A Preliminary Analysis of Spatial Variability of Raindrop Size Distributions during Stratiform Rain Events

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ABSTRACT

The spatial variability of raindrop size distributions (DSDs) and precipitation fields is investigated utilizing disdrometric measurements from the four Precipitation Occurrence Sensor Systems (POSS) and radar reflectivity fields from S-band dual-polarization radar and vertically pointing X-band radar. The spatial cross correlation of the moments of DSDs, their ratio, error in rainfall estimate, and normalization parameters are quantified using a “noncentered” correlation function. The time-averaged spatial autocorrelation function of observed radar reflectivity factor ($Z_e$) is smaller than that of estimated rainfall rate from $Z_e$ because of power-law $R$–$Z$ transformation with its exponent larger than unity. The important spatial variability of DSDs and rain integral fields is revealed by the significant differences among average DSDs and leads to an average fractional error of 25% in estimating rainfall accumulation during an event. The spatial correlation of the reflectivity from POSS is larger than that of $Z_e$ because of larger measurement noise in $Z_e$. The higher moments of DSDs are less correlated in space than lower moments. The correlation of rainfall estimate error is higher than that of estimated rainfall rate and of rainfall rate calculated from DSDs. The correlation of the characteristic number density is low (0.87 at 1.3-km distance), suggesting that the assumed homogeneity of the characteristic number density in space could result in larger errors in the retrieval of DSDs and rain-related parameters. However, the characteristic diameter is highly correlated in space.

1. Introduction

Rain estimates from radar measurements have been important for many applications such as precipitation forecast through extrapolation methods, validation and correction of numerical models, flash flood forecast by combining them with hydrological models, climatological study on precipitation systems, and so on. However, radar quantitative precipitation estimate (QPE) suffers from significant uncertainty due to radar measurement errors (radar calibration and sampling uncertainty), range effects, attenuation, and the variability of raindrop size distributions (DSDs). Among them, the variability of precipitation fields and of DSDs is an important factor in particular at near ranges where other sources of uncertainty are not severe. In addition, this variability is highly related to the variation of microphysical processes (Fujiwara 1965; Joss and Waldvogel...
1970; Uijlenhoet et al. 2003b; Steiner et al. 2004; Lee and Zawadzki 2005a). Thus, the understanding of the DSD variability in time and space is essential for understanding microphysical processes.

Previous works try to investigate the spatial and temporal structures of precipitation $R$ with radar reflectivity factor $Z$ or gauge-measured rain accumulation. Zawadzki (1973, 1975) investigated statistical properties of rain fields by analyzing the spatial and temporal autocorrelation function of radar-estimated rainfall fields from observed radar reflectivity factor. For a convective precipitation occurring in Montreal, Canada, the correlation of the rainfall rate followed an exponentially decreasing curve. The decorrelation distances varied from 20 to 35 km, and the normalized Eulerian and Lagrangian decorrelation times were about 32 and 44 min, respectively. Gebremichael and Krajewski (2004), using radar reflectivity data and ground-based tipping-bucket rain gauge data, performed quantitative analysis on the spatial characteristics of the rainfall intensity field to validate the Tropical Rainfall Measuring Mission (TRMM) rainfall estimation algorithms. Krajewski et al. (2003) investigated rainfall correlation modeling at small distances and a more thorough study was performed by Ciach and Krajewski (2006). Datta et al. (2003) analyzed rainfall products of TRMM ground validation (GV) at the resolution of 2 km × 2 km and of ground rain gauges evenly distributed in radar coverage in Florida. They found that the spatial decorrelation lengths of precipitation are 10–20 km depending on areas and precipitation types, and that significant variability exists within a radar grid (~1 km$^2$). Mirovsky et al. (2004) also showed the significant variability within a single radar measurement area.

In addition to reflectivity and rainfall fields, disdrometric measurements provide full information on the variation of DSDs (including various moments as well as their ratios) and thus may lead to a more complete understanding of physical processes. A sequential observation of DSDs at a single point depicts the physical process occurring in a rain system passing over the disdrometer. Thus, the measured DSDs are an outcome of the microphysical processes (aggregation, riming, coalescence, etc.) and dynamical effects due to updraft and horizontal advection.

Numerous studies investigated the DSD variability, mostly in terms of the $R$–$Z$ relationship, using the time series of DSDs from different precipitation types (convective versus stratiform) and microphysical processes (Fujitwara 1965; Joss and Waldvogel 1970; Uijlenhoet et al. 2003b; Lee and Zawadzki 2005a, and references therein). Uijlenhoet et al. (2003b) showed the variation of the $R$–$Z$ relationship along a squall line. Steiner et al. (2004) investigated the dominant control factors (size or number density) of the DSD variability. Lee and Zawadzki (2005a) analyzed a large DSD dataset observed in Montreal by the Precipitation Occurrence Sensor System (POSS) for 5 yr and concluded that the DSD variability goes beyond traditional stratiform and convective rain. They showed that the most DSD variability comes from within a storm and thus an identification of governing microphysical processes within a storm is essential to improve the accuracy in radar QPE. Also, C. K. Lee et al. (2007) showed the intrastorm variability of the DSDs, likely caused by physical processes above the melting layer, and showed the distinctive change of $R$–$Z$ relationships.

It is natural to expect the DSD variability in space. However, no extensive study has been conducted until recently because of limited observations. Mirovsky et al. (2004) performed a pioneering work on the analyses of the spatial variability of DSDs within 1 km$^2$. Although they could not explicitly determine quantitative variability due to the uncertainty among different disdrometers, the results clearly showed the significant spatial variability within 1 km$^2$. The following works by Tokay and Bashor (2007) show the extreme variability within 1.7 km distance [over 10% (37%) of mean difference (standard deviation) in event total rain accumulation] with three Joss–Waldvogel disdrometers. The present study is an extension of these works that were mainly focused on the variability of $R$ and $Z$. This study shows the spatial variability of various moments, their ratio, and the error in $R$ estimate.

The goal of the present study is to perform a preliminary analysis of the spatial variability of the bulk parameters of DSDs and their ratio from observations of disdrometers spaced over distances extending from 1.3 to 32.6 km. These parameters are the moments of DSDs, rainfall rates calculated from DSDs and estimated from $Z$, the ratio of two rainfall rates, and normalized characteristic number density and diameter [see Eqs. (1)–(8) for their definition]. The spatial variability is analyzed by computing spatial auto- and cross-correlation.

To reveal the spatial variability of the DSDs, a specifically designed disdrometer network is desired. Since this variability is expected to be scale dependent an evenly distributed disdrometer network and a long-term observation with this network would be a good choice to investigate the DSD variability for a clearly defined range of scales. Optimized spacing of disdrometers within the network could cover a broader range of scales of variability. However, the disdrometers used in the present study deployed for a winter icing experiment, the Alliance Icing Research Study II (AIRS II:
Isaac et al. 2001, 2005), so that the locations of POSS were not designed for the study of the spatial variability of DSDs. Thus, obtained datasets are insufficient to fully understand the spatial variability. The spatial coverage and separation distance are limited. In addition, it is desired to collocate all four POSSs to perform the cross calibration. But this was not possible due to many constraints. Thus, the cross calibration is performed with limited available datasets. Nevertheless, the current datasets show some interesting characteristics and points the way to further studies.

In section 2, the data and instruments used are described. Several integral parameters and their ratio and “noncentered” spatial correlation are defined in section 3. The results and conclusions are presented in sections 4 and 5.

2. Instrumentation and data

The data used are composed of DSDs from four POSSs and radar reflectivity fields from McGill S-band dual-polarization radar and vertically pointing X-band radar (VertiX) during AIRS II. The locations of instruments are shown in Fig. 1. The relative distances between two POSSs are $d_{12} = 1.3$, $d_{23} = 15.5$, $d_{31} = 29.3$, $d_{23} = 30.5$, $d_{14} = 31.8$, and $d_{24} = 32.6$ km. Here, $d_{ij}$ is defined as a distance between $i$th and $j$th POSSs. Since the separation distance of four pairs ($d_{13}$, $d_{23}$, $d_{14}$, and $d_{24}$) is similar, the mean correlation value of these four pairs will be shown. The deployment of four POSSs is not designed to reveal the spatial variability of DSDs. For example, there were no DSD measurements at the distance of several kilometers while there are three pairs of POSSs around 30-km separation distance. In addition, the four points in space are not sufficient to fully resolve the DSD variability at various different scales.

McGill S-band dual-polarization radar collects 24 plan position indicator (PPI) images every 5 min from which the constant altitude plan position indicator (CAPPI) at 1.3-km height is derived using the method suggested by Marshall and Ballantyne (1975). The ground echoes and anomalous propagation are eliminated before making CAPPI (Cho et al. 2006). Although the brightband (BB) contamination is not significant in the cases we selected, the correction of vertical profile of reflectivity is applied (Bellon et al. 2005, 2007). The CAPPI has a coverage of 120 km by 120 km and a resolution of 1 km by 1 km by 5 min. A total 1440 CAPPI images were analyzed to investigate the spatial structure of rain fields.

POSS was developed to identify the current weather, in particular precipitation types, and later software was developed to retrieve DSDs from 1-min average Doppler power spectra (Sheppard 1990). The drop number densities $[N(D)]$ are obtained at 34 diameter bins from 0.34 to 5.34 mm. The diameter interval of each bin increases with increasing size from $\Delta D = 0.05$ mm at mean drop size $D = 0.34$ mm to $\Delta D = 1.84$ mm at $D = 5.34$ mm. The sampling volume of a POSS is three orders of magnitude larger than that of a conventional disdrometer (i.e., Joss–Waldvogel disdrometer). Thus, measured DSDs are less affected by the undersampling uncertainty. An extensive validation of POSS as a disdrometer by comparison with other instruments is provided by Sheppard and Joe (1994), Campos and Zawadzki (2000), and Lee and Zawadzki (2005b).

DSD measurements from POSS site P1 are extensively verified with an optical disdrometer and a Joss–Waldvogel disdrometer (Campos and Zawadzki 2000; Lee and Zawadzki 2005b). As a basic step to remove possible systematic bias among POSSs after hardware calibration, collocated measurements of DSDs are required. However, several technical issues such as accessibility to the site and availability of individual units make it impossible to collocate them for a certain time period. P4 was located within an international airport and it was not possible to move different units into the airport or vice versa. P1 and P3, which are owned by McGill University, were collocated at the P3 site for longer than a year. During the AIRS II experiment, P1 moved to the P1 site and then later to the P2 site because of closure of the P1 site. This allows cross calibration of P1, P2, and P3 by using P1 as a reference, although their data are not collected at the same location and period. The mean bias factor is derived by a ratio $[N_{P1}(D)/N_{P3}(D)]$ of average DSDs from P1 to that from P3 $[or N_{P1}(D)/N_{P2}(D)]$ for P1 and P2. The
correction is done by multiplying this size-dependent factor by measured DSDs from P3 (or P2). The size of the calibration samples is about 32,340 DSDs for the pair of P1 and P3 and 900 DSDs for that of P1 and P2. Results show that the systematic error is minimal (the factor is close to 1). P4 could not be collocated with P1 because of the described technical issues. However, the previous study by Lee and Zawadzki (2006) shows that the mean difference in radar calibration error derived from P1 and P4 is about 0.5 dB. This indicates that the systematic bias between P1 and P4 should be smaller than this value. However, cross calibration by collocated observations could improve the agreement between POSSs.

Several rain events were observed during the AIRS II experiment. However, based on the selection criteria (rain at all four sites and minimal brightband height above 1.5 km), we have selected four stratiform rain events that lasted longer than 4 h: 1) 0400–0900 UTC 13 November, 2) 0630–2300 UTC 19 November, 3) 2000–2400 UTC 28 November, and 4) 1500–1930 UTC 11 December 2003. The total number of 1-min DSDs during the four events is about 1800 from each site. The bright bands (BB) are clearly identified during these four events from the time–height profiles of reflectivity and vertical Doppler velocity measured by VertiX such as shown in Fig. 2. The intensity of BB varies with time although its height is almost constant. This example shows some degree of riming above BB as shown by the high vertical Doppler velocity larger than 2 m s\(^{-1}\) as well as upward motions, in particular at 2120 UTC.

The total rainfall accumulation from POSS in Table 1 shows significant differences among POSSs (total accumulation from 64 to 92 mm), illustrating extreme variability of rain even after accumulating 30 h. Total accumulation shows over 22% difference at a 1.3-km distance. This demonstrates an extreme variability at a smaller scale. We need to note that P1 and P2 are cross calibrated with collocated measurements. In addition, the average reflectivity shows opposite results. Thus, this difference should be interpreted as physical variability rather than instrumental systematic bias. However, we cannot completely rule out the possible miscalibration since the number of samples that is used for cross calibration is relatively smaller (≈900 DSDs).

3. Methodology

a. Integral parameters of DSDs

When we measure the number density of drops \(N(D)\) (m\(^{-3}\) mm\(^{-1}\)) at given diameter intervals \((dD; \text{mm})\), the \(n\)th moment of the DSD \(\langle M_n \rangle\) is defined as follows:

\[
M_n = \int_{D_{\text{min}}}^{D_{\text{max}}} D^n N(D) \, dD,
\]

where \(D_{\text{min}}\) (mm) and \(D_{\text{max}}\) (mm) are minimum and maximum diameters observed by a POSS, and \(D\) (mm) is the drop diameter centered at each diameter interval \(dD\). The lower moments are noisy due to the instrumental uncertainty such as truncation at smaller diameters and possible wind effects. Since the sampling volume of POSS is three orders of magnitude larger than conventional disdrometers, the high-order moments are less affected by undersampling effects. On the other hand, higher moments could be biased or noisy due to truncation at larger drops. Thus, the use of higher moments such as the 7th or 8th is not recommended in analysis. Here, we have used the 3rd to 6th moments to minimize the effects of the measurement uncertainty in further data analysis.

In the absence of updrafts and downdrafts, the rain rate \(R_D\) (mm h\(^{-1}\)) and radar reflectivity factor \(Z\) (mm\(^6\) m\(^{-3}\)) can be calculated from measured DSDs using the following equations:

\[
R_D = \frac{\pi}{6} \rho \int_{D_{\text{min}}}^{D_{\text{max}}} D^3 \nu(D) N(D) \, dD \quad \text{and} \quad (2)
\]

\[
Z = \int_{D_{\text{min}}}^{D_{\text{max}}} D^5 N(D) \, dD, \quad (3)
\]

where \(\rho\) is the density of water and \(\nu(D)\) is the terminal fall speed in still air according to Gunn and Kinzer (1949). When we assume a power-law relationship between the fall speed and drop diameter \(\nu(D) \sim D^{3.67}\), then \(R \sim M_3^{0.67}\). The radar reflectivity is usually expressed in dBZ using the transformation \(10 \log Z\). On the other hand, the rainfall intensity \(R_Z\) can be obtained from the derived radar reflectivity \(Z\) with a known \(R\)–\(Z\) relationship:

\[
R_Z = (Z/a)^{1/b}, \quad \text{(4)}
\]

where \(a\) and \(b\) are the coefficient and exponent of the \(R\)–\(Z\) relationship \(Z = aR^b\). The accuracy of the rainfall estimation depends on the \(R\)–\(Z\) relationship used and the variability of DSDs. Lee and Zawadzki (2005a) showed that the \(R\)–\(Z\) relationships vary systematically with different microphysical processes and this variation leads to a systematic bias in rain estimation. Furthermore, in addition to the systematic bias, the random error remains within a quasi-homogeneous microphysical period due to the inhomogeneity of processes, measurement noises, and drop sorting effects (Lee and
Zawadzki 2005b). We can define the uncertainty of rain estimation $X_R$ by the ratio of $R_D$ and $R_Z$:

$$X_R = \frac{R_D}{R_Z}. \quad (5)$$

Thus, the spatial and temporal structures of $X_R$ represent the error structures in rain estimation due to the variability of DSDs. To a certain extent, these structures reflect the variation of microphysical processes. The paper by G. W. Lee et al. (2007) defines $X_R$ in dB units and shows that the temporal coherence of $X_R$ could last longer than 1 h. Here, we will investigate the spatial structure of $X_R$.

**TABLE 1.** Rainfall accumulation (mm) during the events from the four POSSs.

<table>
<thead>
<tr>
<th>No.</th>
<th>Periods (UTC)</th>
<th>Total no. of samples</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0400–0900 13 Nov 2003</td>
<td>300</td>
<td>29.9</td>
<td>20.0</td>
<td>11.4</td>
<td>28.3</td>
</tr>
<tr>
<td>2</td>
<td>0630–2300 19 Nov 2003</td>
<td>990</td>
<td>23.1</td>
<td>23.4</td>
<td>12.2</td>
<td>17.0</td>
</tr>
<tr>
<td>3</td>
<td>2000–2400 28 Nov 2003</td>
<td>240</td>
<td>16.7</td>
<td>11.0</td>
<td>27.9</td>
<td>26.4</td>
</tr>
<tr>
<td>4</td>
<td>1500–1930 11 Dec 2003</td>
<td>270</td>
<td>22.8</td>
<td>16.6</td>
<td>12.2</td>
<td>17.5</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>1800</td>
<td>91.6</td>
<td>71.0</td>
<td>63.7</td>
<td>89.2</td>
</tr>
</tbody>
</table>

**FIG. 2.** Time–height profiles of (top) radar reflectivity and (bottom) vertical Doppler velocity observed by VertiX during the stratiform rain event on 19 Nov 2003. VertiX is located at the P2 site.
b. Normalization of DSDs

The normalization of DSDs was used for a compact representation of DSDs (Sekhon and Srivastava 1971; Testud et al. 2001; Illingworth and Blackman 2002). More recently, the DSD variability has been described by the concept of a scaling law (Sempere-Torres et al. 1994, 1998; Uijlenhoet et al. 2003a; Lee et al. 2004). Furthermore, Lee et al. (2004) have given a unified representation of previously suggested DSD normalizations with a single- or double-moment scaling based on the widely used power law between moments. The general form of double-moment normalized DSDs can be written in the following form (Lee et al. 2004):

\[ N(D) = M_i^{j+1} M_j^{j+1} h(x_2) \quad \text{with} \quad x_2 = DM_i^{j+1} M_j^{j+1}, \] (6)

where the \(i\)th and \(j\)th moments \((M_i\) and \(M_j)\) are used for the normalization and \(h(x_2)\) is a generic shape of DSDs. Then, we can define the characteristic number density and diameter that normalize \(N(D)\) and \(D\), respectively:

\[ N'_0 = M_i^{j+1}/M_j(j-i) \quad \text{and} \quad D'_n = (M_i/M_j)^{1/(j-i)}. \] (7)

Thus, the general form can be rewritten into \(N(D) = N'_0 h(D/D'_n)\). This equation describes the existence of the generic shape of DSDs when all the DSD variability is controlled by two parameters \((N'_0\) and \(D'_n\) or \(M_i\) and \(M_j)\). In this study, we will concentrate on the spatial variability of two parameters, namely the characteristic diameter and number density. The \(M_3\) and \(M_6\) are used as normalizing parameters and within the scaling framework completely define the DSD variability.

c. Correlation

The correlation coefficient is a measure of linear dependency between a pair of random variables. The sample Pearson product-moment correlation coefficient has been commonly used to estimate the population coefficient. It has been argued that estimation of the correlation coefficient suffers from several shortcomings (Habib and Krajewski 2002). The homogeneity and stationarity of the distribution should be satisfied to have a statistically meaningful Pearson correlation coefficient. In general, rainfall fields do not have these properties. All moments can be ill defined in nonstationary processes. The correlation function of the nonstationary process is dependent on the location of the considered pattern, not only on the lag, and because of this one can say that it is not well defined (not as well defined as for the stationary process). Thus, we may need to define a quantity that is equivalent to the Pearson correlation function for nonstationary processes in the sense that it gives us a measure of space (or time) independence. Zawadzki (1973) defined a “noncentered” correlation that avoids subtraction of the mean. He showed that this correlation is independent of the area taken as long as it contained the entire precipitation pattern. The advantage of this definition is that it gives a nonarbitrary measure and hence it is useful even if it does not remove all the difficulties when we deal with nonstationary processes.

We use the following definition of spatial autocorrelation \((\rho)\) and cross correlation \((c)\) used by Zawadzki (1973):

\[ \rho(\alpha, \beta, t) = \frac{\sum Y_1(x, y, t)Y_1(x + \alpha, y + \beta, t)}{\left[ \sum Y_1(x, y, t)^2 \sum Y_1(x + \alpha, y + \beta, t)^2 \right]^{0.5}} \] (9)

and

\[ c(\alpha, \beta, t) = \frac{\sum Y_1(x, y, t)Y_2(x + \alpha, y + \beta, t)}{\left[ \sum Y_1(x, y, t)^2 \sum Y_2(x + \alpha, y + \beta, t)^2 \right]^{0.5}}, \] (10)

where \(\Sigma\) indicates the summation in space. From these definitions, the time-averaged values can be obtained. For the point measurements such as disdrometric measurements, the summation is done in time to derive one representative spatial correlation throughout an interest period; \(Y_1\) and \(Y_2\) are the parameters of interest. We call Eqs. (9) and (10) noncentered correlations in which the mean value is not subtracted.

4. Results and discussion

a. Radar fields

Figure 3 shows the temporal average of the radar reflectivity factor measured by the S-band radar \(Z_r\) (left column) and that of the estimated rainfall rate \(R_Z\) (right column) from \(Z_r\). The color scales of \(Z_r\) and \(R_Z\) are matched with respect to \(Z = 210 R^{1.47}\) for the comparison. The scale \(R_Z\) is first derived from \(Z_r\) using the climatological \(R–Z\) relationship \(Z = 210 R^{1.47}\) (Lee and Zawadzki 2005a) and then they are averaged in time. A significant spatial gradient exists particularly in the first two cases. The average storm motion derived from the cross-correlation technique was about 10–15 m s\(^{-1}\) toward the NNE to NE direction. It is evident that the mean fields are not always aligned with the mean
motion of precipitation fields. A strong gradient exists along the direction of echo motion for the second and fourth cases while the gradient perpendicular to the motion is present in the first case. The average $Z_e$ and $R_{Ze}$ fields show a slight difference due to the power-law transformation followed by averaging.

The spatial variability of $Z_e$($\rho_{Z_e,\text{TASACF}}$) and $R_Z$($\rho_{R_Z,\text{TASACF}}$) is shown in Fig. 4 as the time-averaged spatial autocorrelation function $\rho_{\text{TASACF}} = 1/n \sum_{i=1}^{n} p(\alpha, \beta, t_i)$. The $R_Z$ is estimated from $Z_e$ using the climatological $R-Z$ relationship, $Z = 210R^{1.47}$. The major axis well aligns with the mean echo motion except for the first case in which it locates in the northwest to southeast direction. Zawadzki (1973) showed an exponential decrease of the correlation function with distance and Habib and Krajewski (2002) fitted the
Fig. 4. TASACF of (left) equivalent radar reflectivity $Z_e$ and (right) radar-estimated rainfall rate $R_e$. The symbol “+” indicates the relative direction and distance for each pair of POSSs: (a),(b) 13 Nov, (c),(d) 19 Nov, (e),(f) 28 Nov, and (g),(h) 11 Dec 2003.
correlation function with a three-parameter exponential model. Later, studies showed the flattening of the correlation function at shorter distance (<1 km) and furthermore proposed a modified exponential model to describe the derived correlation function (Krajewski et al. 2003; Gebremichael and Krajewski 2004). In our cases, the contours of correlation are confined within 10–15 km and are coarse beyond this distance. This indicates a rapid drop of correlation at shorter distances and slower drop at farther distances. This trend illustrates a slower decrease of mean autocorrelation at near ranges than the exponential function and a more rapid decrease at far ranges (see thin solid lines in Fig. 7). (A power-law model should be a linear line in log–log graph.) The decorrelation distance of $Ze$ is from about 43 km (first case) to 120 km (fourth case). These values are significantly longer than that of 20–35 km for convective rain in Montreal (Zawadzki 1973) and that of 5–10 km for convective rain in central Florida (Krajewski et al. 2003). The degree of anisotropy also increases with distance.

One interesting result is the increase of time-averaged spatial autocorrelation function (TASACF) by transforming $Ze$ into $RZe$ (i.e., see Figs. 4a,b). We can find the explanation for this from the reduction in the spatial variability of the estimated rain field by the $R-Z$ transformation. The variation of the $Ze$ field is an outcome of the natural variability, which includes all the variability of DSDs. When the exponent of the $R-Z$ relationship is larger than 1 (1.47 in this case), the variability of estimated rain fields is always smaller than that of $Z$. The correlation of $Ze$ is the same as that of $RZe$ when the exponent is unity.

b. Average DSDs

The average DSDs are derived during each event at each site (Fig. 5). A significant difference exists among average DSDs, indicating the spatial variability of DSDs and rain integral fields. Miriovsky et al. (2004) demonstrated an extreme variability of reflectivity within an area of 1 km$^2$. Here, the average DSDs from the two sites (P1 and P2) that are 1.3 km apart show discernable variation. In general, the first two cases show the parallel displacement of DSDs along the y axis with increasing $RD$ or $Z$. This indicates that the variability of DSDs is controlled by the variation of the characteristic number density. However, the last two cases show the decrease (increase) of number concentration at smaller (larger) size with increasing $Z$. Detailed investigation of VertiX data reveals that the first two cases are characterized by a strong riming process indicated by the vertical Doppler velocity larger than 2 m s$^{-1}$ and frequent upward motion. In particular, the first case shows the updraft of 1–2 m s$^{-1}$ throughout the period and the Doppler velocity reaches 3–4 m s$^{-1}$.
during the second period. In the third case, the larger Doppler velocity is less evident except for the first 1 h and intermittent short period. The last case shows strong bright band and Doppler velocity mostly less than 2 m s\(^{-1}\). This may be indicative of low density particles resulting from a dominant aggregation process. The variation of average DSDs in space is consistent with microphysical characteristics suggested by the vertical structure of \(Z_e\) and vertical Doppler velocity.

The variation of average DSDs is quantified in terms of the \(R–Z\) scatterplot in Fig. 6. A similar analysis was used to quantify the DSD variability between days or between different microphysical processes (Lee and Zawadzki 2005a). The four different symbols indicate the four cases (\(\square\): first case, *: second case, \(\triangle\): third case, and \(\times\): fourth case). The degree of scatter illustrates the variability of event-average DSDs in space. The maximum difference in \(Z\) within an event is about 6 dB. The \(Z\) gradient between two sites 1.3 km apart reaches about 1.2 dB. After applying a temporal average over 4 h, the variation is still significant and leads to the average fractional error (AFE) of 25% where \(\text{AFE} = \frac{1}{k} \Sigma |R_D - R_Z|/R_D\). Here, \(k\) is the number of data points. This quantity illustrates the amount of error in rain accumulation over 4 h with a single \(R–Z\) relationship (\(Z = 245R^{1.26}\)) due to the DSD variability or the variation of the \(R–Z\) relationship.

c. DSD moments and characteristic parameters

In this section we investigate the spatial variability of DSD moments and their characteristic parameters using disdrometric measurements from the four POSSs. The azimuthal average values of TASACFs (\(\langle p_{Z_e,TASACF}\rangle\) and \(\langle p_{RZ_e,TASACF}\rangle\)) are used as references and indicated as thin solid line in Figs. 7–10.

The sampling volume of POSS varies with size \([V(D) = 0.3–190 \text{ m}^3 \text{ s}^{-1}]\) and the maximum sampling volume for 1 min is less than \(1.14 \times 10^4 \text{ m}^3\) (Sheppard 1990; Sheppard and Joe 1994). On the other hand, the typical sampling volume of radar is about 0.1–1 km\(^3\). In this study, we have used radar data at 1 km by 1 km. By considering the time period of sampling and average storm motion, POSS obtains the average DSD at a small segment of about 1.5-km length. Thus, the sampling difference between two instruments is about five orders of magnitude. The large sampling volume definitely reduces the small-scale variability of rain so
that we may expect larger autocorrelation values. In addition, the measuring heights are different: radar at 1.3 km and POSS at ground. Lee and Zawadzki (2005b) showed the increase of variability of DSDs by simulating the drop sorting. The exponential DSDs were assumed at the cloud base and the $R-Z$ scatter-plot at ground was investigated by size sorting of individual falling drops. They showed an increase of the degree of scatter in the $R-Z$ plot because of size sorting. This leads to a decrease of the spatial correlation at ground.

The comparison of correlation from POSS $Z$ and radar $Z_e$ is shown in Fig. 7a. The thick solid line indicates the spatial cross correlation of $Z_e(cZ_e)$ from the four POSSs that is derived from Eq. (10) with the values of $Y$ from four POSSs. The dashed line is the spatial cross correlation of radar-observed $Z_e(cZ_e,point)$ at the four POSS sites. The dotted line is the value of TASACFs ($cZ_e,avg$) at the locations indicated by the plus symbol in Fig. 4. That is, the $cZ_e,avg$ (or $cRZe,avg$) is the spatial average value of the spatial autocorrelation of radar $Z_e$ (or $RZe$) from all data points that have the same distance and direction as the pairs of POSSs. The thin solid line indicates the azimuthally averaged value of TASACF, $\langle p_{Z_e,TASACF} \rangle_i$. The correlation values for pairs of POSSs around 30 km are averaged and minimum and maximum values are shown in Table 2. These minimum and maximum values should provide an idea of the uncertainty in correlation values. Opposite to the above physical explanations (difference in sampling volumes and drop sorting), the comparison shows $cZ > cZ_{e,point}$. This may be due to the larger radar measurement noise of the rapid scanning McGill radar ($\sim$6 rpm). This measurement noise is slightly alleviated by the spatial average of correlation values as shown by $cZ_{e,avg} > cZ_{e,point}$. But it is still worthwhile to note that the ground-based POSS $Z$ has higher correlation. The correlation of estimated rainfall rate from POSS $Z(cR_Z)$ is shown in Fig. 7b with the correlations of radar rain estimate. The climatological $R-Z$ relationship is used. The $cR_Z$ is again larger than $cRZe,point$ because of the large radar measurement noise and the reduction of variability by power-law transformation.

Two collocated POSSs allow one to obtain the degree of noise (or nugget values) in calculated cross-correlation values of the DSD-related parameters. Table 3 shows the cross-correlation values derived from two collocated POSSs. The difference between these values and the unity gives the nugget values that are associated with
the degree of noise. The higher the moment order, the smaller the noise. The characteristic diameter has the smallest noise while the largest noise is shown in the characteristic number density. The cross correlation derived from two POSSs at different locations should be lower than actual values without measurement noises. The nugget values are added in all spatial correlation values shown afterward to consider the decrease of correlation values due to measurement noise.

Higher-order moments are obtained with larger weight at bigger drops than lower-order moments. Thus, the spatial correlation of different moments should provide an insight as to whether the DSD variability is controlled by smaller or larger diameters. It is interesting to see the correlation of moments showing different trends with ranges (Fig. 8). The cross correlations of different moments, $M_i$, in this figure are derived from Eq. (10). In general, the higher moments are less correlated than lower moments ($c_{M_i} < c_{M_j} > c_{M_k} > c_{M_l}$). This result implies that the larger drops are more variable than smaller drops at larger separation distances. Furthermore, the larger drops are the more important controlling factor of the natural variability of precipitation fields at longer distances. The slightly higher correlation of the higher moment is shown at the separation distance below 2 km. More extensive analysis with large datasets is required to investigate the physical importance of this higher correlation at shorter distance.

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{Z_0, Z_0}$</td>
<td>0.53</td>
<td>0.56</td>
<td>0.59</td>
</tr>
<tr>
<td>$c_{R_0, Z_0}$</td>
<td>0.68</td>
<td>0.70</td>
<td>0.73</td>
</tr>
<tr>
<td>$c_{Z_0, Z_0}$</td>
<td>0.54</td>
<td>0.55</td>
<td>0.56</td>
</tr>
<tr>
<td>$c_{R_0, Z_0}$</td>
<td>0.64</td>
<td>0.65</td>
<td>0.66</td>
</tr>
<tr>
<td>$c_{R_0}$</td>
<td>0.72</td>
<td>0.73</td>
<td>0.73</td>
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<tr>
<td>$c_{Z_0}$</td>
<td>0.62</td>
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</tr>
<tr>
<td>$c_{X_0}$</td>
<td>0.80</td>
<td>0.84</td>
<td>0.88</td>
</tr>
<tr>
<td>$c_{M_1}$</td>
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<td>0.67</td>
<td>0.71</td>
</tr>
<tr>
<td>$c_{M_2}$</td>
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<td>0.67</td>
<td>0.74</td>
</tr>
<tr>
<td>$c_{M_3}$</td>
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<td>0.64</td>
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<tr>
<td>$c_{M_4}$ ($c_Z$)</td>
<td>0.59</td>
<td>0.60</td>
<td>0.61</td>
</tr>
<tr>
<td>$c_N$</td>
<td>0.50</td>
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<td>0.66</td>
</tr>
<tr>
<td>$c_{R_0}$</td>
<td>0.84</td>
<td>0.85</td>
<td>0.86</td>
</tr>
</tbody>
</table>

We now compare the spatial cross correlation of rainfall-related parameters ($R_D, R_Z$, and their ratio $X_R$) from POSSs. As shown in Fig. 9, the spatial cross-correlation values of $R_Z$ are larger than $R_D$, $c_{R_Z} > c_{R_D}$. Furthermore, Figs. 7a and 9 show $c_{R_0} > c_{R_0}$ > $c_{Z}$. That is, the true correlation of rainfall intensity fields is bounded by $c_{R_0}$ and $c_{Z}$. This is consistent with a significant reduction of the variability in rainfall intensity fields by the $R-Z$ transformation using a single relationship. While the $R-Z$ transformation reduces the spatial variability of $R_Z$ fields, the reduced variability should cause the increase in the difference in $R_Z$ and $R_D$. That is, the larger the variation of $R-Z$ relationship in space, the larger the difference between $c_{R_0}$ and $c_{R_0}$; $c_{X_0}$ is larger than $c_{R_0}$ and $c_{Z}$. Thus, no correlation of fields ($Z$ or $R$ obtained from $Z$) represents the actual error structure of the rain estimate [for an alternative discussion of this point the reader is referred to C. K. Lee et al. (2007)].

The double-moment normalization of DSDs is widely used to understand the microphysical processes and to retrieve DSDs or rainfall rate from dual-polarimetric radar measurements (Sekhon and Srivastava 1971; Testud et al. 2001; Illingworth and Blackman 2002; Lee et al. 2004; Bringi et al. 2003). In particular, the characteristic number density is assumed constant in space to retrieve other rainfall-related parameters (Testud et al. 2000; LeBouar et al. 2001; Bringi et al. 2003). Figure 10 addresses the question of whether the assumption is valid in the stratiform rain events analyzed here. The $c_{N}$ is relatively low (about 0.87) at the separation distance of 1.3 km even after taking into account the instrumental noise [the correlations of $N_0$ ($D_m$) from the collocated POSSs are 0.941 (0.998)]. However, it slowly decreases with separation distance. When the characteristic number density is retrieved by assuming constant values at about 15 km (distance between P3 and P4 site), significant natural variability is neglected because of its low spatial correlation (0.68). Although the Marshall and Palmer (1948) model distribution shows constant number density, this result illustrates the highly variable nature of the characteristic number density even within stratiform rain events (Waldvogel 1974; Huggel et al. 1996). In addition, the $c_{D_m}$ is much larger than $c_{N}$, indicating smaller variability of the characteristic diameter in space.

5. Conclusions

We have investigated the spatial variability of DSDs and precipitation fields using disdrometric measurements from the four POSSs and radar reflectivity fields from McGill S-band dual-polarization radar and vertically pointing X-band radar deployed around the Montreal
area. Four stratiform rain events lasting longer than 4 h were selected. The total number of 1-min DSDs from each POSS was about 1800 and 1440 CAPPIs from S-band radar were analyzed. The dataset used here is limited. The disdrometer network was not specifically designed for the current study and questions related to the anisotropy of the fields could not be resolved. Nevertheless, in a first attempt at studying the spatial variability of DSDs at various scales we have quantified the spatial cross correlation of the moments of DSDs, their ratio, the uncertainty in $R$ estimate, and the normalization parameters. Because of the nonstationary and inhomogeneous nature of rain fields, the “non-centered” correlation defined by Zawadzki (1973) was used. The major findings are as follows:

1) The time-averaged spatial autocorrelation function (TASACF) of $Z_{cr}$, $\rho_{Z_{cr},TASACF}$, and estimated rainfall rate, $\rho_{R_{cr},TASACF}$, are not always elongated along the storm motion. The $R-Z$ transformation diminishes the variability of DSDs. Thus, $\rho_{R_{cr},TASACF} > \rho_{Z_{cr},TASACF}$, indicating that the spatial correlation of true $R$ in the scale of 1 km$^2$ should be between $\rho_{Z_{cr},TASACF}$ and $\rho_{R_{cr},TASACF}$. This provides a guideline of the correlation structure of true $R$ fields.

2) Significant difference exists among average DSDs, indicating the significant spatial variability of DSDs and rain integral fields. The discernable variation has been shown even for 1.3-km distance. This variation leads to the average fractional error of 25% in the estimate of rainfall accumulation during the event. Thus, this illustrates the extreme DSD variability in space.

3) The comparison of the spatial correlation of POSS $Z$ and radar $Z_r$ shows $c_{Z} > c_{Z_r,point}$. We expected the opposite result because of the difference in sampling volumes and drop sorting effect and attributed this to the large radar measurement noise due to the rapid scanning of the McGill radar (~6 rpm). The $c_{R_{cr}}$ is again larger than $c_{R_{cr},point}$.

4) The lower moments of DSDs are slightly more correlated than higher moments ($c_{M_9} < c_{M_6} < c_{M_4} < c_{M_2}$). This result implies that the larger drops are more important factors that increase the natural variability of precipitation fields.

5) The spatial cross correlation of rainfall-related parameters ($R_D$, $R_Z$, and their ratio $X_R$) shows $c_{X_R} > c_{R_D} > c_{R_Z} > c_Z$. Thus, the true correlation of rainfall intensity fields is bounded by $c_{R_D}$ and $c_Z$. The correlation of rainfall estimate error ($c_{X_R}$) is higher than that of estimated or true rainfall rate ($c_{R_D}$ or $c_{R_Z}$).

6) Although Marshall and Palmer (1948) model DSD implies constant number density, the correlation of the characteristic number density illustrates its highly variable nature even within stratiform rain events. The $c_{N_i}$ is relatively low (about 0.87 after taking into account the nugget value) at the separation distance of 1.3 km. This means that the number density is inherently variable. Also, $D_m$ is highly correlated in space ($c_{D_m} > 0.87$ up to 30 km).

We have expanded the investigation on the spatial variability of DSDs by examining $R$, $Z$, the moments of DSDs, their ratio, and the characteristic number density and diameter. However, our analysis is limited by the available datasets during AIRS II. It would be very interesting to examine how the spatial correlation varies with precipitation types, and more importantly with microphysical processes. In addition, we have derived the spatial cross correlation from two time series without a time lag. However, the storm moves along winds and the DSDs evolve as well. Thus, it would be very interesting to investigate the separation of temporal and spatial variability by deriving the correlation with time lags from datasets along the direction of dominant storm motion. The current datasets were not sufficient to explore this issue.

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