-limited signal can remain free of aliasing. With two smoothers each appropriate for its respective satellite and a denser than warranted regular grid, it is clear from sections 2 and 3 that every source that causes inherent aliasing when each satellite dataset stands alone is still going to cause inherent aliasing when the datasets are combined, but the inherent aliasing occurs at half the amplitude when the two datasets are combined rather than standing alone.

Thus by its strict definition, the resolving power is actually reduced when datasets are combined, not enhanced. However, since the inherent aliasing occurs at a quarter of the energy level as previous, let $P$ or $E$ be the total energy of the inherent aliasing when T/P or ERS stands alone. Then the total energy of the inherent aliasing when they are combined is $P/4 + E/4 = (1/2)(P + E)/2$, or half of the average inherent aliasing level of T/P and ERS. Hence one is clearly better off using the combined datasets.

5. Extension of the results to more than two uncoordinated satellites

The extension to more than two satellites is obvious. Thus if we smooth $N$ uncoordinated satellites with $N$ different smoothers (each appropriate for the sampling characteristics of its intended satellite) onto a common regular grid that uses the densest midpoint grid of the $N$ satellite with a sampling period that is the shortest of the $N$ repeat periods, then the smoothing coefficients for each satellite are reduced to $1/N$ of the values appropriate for the single-satellite case, assuming equal weighting for all satellites. Hence the overall energy level of combined aliasing is also reduced to $1/N$ of the average aliasing level of the individual satellites.

The combined resolving power only resolves the spectral space that is common to all the spectral spaces that are resolved by individual satellites if one sticks to the strict definition of the resolving power. Of course, the resolving power of the combined case has little physical meaning other than to give the largest (which turns out to be the smallest possible) bandwidths within which a band-limited signal can remain aliasing free.

6. The strategy for combining the LS results obtained individually from the uncoordinated satellites

It is clear that the LS approach has many desirable qualities, such as not having any artificial aliasing and having done the spectral analysis already as data are processed (in contrast to the smoothing approach). It turns out that a strategy can be devised to reap the benefits of reducing the aliasing by combining the LS results in spectral terms. Suppose there are $N$ uncoordinated satellites and that the LS results are obtained for each individual satellite. The strategy is to average the spectral results wherever (in the spectral space) there is more than one result. Since each satellite gives a resolved spectral space, the repetition ranges from 1 to $N$. Wherever (in the spectral space) there are $N$ results, the energy level of the combined aliasing is reduced by a factor of $1/N$ (note that the aliasing comes from different places in the spectral space for the $N$ satellites). Thus the benefit ranges from a factor of $1/N$ reduction of the energy level of the combined aliasing in the spectral space that is common to all $N$-resolved spectral spaces (i.e., giving the combined resolving power) to none at places on the spectral space where there is only one LS result.

7. Conclusions

It is better to combine altimetric satellite datasets even when the satellites are uncoordinated because there is a factor of $N$ reduction of the overall inherent aliasing energy level by combining $N$ datasets. By its strict definition, the resolving power of the combined datasets actually is reduced to resolve only the spectral space that is common to all the spectral spaces resolved by individual satellites. However, the combined resolving power in this case is not very meaningful.

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APPENDIX A

Coordinated and Uncoordinated Altimetric Satellites

The exact-repeat sampling of any altimetric satellite is characterized by four constant parameters: $X$, the east–west distance between parallel satellite ground tracks; $Y$, the north–south distance between parallel tracks; $T$, the repeat period, after which the sampling pattern is retraced anew; and $\tau$, the sampling time dif-
ference between adjacent parallel tracks. T04 has shown that this sampling resolves a spectral space delimited by Nyquist wavenumbers of \( k_c = 2\pi/X \) and \( l_c = 2\pi/Y \) and a frequency of \( \omega_c = \pi/T \). T06 has shown how the aliasing occurs and that \( \tau \) is instrumental in the temporal aliasing.

Of course, these four parameters are determined by orbital mechanics. Only a limited number of combinations are possible physically. The term “coordinated satellites” conveys the meaning that these satellites work together to enhance the resolution of the collective sampling. T04 in his section 5, discussed various ways of arranging a constellation of satellites (all sampling with the same sampling parameters of \( X, Y, T, \) and \( \tau \)) to work together to sample with a new set of \( X', Y', T', \) and \( \tau' \) (clearly \( X' \leq X, Y' \leq Y, \) and \( T' \leq T \)), thus appearing as if the collective sampling were carried out by a single satellite with supernatural ability.

An actual physical example of coordination is the Tandem Mission with TOPEX/Poseidon and Jason-I, the combination of which results in \( X' = X/2, Y' = Y/2, \) and \( T' = T \). Even though a constant \( \tau \) could have been arranged, \( \tau' \) for the Tandem Mission is not a constant, which has serious implications for the temporal aliasing (Tai 2008, manuscript submitted to J. Atmos. Oceanic Technol.) that result in the temporal aliasing reverting back to the spatial resolution characterized by \( X \) and \( Y \), rather than by \( X' \) and \( Y' \).

APPENDIX B

The Resolving Power

The concept of the resolving power is so important that T04 (section 2) has devoted a section to explain the meaning of the resolving power. Because of its importance, we will take this opportunity to give it another try.

In information, communication, and interpolation theories, the question has been raised as to how densely a signal has to be sampled in order to represent the signal faithfully. The result is a theorem sometimes called the Nyquist–Shannon sampling theorem or just the sampling theorem [e.g., see the textbook by Bracewell (1986)]. It simply states that the size of the sampling interval gives the greatest signal bandwidth (i.e., the Nyquist frequency or wavenumber) under which the signal can be reproduced faithfully from the sampling; that is, the signal is fully resolved by the sampling. In other words, a band-limited signal with its bandwidth less (greater) than the Nyquist can (cannot) be reproduced exactly (rather, it suffers from aliasing).

The altimetric sampling is obviously not the one-dimensional sampling at constant intervals textbook sampling. However, our definition for the resolving power needs to be consistent with the sampling theorem. The following definition certainly fulfills this requirement. That is, the resolving power of the altimetric aliasing is the largest bandwidth that a band-limited signal can possess without suffering from aliasing.

APPENDIX C

The Smoothed Data Combining T/P and ERS Distributed by Archiving, Validation, and Interpretation of Satellite Oceanographic Data (AVISO)

Le Traon and colleagues have combined the data from T/P (and follow-on satellite) and ERS (and follow-on satellite), smoothing them onto a 1/4° by 1/4° by 10-day regular grid. They have adopted the objective analysis methodology (also known as the objective mapping in oceanography, the least squares collocations in geodesy, or more generally the Gauss–Markov theorem in estimation theory). This dataset has generated a wide following and support among altimeter users, very much deservedly so. However, it does not mean that this dataset cannot be improved upon. It is in this spirit that we raise a few questions about the dataset.

First, does the optimal estimation theory give the best results? The Gauss–Markov theorem treats the entities involved as random variables, then uses the cross correlation of the observed and the estimated as the basis for the optimal estimation. In this case the cross correlation is more or less the autocovariance function of altimetric sea level. The Le Traon approach can be treated as a smoothing operation. That is, the smoothing coefficient used in this paper is the value of the autocovariance function subjected to the normalization constraint expressed in this paper as Eq. (4). In trying to reconcile the sampling theory with the estimation theory, the only connection seems to be that the decorrelation scale is related to the bandwidth. The following example further casts doubts on the meaning of the “best.” From the Nyquist–Shannon sampling theorem, a band-limited signal can be recovered exactly from the sampled values as long as its bandwidth is less the Nyquist [the interpolation formula is given in many textbooks, such as in Bracewell (1986)]. In this case, the estimation theory clearly cannot be used to recover the signal exactly even if given the exact autocorrelation function because there is only one interpolation formula, but by varying the bandwidth, one can get many autocovariance functions. Thus one may have to con-
clude that what is best in the sampling theory may not be the best in the estimation theory.

Second, the autocovariance function of the altimetric sea level (like the spectra of sea level) varies geographically and temporally. Thus the estimation has to be suboptimal. Third, the autocovariance function gives a bandwidth that may or may not be resolved by a satellite. The resulting artificial aliasing as detailed in sections 2 and 3 of this paper argues for the multismoother approach.

REFERENCES