Evaluation of Turbulence Closure Models for Large-Eddy Simulation over Complex Terrain: Flow over Askervein Hill

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ABSTRACT

The evaluation of turbulence closure models for large-eddy simulation (LES) has primarily been performed over flat terrain, where comparisons with theory and observations are simplified. The authors have previously developed improved closure models using explicit filtering and reconstruction, together with a dynamic eddy-viscosity model and a near-wall stress term. This dynamic reconstruction model (DRM) is a mixed model, combining scale-similarity and eddy-viscosity components. The DRM gave improved results over standard eddy-viscosity models for neutral boundary layer flow over flat but rough terrain, yielding the expected logarithmic velocity profiles near the wall. The results from the studies over flat terrain are now extended to flow over full-scale topography. The test case is flow over Askervein Hill, an isolated hill in western Scotland, where a field campaign was conducted in 1983 with the purpose of capturing wind data representing atmospheric episodes under near-neutral stratification and steady wind conditions. This widely studied flow provides a more challenging test case for the new turbulence models because of the sloping terrain and separation in the lee of the hill. Since an LES formulation is used, a number of simulation features are different than those typically used in the Askervein literature. The simulations are inherently unsteady, the inflow conditions are provided by a separate turbulent flow database, and (uniquely herein) ensemble averages of the turbulent flow results are used in comparisons with field data. Results indicate that the DRM can improve the predictions of flow speedup and especially turbulent kinetic energy (TKE) over the hill when compared with the standard TKE-1.5 model. This is the first study, to the authors' knowledge, in which explicit filtering and reconstruction (scale similarity) and dynamic turbulence models have been applied to full-scale simulations of the atmospheric boundary layer over terrain. Simulations with the lowest level of reconstruction are straightforward. Increased levels of reconstruction, however, present difficulties when used with a dynamic eddy-viscosity model. An alternative mixed model is proposed to avoid the complexities associated with the dynamic procedure and to allow higher levels of reconstruction; this mixed model combines a standard TKE-1.5 eddy-viscosity closure with velocity reconstruction to form a simple and efficient turbulence model that gives good results for both mean flow and turbulence over Askervein Hill. The results indicate that significant improvements in LES over complex terrain can be obtained by the use of mixed models that combine scale-similarity and eddy-viscosity components.

1. Introduction

Large-eddy simulations of atmospheric boundary layer flows rely heavily on the quality of the chosen turbulence closure scheme because of limited grid resolution and density-stratification effects. This is especially true for neutrally or stably stratified flows, where the contribution of the turbulence model dominates that of the resolved terms (see the discussions in Sullivan et al. 1994; Kosović 1997). The evaluation of turbulence closure models for large-eddy simulation (LES) has primarily been performed over flat terrain, where comparisons to theory and observations are simplified. In previous work, we examined neutral, rotation-influenced, large-scale boundary layer flow over flat terrain. We introduced a new closure model approach, which was able to bring the simulated flow fields into agreement with theoretical

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expectations in the near-wall region, thus addressing a long-standing difficulty in simulations of the boundary layer (Chow et al. 2005). The current paper extends our analysis to consider neutrally stratified flow over an isolated hill, which is a much more challenging test case because of the effects of terrain. Our explicit filtering and reconstruction turbulence modeling approach (Chow et al. 2005) is called the dynamic reconstruction model (DRM) and uses series expansions for reconstruction of the resolvable subfilter-scale (RSFS) stresses to create a scale-similarity model. This is combined with the dynamic eddy-viscosity model of Wong and Lilly (1994) and a near-wall stress model for the subgrid-scale (SGS) stresses. This is the first study, to our knowledge, in which reconstruction and dynamic turbulence models have been evaluated in full-scale LES of the atmospheric boundary layer over complex terrain.

Several studies have been performed over simple hills to evaluate the performance of different turbulence models; however, most are done at laboratory scales because of the availability of experimental data for comparison (Brown et al. 2001; Allen and Brown 2002; Besio et al. 2003). Such simulations are convenient because they have clearly defined boundary conditions and are generally well-resolved numerically because of the lower Reynolds number conditions. As our interest is in improving the performance of mesoscale atmospheric flow simulations, we have instead chosen to simulate flow over Askervein Hill, a relatively isolated hill located along the west coast of South Uist Island, Scotland. The Askervein Hill project collected velocity and turbulence data that provide a unique dataset for comparison to numerical simulations (Taylor and Teunissen 1987). Similar observational datasets are also available from field campaigns performed at Black Mountain (Bradley 1980), Cinder Cone Butte (Laver et al. 1982; Strimaitis et al. 1982), Blashavall Hill (Mason and King 1985), and Kettles Hill (Salmon et al. 1988), among others. Askervein Hill was selected for this study because turbulence measurements are available for comparison and because this flow has been extensively modeled by other researchers (e.g., Raithby et al. 1987; Kim and Patel 2000; Lopes and Palma 2002; Castro et al. 2003; Undheim et al. 2006; Bechmann 2006; Lopes et al. 2007). The goal of this work is to evaluate the new reconstruction turbulence closure methods previously presented in Chow et al. (2005) for large-eddy simulation of flow over complex terrain.

The Askervein flow dataset has been the object of several recent studies related to wind power prediction. The development of accurate wind energy prediction models for flow over complex terrain has been notoriously difficult as a result of the representation of steep topography, unsteadiness in the flow, poor performance of turbulence models, and lack of adequate field data for validation, among other factors. These same challenges apply to weather prediction over complex terrain. Because Askervein is a relatively isolated hill and the dataset includes steady and neutral flow conditions, the Askervein flow case is commonly setup with semi-idealized conditions. Thus, steady inflow conditions, constant surface roughness, and neutral stability are specified without regard to any other external forcings. Eidsvik (2005) used Askervein flow data to validate a Reynolds-averaged Navier–Stokes (RANS) finite-element code designed for wind power estimation. Prospathopoulos and Voutsinas (2006) used a RANS solver and investigated the sensitivity of the results to surface roughness, grid spacing, and the numerical domain size. Sensitivity of RANS results to these parameters was also studied by Castro et al. (2003). Undheim et al. (2006) studied the effects of grid spacing, incident wind direction, and resolution of topographic data, also with a RANS solver. The steady RANS calculations used in these previous studies could not capture the intermittent separation observed in the field. Castro et al. (2003) were able to capture the intermittency by using unsteady RANS (URANS) with a high-order time-dependent formulation. In all of these studies, general mean characteristics of the flow were well predicted, but most of the previous studies were unable to capture the effect of the hill on observed turbulent kinetic energy, as described further below.

The use of LES over complex terrain has been limited as a result of questions about the performance of turbulence models and the choice of lateral boundary conditions (Moeng et al. 2007). We have previously applied LES and the closure models discussed in this paper in simulations of highly complex flow in the Riviera Valley, Switzerland, but that work did not focus on the performance of the turbulence models. Thus, the semi-idealized nature of the Askervein flow case is attractive for our purposes. Some results from our LES study were first presented by Chow and Street (2004). Lopes et al. (2007) have also investigated flow over Askervein Hill using a hybrid RANS–LES model, though their focus was on grid aspect ratio and effects on recirculation in the lee of the hill. The results from our semi-idealized investigation will be applicable to high-resolution mesoscale weather prediction, wind power prediction, and other flow simulations over complex terrain.

Because we use an LES formulation, a number of features in our flow setup are different than those typically used in previous work and thus present new contributions to both the Askervein Hill literature and
toward the extension of LES to complex terrain. Namely, our simulations are inherently unsteady, and the inflow conditions are provided by a separate turbulent flow database (see also Lopes et al. 2007); we use explicit filtering, and the turbulence models do not exclusively rely on an eddy-viscosity assumption; and ensemble averages are used to compare with observation data. These model features are described in further detail below. Last, an alternative mixed turbulence model is proposed, which is simple to implement yet preserves the required features and benefits of our explicit filtering and reconstruction approach.

2. Dynamic reconstruction turbulence closure approach

LES separates resolved, larger-scale motions from smaller, subfilter-scale (SFS) turbulent motions using a physical length scale, the width of the explicit spatial filter. The large scales are directly simulated, while the effect of subfilter scales on these large resolved scales must be modeled. The presence of a numerical grid divides the SFS motions into resolved and unresolved portions. The RSFS motions can be reconstructed using a scale-similarity approach, while the unresolved subfilter-scale motions (also called SGS) must be modeled [see Gullbrand and Chow (2003) and Chow et al. (2005) for more details]. Both the RSFS and SGS models must be based on knowledge of the resolved-scale behavior alone.

Reconstruction modeling of the RSFS stresses requires the definition and application of an explicit filter in the LES computation, as described below. In contrast, traditional LES treats the discretization on the grid as an implicit filter operation, but the nature of the filter is unknown and different for each term in the equations, making reconstruction difficult. Standard closures for atmospheric flows use eddy-viscosity models and ignore the interaction of filtering and discretization operators. Furthermore, at large scales, the subfilter scales are probably not isotropic, as assumed in eddy-viscosity turbulence models. A well-known problem in traditional simulations of the atmospheric boundary layer is the lack of agreement with similarity theory in the near-wall region. Use of explicit filtering and reconstruction, combined with dynamic closure and near-wall stress models, led to much-improved agreement of LES results and similarity theory in neutral boundary layer flow (Chow et al. 2005). Explicit filtering and reconstruction are especially useful for reducing numerical errors in the context of finite volume or finite difference codes; spectral methods do not require explicit filtering and reconstruction (Winckelmans et al. 2001) but are not suitable for flows over complex geometries.

The LES governing equations on a discrete grid can be obtained by applying an explicit filter (overbar) and a discretization operator (tilde) to the Navier–Stokes and continuity equations, here in compressible form:

\[
\frac{\partial \bar{\rho} \bar{u}_i}{\partial t} + \frac{\partial \bar{\rho} \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} - \rho g \delta_{ij} + \rho c_{mf} \bar{u}_m - \frac{\partial \bar{\rho} \bar{\tau}_{ij}}{\partial x_j} \tag{1}
\]

\[
\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho} \bar{u}_j}{\partial x_j} = 0. \tag{2}
\]

Here, \(\bar{u}_i\) are the resolved velocity components, \(\bar{p}\) is the pressure, \(\bar{\rho}\) is the density, and \(f\) is the Coriolis parameter. The filtering and discretization operators are applied as Favre filters to separate the density and velocity fields, and viscous terms are neglected. While the discretization effects are different for every term in the equation (as a result of the various finite-difference schemes used), the same explicit filter (a top-hat filter of width \(\Delta\) that is twice the grid spacing) is applied to all variables. It is assumed that the filtering and discretization operations commute with the spatial derivatives, which is true for spatially homogeneous filters and holds approximately for small (<10%) grid stretching ratios (Ghosal and Moin 1995).

We define the total SFS stress as

\[
\tau_{ij} = \bar{u}_i \bar{u}_j - \bar{\bar{u}}_i \bar{u}_j \tag{3}
\]

and decompose this into resolvable (RSFS) and unresolved (SGS) portions:

\[
\tau_{SFS} = \tau_{ij} = (\bar{u}_i \bar{u}_j - \bar{\bar{u}}_i \bar{u}_j) + \left( \tau_{RSFS} - \tau_{SGS} \right). \tag{4}
\]

The partitioning of SFS motions into RSFS and SGS portions facilitates an understanding of the roles of various turbulence model components. The first pair of terms on the right-hand side is the subgrid-scale stresses \(\tau_{SGS}\). They depend on scales beyond the resolution domain of the LES and contain the unclosed nonlinear term \(\bar{u}_i \bar{u}_j\), which must be modeled. The last pair of terms is the filtered-scale stresses \(\tau_{RSFS}\), which depend on the differences between the exact and filtered velocity fields within the resolution domain. This resolvable subfilter-scale component \(\tau_{RSFS}\) can theoretically be reconstructed because it is a function of \(\bar{\bar{u}}_i\), which can be obtained by deconvolution (inverse filtering). The reconstruction terms allow backscatter of energy from the subfilter to the resolved scales (see Zang et al. 1993).

Reconstruction using the van Cittert iterative series expansion method [from the approximate deconvolution
method (ADM) of Stolz et al. (2001) [provides an estimate \( \overline{\mathbf{u}}_i \) of the unfiltered velocity \( \overline{\mathbf{u}}_i \) in terms of the filtered velocity \( \overline{\mathbf{u}}_i \); this is used to calculate the SGS stresses. The unfiltered quantities can be derived by a series of successive filtering operations \( G \) applied to the filtered quantities with

\[
\overline{\mathbf{u}}_i = \overline{\mathbf{u}}_i + (I - G)\overline{\mathbf{u}}_i + (I - G)[(I - G)\overline{\mathbf{u}}_i] + \cdots, \tag{5}
\]

where \( I \) is the identity operator and \( G \) is the explicit filter. Thus

\[
\tau_{\text{RSFS}} = \overline{\mathbf{u}}_i^2 \overline{\mathbf{u}}_j - \overline{\mathbf{u}}_i \overline{\mathbf{u}}_j, \tag{6}
\]

where \( \overline{\mathbf{u}}_i^2 \) represents the truncated series expansion in Eq. (5). The SGS contribution is provided by an eddy viscosity model,

\[
\tau_{\text{SGS}} = -2\nu_T \overline{S}_{ij}, \tag{7}
\]

where \( \nu_T \) is the eddy viscosity and \( \overline{S}_{ij} = (1/2)(\overline{\partial u_i/\partial x_j + \overline{\partial u_j/\partial x_i}}) \) is the resolved strain rate tensor. Here the dynamic Wong–Lilly (DWL) model (Wong and Lilly 1994) is used to obtain the eddy viscosity. The DWL is written as

\[
\tau_{\text{SGS}} = -2C_r \Delta^{4/3} \overline{S}_{ij}, \tag{8}
\]

where \( \Delta \) is the explicit filter width. The coefficient \( C_r \) is determined dynamically by applying a local test filter at twice the explicit filter width [details are given in Chow et al. (2005)]. Because the dynamic procedure tends to underestimate the eddy viscosity in the near-wall region, a near-wall stress model, based on the canopy-stress model of Brown et al. (2001), is used to augment the SGS stress provided by the DWL model. The near-wall stress can be represented by integrating a drag forcing term to provide extra stress at the few grid points near the ground surface:

\[
\tau_{i, \text{near-wall}} = -\int C_r a(z) \overline{u_i} \overline{u_i} dz. \tag{9}
\]

Here, \( C_r \) is a scaling factor and the function \( a(z) \) allows for a smooth decay of the forcing function as a specified cutoff height \( h_c \) is approached. The function \( a(z) \) is set equal to \( \cos^2(\pi z/2h_c) \) for \( z < h_c \), and is zero otherwise. The integration constants are chosen so that \( \tau_{i, \text{near-wall}} = 0 \) at the top of the enhanced stress layer. The recent work of Brown et al. (2001), Nakayama et al. (2004), Cederwall (2001), and our successful previous simulations of the neutral boundary layer over flat terrain (Chow et al. 2005) suggest that this form is both physically and mathematically reasonable. All the simulations using DRM and DWL include the near-wall stress model, with a proportionality factor, \( C_r = 0.5 \), and a layer height, \( h_c = 4\Delta x = 140 \text{ m} \), for our grid. This is equivalent to the minimum well-resolved horizontal eddy size beneath the filter width. With \( C_r = 0.5 \), the near-wall stress model contributes an amount equal to half the wall stress at the first grid point above the wall.

The near-wall stress is directly added to the \( \tau_{i, \text{SGS}} \) SGS terms from the DWL contribution. Thus the complete DRM (Chow et al. 2005) is a mixed model for the total SFS stress consisting of the RSFS and SGS components, which are scale-similarity and eddy-viscosity terms, respectively, with near-wall enhancement:

\[
\tau_{ij} = \left(\overline{\mathbf{u}}_i^2 \overline{\mathbf{u}}_j - \overline{\mathbf{u}}_i \overline{\mathbf{u}}_j \right) - 2C_r \Delta^{4/3} \overline{S}_{ij} + \tau_{i, \text{near-wall}}. \tag{10}
\]

The level of reconstruction \( n \) is determined by the number of terms \( (n + 1) \) in the series expansion in Eq. (5); for example, level 0 reconstruction includes one term in the series and is denoted as DRM–ADM0. Level 0 reconstruction is similar to the scale-similarity model of Bardina et al. (1983). The DWL can also be used alone, in which case the contribution of the RSFS terms is ignored. As discussed in section 5, reconstruction of levels greater than zero leads to terrain-induced instabilities. Modifications are proposed to allow for higher levels of reconstruction.

3. Model setup

Our simulations use the Advanced Regional Prediction System (ARPS), developed at the Center for Analysis and Prediction of Storms at the University of Oklahoma. Intended mainly for mesoscale and small-scale atmospheric simulations, ARPS is formulated as an LES code and solves the three-dimensional compressible, nonhydrostatic-filtered Navier–Stokes equations. Details can be found in Xue et al. (2000, 2001, 2003), so we only mention the relevant settings for this application.

Fourth-order spatial differencing is used for the advection terms. Temporal discretization is performed using a mode-splitting technique to accommodate high-frequency acoustic waves. The large time steps \( (\Delta t) \) use the leapfrog method. First-order forward–backward explicit time stepping is used for the small time steps \( (\Delta t) \), with terms responsible for vertical acoustic propagation treated implicitly. Simulations were performed in parallel [using a message passing interface (MPI) library] on IBM SP Power5 processors. Moist processes were not activated for these simulations.
a. Grid setup and surface boundary conditions

Topographic data for Askervein were provided by Walmsley and Taylor (1996) at approximately 25-m horizontal resolution. Elevation contours are shown in Fig. 1. The grid was centered at $57^\circ 8^\prime 11^\prime 9^\prime N$, $2^\circ 7^\prime 8^\prime 22^\prime W$ and was rotated 60° clockwise to align the $x$ axis parallel to the incoming 210° winds. Elevations were interpolated to $\Delta x = \Delta y = 35$-m horizontal resolution using $nx = ny = 163$ grid points in each horizontal direction to cover a 5600-m square domain (ghost points are included in the total number of grid points; domain size in the $x$ direction, for example, is calculated with $(nx - 3)\Delta x = 5600$ m). In the vertical, 59 points are used; the minimum grid spacing is 5 m at the ground surface and is stretched using a hyperbolic tangent function to yield an averaging spacing of 12.5 m over the 700-m vertical extent of the domain.

Simulations were also performed with a 1000-m domain height but with very little difference in the results, so they are not shown. The vertical resolution is intentionally coarser than most recent Askervein studies (e.g., Undheim et al. 2006; Lopes et al. 2007), because finer resolution is not practical for real atmospheric flows over complex terrain. It is also important to maintain an aspect ratio of horizontal to vertical grid spacing that is suitable for LES (see Chow et al. 2005). The grid is staggered in all three directions, with velocities defined at cell faces and scalar quantities at cell centers. The large time step is 0.25 s and the small time step is 0.025 s for all simulations. We use a roughness value of $Z_0 = 0.03$ m and apply a log-law bottom boundary condition (by specifying the surface stress), as done by Raithby et al. (1987) and Castro et al. (2003). A land surface model is not used, so there are no heat or moisture fluxes at the surface.

b. Observation data

Following the examples given by Raithby et al. (1987) and Castro et al. (2003), our simulation results are compared with field measurement datasets listed in Table 1 collected between 1200 and 1700 (British summertime = UTC + 1 h) on 3 October 1983 and reported by Taylor and Teunissen (1985). These observation periods had Richardson numbers between $-0.0038$ and $-0.011$ (very slightly unstable); therefore, the atmosphere can be considered approximately neutrally stratified. The moderate-to-strong winds were from the southwest ($210^\circ$ clockwise from north) during this period. A long rain shower occurred earlier in the morning, and low clouds were present at approximately 300 m above ground level (AGL; at less than 300 m AGL over the hills). This perhaps indicates the presence of a stable layer at about 300 m. The observed mean flow data were averaged in time over 10 min and turbulence data were calculated over 30 min (Taylor and Teunissen 1987). Intermittent flow separation was observed in the lee of the hill. Given the relatively constant wind speed and direction during these observation periods, simulations in this work are compared to various quantities in this dataset without reference to the specific dataset name.

c. Turbulence model options

Reference simulations are performed using the standard 1.5-order turbulent kinetic energy (TKE) closure (Deardorff 1980; Moeng 1984) in ARPS. The TKE-1.5 model solves an equation for the TKE to determine the velocity scale for use in an LES-type eddy-viscosity formulation. The model can be used for LES as long as the chosen length scale is proportional to the filter width, as is done in ARPS (Deardorff 1980; Moeng 1984). The TKE-1.5 results are compared with results from the DWL and the DRM (with various reconstruction levels) described above.

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**Fig. 1.** Elevation contours of topography (m) for Askervein Hill simulations, rotated 60° clockwise from north. Contour interval is 12 m. The RS is located on line A at the left edge of the domain.

**Table 1.** Observation datasets used for comparison with simulations, with reference values measured at RS. All data are from 3 Oct 1983. Richardson number and British summer time are noted as Ri and BST, respectively.

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Time (BST)</th>
<th>Mean wind (m s$^{-1}$)/direction</th>
<th>Ri</th>
</tr>
</thead>
<tbody>
<tr>
<td>MF03-D</td>
<td>1400–1700</td>
<td>8.9/210</td>
<td>−0.0110</td>
</tr>
<tr>
<td>TU03-A</td>
<td>1200–1300</td>
<td>9.8/210</td>
<td>−0.0038</td>
</tr>
<tr>
<td>TU03-B</td>
<td>1400–1700</td>
<td>8.9/210</td>
<td>−0.0074</td>
</tr>
<tr>
<td>TK03</td>
<td>1500–1700</td>
<td>—/205</td>
<td>−0.0040</td>
</tr>
</tbody>
</table>
As discussed below, reconstruction of levels greater than zero (i.e., more than the Bardina term) leads to terrain-induced instabilities. Modifications to the SGS model component are proposed to allow for higher levels of reconstruction. Namely, the TKE-1.5 model is partnered with the RSFS terms to produce a relatively simple (and effective) model, which includes higher levels of reconstruction and hence transfer of energy between the resolved and subfilter scales (see Zang et al. 1993). A summary of the simulations performed is given in Table 2.

In addition to the turbulence model, fourth-order computational mixing is used to damp high-frequency motions that can build up as a result of nonlinear interactions; this can be considered a type of hyper-viscosity and is applied in computational space. ARPS also includes a divergence damping term to control acoustic noise. The effect of both of these damping terms has been investigated, and the coefficients have been set to give the minimum amount of mixing required for stability (Chow 2004).

d. Lateral boundary conditions and turbulence database

To provide a realistic turbulent inflow, a separate neutral boundary layer simulation with periodic boundary conditions, geostrophic forcing, and flat terrain is performed, and data are extracted from a slice in the domain at every time step. This “turbulence database” is obtained from simulations similar to those performed by Chow et al. (2005). Here we use the DWL closure, which provides a good representation of the logarithmic velocity profile expected in a neutral boundary layer. The grid size for this periodic case is (83, 163, 59) with 35-m horizontal and 12.5-m average vertical spacing, and 5-m minimum vertical resolution, covering a $2800 \times 5600 \times 700 \text{ m}^3$ domain. The time steps are the same as the Askervein Hill simulations. The reference elevation for ARPS is set to 10 m MSL, so that the terrain and hence the pressure match those at the inflow to the Askervein domain. This turbulent dataset is then used to specify the inflow velocity at every time step on the western side of the Askervein domain [similar to Lund et al. (1998), Yuan et al. (1999), and Lopes et al. (2007)].

The flow throughout the domain is thus fully turbulent. In contrast, if the flow is driven by constant inflow boundary conditions with random perturbations (as typically done in the literature), then the flow does not become fully turbulent over the short length of the domain. Thus simulations using a constant inflow did not give satisfactory results with an LES formulation.

The north and south boundaries are set to be solid free-slip walls. At the east boundary, zero-gradient conditions are applied. The initial conditions are set to a constant logarithmic profile and neutral stratification. Severe oscillations were initially observed when the turbulent inflow data were imposed, because disturbances at the boundary propagated quickly through the pressure field into the initially uniform flow fields. This was corrected by using the pressure detrending option in ARPS, which sets the domain-wide mean perturbation Exner function to zero to control pressure drift (usually as a result of boundary condition effects). The effects of the detrending on the flow solution are small; the magnitude of the pressure appears only in the relatively small pressure perturbation contribution to the buoyancy term (Klemp and Wilhelmson 1978; Xue et al. 1995).

4. Comparison with observations

a. Ensemble simulations and inflow conditions

While not previously well-explored for LES, an ensemble framework has the potential to improve statistical comparisons with observations, because the fluctuating turbulent motions represented by LES should not be treated deterministically. Thus, LES of atmospheric flows is better suited to an ensemble approach in which several simulations are performed with slight perturbations to obtain a collection of simulated fields, which provide higher skill in their average result (Mass 2003; Hamill et al. 2000). The spread of the ensemble results indicates the uncertainty in the mean and can provide probabilistic information on expected forecast conditions if the spread in ensemble members is adequate. According to Nutter et al. (2004), perturbations of

<table>
<thead>
<tr>
<th>Run name</th>
<th>SGS model</th>
<th>RSFS reconstruction level</th>
<th>Explicit filtering</th>
<th>Near-wall stress model</th>
</tr>
</thead>
<tbody>
<tr>
<td>TKE-1.5</td>
<td>TKE 1.5-order</td>
<td>—</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>DWL</td>
<td>Dynamic Wong–Lilly</td>
<td>—</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>DRM–ADM0</td>
<td>Dynamic Wong–Lilly</td>
<td>0</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>TKE–ADM0</td>
<td>TKE 1.5-order</td>
<td>0</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>TKE–ADM1</td>
<td>TKE 1.5-order</td>
<td>1</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>TKE–ADM2</td>
<td>TKE 1.5-order</td>
<td>2</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>
the lateral boundary conditions are a very efficient way to produce large enough spread in the ensemble group.

The turbulence inflow data is by construction fluctuating and time dependent, as it results from a separate LES computation (see previous section). The driving pressure gradients for the inflow database were selected such that the time-averaged inflow data correspond well with observations in the field. There are, however, larger-scale time fluctuations that persist, meaning that inflow data extracted at different time intervals will give different mean profiles, though still generally in good agreement with the observations. Fortunately, the time-dependent nature of the inflow data is consistent with the time-dependent flow observed in the field. In our simulations, we take advantage of the time dependence of our inflow conditions to generate an ensemble of simulation results. Thus we perform three runs using different sets of inflow data. In each case, the run is initialized at different points in time and allowed to spin up for 2700 s. Data are averaged over an additional 900 s (from 2700 to 3600 s run time) for both mean velocity and turbulence statistics. Data are collected at 30-s intervals during this averaging period. Results using four ensemble members were very similar and therefore not worth the extra computational cost for all comparison runs. The simulated wind profiles at the reference site (RS; see Fig. 1) from each ensemble member are plotted in Fig. 2; the velocities agree well with observations and are on average logarithmic, but they certainly fluctuate from that instantaneously. Figure 3 shows time–height contours of the inflow data at point RS (with 30-s resolution); each subplot indicates the data used to drive each of the three ensemble member simulations during the 900-s averaging period for statistics. The time-averaged profile at RS from each set was shown in Fig. 2. The variability of each ensemble member can be seen; the effects of this variability on other flow properties are described below. Note that multimodel ensembles are often performed, where simulations with different turbulence models (in this case) would be used to obtain an ensemble result; this may be useful in an operational setting, but it is not performed here because our focus is on differences between the turbulence models.

b. Mean winds

Observations along lines A and AA (43°, northeast–southwest) and along line B (133°, southeast–northwest) in Fig. 1 are compared with the corresponding ensemble-averaged quantities from the three-dimensional simulated velocity fields. The interpolation schemes in the ARPS postprocessing plotting software were used to extract the simulation outputs along these cross sections. Figure 4 shows the observed and simulated wind speedup ratio at 10 m above the ground along line A. Observation data are not available more than 400 (line A) or 600 m (line A–A) downwind of the hill top (HT). The fractional wind speedup ratio provides the most straightforward comparison of the various model results and is defined as

$$\Delta S = \frac{S(z) - S_{RS}(z)}{S_{RS}(z)},$$

where $S$ is the horizontal wind speed and $S_{RS}$ is at the reference site. The speedup is a useful nondimensional measure often used in wind engineering for siting of wind turbines and thus has become a standard metric for the Askervein Hill dataset. All the turbulence models slightly underpredict the speedup at the hill top, with the TKE-1.5 results better than the rest. The greatest difference among the models is, however, in the lee of the hill, where intermittent separation was observed in the field (Raithby et al. 1987) and found in unsteady simulations of Castro et al. (2003), Chow and Street (2004), and Lopes et al. (2007). The TKE-1.5 model fails to produce the observed flow deceleration, whereas the DWL and particularly the DRM–ADM0 results are much better. Results are comparable to the hybrid LES–RANS results of Lopes et al. (2007) and the LES results of Bechmann (2006), both of which studies used significantly higher grid resolution than used here. Mean absolute and root-mean-square errors (RMSE) for the speedup along line A relative to observations are given in Table 3. The variability in the results from different ensemble members is shown for three sets of TKE-1.5 and DRM–ADM0 each, together with their respective ensemble averages in Figs. 5 and 6.
Similar speedup results are found along line A–A (Fig. 7), though with less variability among model results, as observed in previous studies (e.g., Castro et al. 2003). The wind direction deviation from $210^\circ$, $\Delta \phi$, is shown in Fig. 8. None of the models completely agrees with the observed wind directions, but the DWL and DRM–ADM0 results again show improvement in the deviation observed in the lee of the hill. Wind speedup comparisons along line B shown in Fig. 9 also show generally good agreement.

Vertical profiles of the wind speedup ratio are shown at the HT in Fig. 10. The speedup ratio at the hill top is underestimated (as seen in Fig. 4). The speedup profile is well reproduced by all the turbulence models, with the shape slightly better represented by the dynamic models.

Figures 11 and 12 show instantaneous vertical cross sections from one realization of the DRM–ADM0 simulations (set 3) of the flow over Askervein to illustrate the intermittent separation observed. In Fig. 11a a “gust” event is visible as the winds sweep down the lee side of the hill. This contrasts with Fig. 12, where a separated flow region is observed in the lee of the hill. Intermittent separation is a challenge for numerical simulations, which are particularly sensitive to the formulations chosen for the wall model and boundary conditions. The recirculation contributes to the strong deceleration observed on average in the wind speedup curves (Fig. 4).

Clearly, accurate prediction of the intermittent separation is related to the ability to predict the wind speedup. The TKE-1.5 results did not exhibit these recirculation patterns (not shown), so the speedup ratio is overpredicted on the lee side of the hill (see Fig. 4).

c. Turbulence

Comparing turbulent quantities from LES and from observations in the field can be complicated because of
the different space and time averaging techniques used. The representation in LES is by definition filtered in space, at least over the dimensions of the grid cell. The measurements in the field are obtained at one specific location and averaged over time. We define the spatially filtered Reynolds stress tensor

$$\overline{\tau}_{ij} = \overline{\tau}_{ij} + \langle \tau_{ij} \rangle$$

(12)

to calculate the normal stresses and shear stresses, consisting of the familiar resolved ($c_{ij} = \overline{\tau}_{ij} - \langle \tau_{ij} \rangle$) plus subfilter ($\tau_{ij}$) contributions. For computational efficiency and data storage reasons, time averages (denoted by $\langle \rangle$) are performed over 15 min (900 s) using LES data at 30-s intervals. Observation data are averaged over 30 min. No significant difference was found when data were extracted at finer intervals or when longer time averaging periods were used.

Figure 13 compares computed and observed normalized TKE along line A. The TKE normalized by $S_{RS}^2$ is also known as the turbulence intensity, which we denote as $k^*$. The predictions from the DRM–ADM0 and DWL are clearly superior to the TKE-1.5 results and likewise much better than results from most previous RANS studies (however, see Castro et al. 2003). The TKE results are comparable to the higher-resolution hybrid LES–RANS results of Lopes et al. (2007) and LES results of Bechmann (2006). Mean absolute and root-mean-square errors for speedup and turbulence intensity from our results are given in Table 3.

Figures 14 and 15 compare simulated $uw$ and $vw$ stresses with observations. The stresses have been rotated to be aligned with line A. The $vw$ stress comparisons are quite good, but significant differences are observed in the $uw$ plots. A similar discrepancy between model results and observations was found by Lopes et al. (2007), who noted that the observed values of the $uw$ stress were unusually low given the relative values of

<table>
<thead>
<tr>
<th>Run name</th>
<th>$\Delta S$ MAE</th>
<th>$\Delta S$ RMSE</th>
<th>$k^*$ MAE</th>
<th>$k^*$ RMSE</th>
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<tr>
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<td>0.0098</td>
</tr>
<tr>
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<td>0.0069</td>
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<tr>
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<td>0.0958</td>
<td>0.0065</td>
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<tr>
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<td>0.0840</td>
<td>0.0109</td>
<td>0.0150</td>
</tr>
</tbody>
</table>
the other stress components. We note that in our results, the contribution of the subfilter-scale stresses is larger when explicit filtering and reconstruction is used; this is consistent with the results from flow over flat terrain, where the SFS stresses increased with increasing reconstruction and the resolved stresses decreased accordingly (Chow et al. 2005).

5. Performance of the dynamic reconstruction models
The above results for wind speedup and turbulent quantities indicate quite good overall agreement between the observations and the simulations using DWL and especially DRM–ADM0. Attempts to directly increase the level of reconstruction were, however, unsuccessful. Using DRM–ADM1 resulted in instabilities that could only be controlled by increasing the fourth-order computational mixing. This had a strong effect on the velocity profiles near the wall, where gradients are largest; velocities slowed down significantly and wind speedup predictions deteriorated. Increasing the strength of the near-wall stress contribution did not improve stability either.

The performance of the dynamic reconstruction model is very sensitive to the calculation of the dynamic coefficient in the Wong–Lilly model. An indication that the dynamic model struggles with flow over terrain can be seen in Fig. 16, showing contours of the dynamic eddy viscosity. Very near the wall \((k = 1, \text{the first grid level above the wall})\), the dynamic coefficient often becomes locally negative, so there is a considerable amount of clipping applied to reset large negative eddy viscosity to zero for stability reasons (see Chow et al. 2005). Further from the wall \((k = 10, 10 \text{ grid levels above the wall})\), the percentage of clipping required is much smaller. Tests of the DWL alone over very complex terrain [Chow (2004), see chapter 8 on simulations in the Riviera Valley] required very large amounts of clipping and ultimately resulted in instabilities. A local test filter is used in our work to determine the dynamic coefficient; the Lagrangian-averaging approach of Meneveau et al. (1996) seems promising for flow over complex terrain and will be pursued in future work.

Iizuka and Kondo (2004) also had difficulty with a dynamic model in simulations over a 2D laboratory-scale hill, where the dynamic Smagorinsky model failed to reproduce the expected recirculation patterns in the lee of the hill. The authors cited the dynamic model’s
underestimation of the eddy viscosity very near the wall as a key reason for the poor performance of the model over terrain. Given that our full-scale hill terrain is neither smooth nor two-dimensional, it is not surprising that we experience further difficulties with the dynamic model. Iizuka and Kondo (2004) proposed a hybrid dynamic–static Smagorinsky model, which uses eddy viscosities from the standard static Smagorinsky model at points near the wall where the eddy viscosity is underpredicted. This hybrid approach augmented the eddy viscosity near the wall and allowed the expected recirculation patterns to form in the lee of their hill. We experimented with this type of hybrid approach but did not find its performance satisfactory in comparisons with the observed mean speedup profiles (Chow and Street 2004).

6. An alternative approach: TKE-1.5 with explicit filtering and reconstruction

An alternative approach is to combine the RSFS model with a different SGS model. This preserves the mathematically consistent explicit filtering framework that separately accounts for the effects of grid discretization and the LES spatial filter, thus limiting the effects of discretization errors. We propose a mixed model that combines the standard TKE-1.5 closure with the ADM reconstruction model for the RSFS stresses:

$$\tau_{ij} = \left( \frac{\bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j}{\tau_{RSFS}^{\tau_{SGS}}} \right) - 2 \nu T \hat{S}_{ij}.$$  \hspace{1cm} (13)
Here, $\nu_T$ is obtained by solving the TKE equation to obtain a characteristic velocity scale, with the turbulent length scale chosen proportional to the filter width. A near-wall stress model is not used as part of the SGS closure, because the TKE-1.5 eddy-viscosity closure is not dynamic and hence not subject to the same difficulties experienced with the dynamic Wong–Lilly model above. Furthermore, the TKE-1.5 model when used by itself tends to be overly dissipative near the ground, so augmentation of the stress is not desirable (see discussion in Chow et al. 2005). Attractive features of this mixed model include its ease of implementation and the use of a scale-similarity model to allow for backscatter and to represent the resolvable SFS stresses. Many atmospheric mesoscale models include a TKE-1.5 closure formulation, which can be easily augmented with the RSFS component simply by defining an explicit filter and implementing the ADM procedure. The mixed model in Eq. (13) is similar to the mixed model originally proposed by Bardina et al. (1983), who combined the lowest-order scale-similarity component with the static Smagorinsky model. Here, we use the TKE-1.5 eddy viscosity component, and we allow for increased levels of reconstruction in the RSFS stress term.

In Fig. 17 we show results from the combined TKE–ADM0 model, which is the TKE-1.5 eddy-viscosity model coupled with ADM0. Increased levels of reconstruction, TKE–ADM1 and TKE–ADM2 are also shown. The TKE–ADM mixed model is able to capture the flow separation and decrease in speedup in the lee of the hill. In addition, it demonstrates good agreement with the turbulence intensity profiles along line A, as seen in Fig. 18. The magnitude of the turbulence intensity predicted in the lee of the hill increases when the reconstruction level increases, but the agreement with the observations is good for all cases shown in Fig. 18. Comparisons of TKE–ADM0 with the standard TKE-1.5 closure and the DRM–ADM0 model are shown for the mean speedup and the turbulence intensity profiles in Figs. 19 and 20. Table 3 shows mean absolute and root-mean-square errors for the TKE–ADM results relative to observations in comparison with those from the TKE–1.5, DWL, and DRM–ADM0 models. These comparisons indicate that the TKE–ADM0 model may be a practical alternative mixed model to use within the explicit filtering and reconstruction RSFS–SGS framework, as the results are considerably better than the standard TKE-1.5 closure. TKE–ADM0 uses the lowest level of reconstruction and thus is easiest to implement and has the lowest computational cost.

Fig. 13. Comparisons of observed turbulence intensity along line A with simulated values using various turbulence closures for three-member ensembles.

Fig. 14. Comparisons of observed $uw$ stress (rotated coordinates) along line A with simulated values using various closures for three-member ensembles.

Fig. 15. As in Fig. 14 but for observed $uw$ stress.
7. Conclusions

Large-eddy simulations of flow over Askervein Hill, an isolated hill in Scotland, were compared with the field observations of Taylor and Teunissen (1987). This flow is a challenging test for the dynamic reconstruction turbulence models, which gave improved results for neutral boundary layer flow over flat terrain in our previous work (Chow et al. 2005). This paper extends our previous work (Chow and Street 2004) and is the first application, to our knowledge, in which reconstruction (scale similarity) and dynamic turbulence models have been studied in full-scale simulations of the atmospheric boundary layer over terrain. While use of a dynamic model sidesteps debate about appropriate coefficient values, such as those needed for the TKE equation (see e.g., Deardorff 1971; Takemi and Rotunno 2003), an alternative mixed model based on a

FIG. 16. Instantaneous contours of eddy viscosity $\nu_T$ (m$^2$ s$^{-1}$) after clipping is applied for DRM–ADM0 at (a) $k = 1$ and (b) $k = 10$. Clipping is indicated by the enclosed white regions.

FIG. 17. Comparisons of observed velocity speedup along line A with simulated values using various turbulence closures involving TKE-1.5 with and without reconstruction for three-member ensembles. The profile of the hill is shown at the bottom of the figure.

FIG. 18. As in Fig. 17, but for turbulence intensity.

FIG. 19. As in Fig. 17 but for TKE-1.5, TKE–ADM0, and DRM–ADM0 closures.
TKE-1.5 closure is also proposed. This work introduces new features for LES applications to atmospheric boundary layer flows, namely, the use of three-member ensemble-averaged simulations and explicit filtering and reconstruction turbulence models that avoid the problems associated with pure eddy-viscosity closures. With LES, our study considers inherently unsteady turbulent flow, which allows intermittent separation in the lee of the hill, as observed in the field [see also the unsteady RANS results of Castro et al. (2003) and the LES of Lopes et al. (2007)].

Simulations with the dynamic reconstruction model at the lowest level of reconstruction (DRM–ADM0) showed improvement for wind speedup ratios over the hill, particularly in the lee of the hill, when compared to results from the standard TKE-1.5 eddy-viscosity model. Predictions of turbulent kinetic energy were significantly improved using the DRM. Results were not as clear for the $uw$ and $yw$ stress components.

Increased levels of reconstruction (beyond level 0) presented difficulties and required modification of the closure model. This was in part because the dynamic procedure underpredicts the stress near the wall over rough surfaces. While all of the simulations using reconstruction also included a near-wall stress component, this was not sufficient to prevent instabilities. We adopted an alternative mixed model consisting of the TKE-1.5 eddy-viscosity model and the velocity-reconstruction scale similarity model. This simple mixed model alleviated difficulties with the dynamic procedure and showed significant improvements over the standard TKE-1.5 closure, indicating that the addition of the RSFS stress to existing standard turbulence models can dramatically change mean flow results. Since most atmospheric codes are already equipped with a TKE-1.5 closure (or similar), conversion to a mixed model by adding the RSFS component should be straightforward.

This work has demonstrated that the explicit filtering and reconstruction turbulence closure framework has the potential to significantly improve future LES applications to the atmospheric boundary layer, both for weather prediction and wind power applications. The results with both DRM and TKE-ADM are quite promising and indicate the need for further study of closure models in flow over complex terrain.

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