Gazing at Cirrus Clouds for 25 Years through a Split Window. Part I: Methodology

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(Manuscript received 26 September 2007, in final form 22 August 2008)

ABSTRACT

This paper demonstrates that the split-window approach for estimating cloud properties can improve upon the methods commonly used for generating cloud temperature and emissivity climatologies from satellite imagers. Because the split-window method provides cloud properties that are consistent for day and night, it is ideally suited for the generation of a cloud climatology from the Advanced Very High Resolution Radiometer (AVHRR), which provides sampling roughly four times per day. While the split-window approach is applicable to all clouds, this paper focuses on its application to cirrus (high semitransparent ice clouds), where this approach is most powerful. An optimal estimation framework is used to extract estimates of cloud temperature, cloud emissivity, and cloud microphysics from the AVHRR split-window observations. The performance of the split-window approach is illustrated through the diagnostic quantities generated by the optimal estimation approach. An objective assessment of the performance of the algorithm cloud products from the recently launched space lidar [Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observation/Cloud-Aerosol Lidar with Orthogonal Polarization (CALIPSO/CALIOP)] is used to characterize the performance of the AVHRR results and also to provide the constraints needed for the optimal estimation approach.

1. Introduction

As the data record from the National Oceanic and Atmospheric Administration’s (NOAA) Advanced Very High Resolution Radiometer (AVHRR) approaches 30 years in length, its relevance as a dataset for studying multidecadal climate variability grows. In particular, the AVHRR has proven capable of producing useful information on global cloudiness. This paper explores a new application of an old approach for the generation of a global multidecadal cloud climatology: the split-window method. The cloud properties of interest here are the cloud-top temperature $T_c$, the cloud 11-$\mu$m infrared emissivity $\varepsilon_c$, and a measure of cloud microphysics, denoted here as $\beta$. The application of this approach is designed for a new version of the AVHRR Pathfinder Atmospheres Extended dataset (PATMOS-x), which is an extension in terms of algorithms, products, and temporal coverage of the PATMOS data described by Jacobowitz et al. (2003) and Stowe et al. (2002). This effort represents the first global and multidecadal application of the split-window algorithm. This algorithm is also run operationally within the NOAA National Environmental Satellite, Data, and Information Service (NESDIS) as parts of the Clouds from AVHRR Extended (CLAVR-x) processing system.

As indicated by the title, the focus of this paper is on cirrus clouds that are defined as semitransparent ice clouds. The split-window approach is certainly applicable to other clouds and is applied to all clouds in PATMOS-x. Because of the higher opacity of most lower-level clouds and the reduced temperature contrast with the surface, the behavior of the split-window approach implemented in the one-dimensional variational data assimilation (1DVAR) analysis in the presence of low-level clouds is more similar to that from a single-channel IR approach. The performance of single-channel IR approaches for $T_c$ estimation has been well documented (Nieman et al. 1993). The technique for determining cloud type and phase for each AVHRR cloudy pixel is given by Pavolonis et al. (2005). This typing algorithm classifies cloudy pixels as being fog, water, supercooled water, cirrus, multilayer, or opaque ice cloud. The fog, water, and supercooled water types are treated as water phase clouds and the others are...
treated as ice phase clouds. The cirrus type is meant to refer to semitransparent single-layer ice clouds. The multilayer cloud type is dominated by cirrus cloud overlying lower water clouds. The opaque ice cloud type consists of optically thick high-level ice clouds.

The goal for this algorithm is to generate climate data records of $T_c$, $\varepsilon_c$, and $\beta$ over the three decades of AVHRR observations. To be relevant, the cloud climatology from PATMOS-x must add to the information provided by the most used and successful satellite cloud climatology, the International Satellite Cloud Climatology Project (ISCCP) (Rossow and Schiffer 1999). During the day, ISCCP uses the visible (VIS)/IR approach for cloud property estimation by estimating $\varepsilon_c$ using an optical depth retrieval performed at 0.63 $\mu$m (VIS) and using the value of $\varepsilon_c$ to estimate $T_c$ from the 11-$\mu$m (IR) radiance. During the night, only the IR radiance is used. These approaches perform very differently for optically thin cloud. The main benefit of the split-window approach relative to the ISCCP approach is that it delivers consistent performance for all solar illumination conditions (including night) and offers an improvement to the ISCCP IR-only nighttime results for optically thin cirrus. Last, the split-window method provides a measure of cloud microphysics that is absent from the ISCCP methodology. While PATMOS-x does not offer the eight times per day sampling of ISCCP, it does provide four times per day sampling (since 1992) with an approach that should offer consistent results at all times of day. Part II of this series will analyze the merits of the climatology derived from the split-window approach as described here.

With the launch of the Geoscience Laser Altimeter System (GLAS) on the Ice, Cloud and Land Elevation Satellite (ICESAT) in 2003 and the Cloud-Aerosol Lidar with Orthogonal Polarization (CALIOP) on the Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observation (CALIPSO) mission in 2006, the National Aeronautics and Space Administration (NASA) has provided unprecedented information on the vertical profiles of cloudiness that can be used to characterize the performance from passive sensors such as the AVHRR. Because of the orbit of CALIPSO, it provided much more collocated data with the AVHRR on NOAA-18 and only data from CALIPSO will be used here. ICESAT does provide additional AVHRR collocation opportunities with the NOAA polar-orbiting satellites in morning orbits and we intend to extend this analysis using ICESAT in the future. Another reason to use CALIPSO is that it flies in formation with the Moderate Resolution Imaging Spectroradiometer (MODIS) and other Earth Observing System (EOS) sensors. Hence, the current period represents a golden age of polar-orbiting satellite cloud remote sensing and we are fortunate that the AVHRR record extends into it. One major goal of this paper is to quantify the performance of the AVHRR relative to these superior observing systems. These results can then be used to determine the confidence of the cloud variability observed over the past three decades by the AVHRR.

This paper is divided into several sections: section 2 describes the physics of the split-window approach. The datasets used in this analysis are described in section 3 and the retrieval methodology is presented in section 4. The procedure used to generate the error estimates for the retrieval is given in section 5 and the estimates of the forward model error are described in section 6. An example of the application of the split-window approach to one scene is given in section 7. Section 8 provides a quantitative comparison of the split-window results relative to those from MODIS and CALIPSO. Section 9 describes the accuracy of the high-cloud amounts generated from the split-window method. Last, section 10 provides our conclusions and plans for future research.

2. The split-window method

While the split-window observations on the AVHRR were originally included for sea surface temperature estimation, their use for the generation of cloud properties commenced early in the life of the AVHRR data record. One of the earliest descriptions of the use of split-window observations for the estimation of $T_c$ and $\varepsilon_c$ is given by Inoue (1985). Inoue (1985) estimated $T_c$ and $\varepsilon_c$ by assuming a fixed value of $\beta = 1.08$. This value was determined through analysis of several AVHRR scenes recorded over the western Pacific Ocean. Inoue found that the use of fixed value of $\beta$ allowed for accurate determination of $T_c$ when $\varepsilon_c > 0.4$. A rigorous study of the behavior of the split-window observations in the presence of cirrus clouds was given by Parol et al. (1991). Determination of $\beta$ from analysis of the $T_{11}$-$T_{12}$ curves from scenes over Europe was performed by Giraud et al. (1997). The more recent study of Cooper et al. (2003) explored the use of split-window measurements for estimating $T_c$, $\varepsilon_c$, and particle size when cloud boundary information is provided by an active sensor such as a lidar and a radar. A similar study applied to contrails was conducted by Duda and Spinhrine (1996) using aircraft-based lidar and radiometer observations.

The split-window method derives cloud properties from two spectrally separated measurements within the 8-13-$\mu$m infrared region. For application to the AVHRR, the measurements used are the 11-$\mu$m ($T_{11}$) and the 12-$\mu$m ($T_{12}$) brightness temperatures. For a single-layer cloud, the cloud properties that control the
split-window observations are the cloud temperature $T_c$, the cloud emissivity $e_c$, and the cloud microphysics. As is commonly done in research involving split-window observations, the cloud microphysics are represented by the parameter $b$, defined as

$$\beta = \frac{\ln(1-e_{c,12})}{\ln(1-e_{c,11})},$$  \hspace{1cm} (1)

where $e_{c,11}$ and $e_{c,12}$ are the cloud emissivities at 11 and 12 μm (Inoue 1985); $\beta$ is a strong function of cloud particle size and phase. Figure 1 shows the variation of $\beta$ computed from Mie theory for ice and water droplets and from bulk nonspherical ice scattering models of Baum et al. (2005). In general for both ice and water clouds, larger values of $\beta$ imply smaller particles. The values of $\beta$ in Fig. 1 are computed solely from the single scattering properties using the method given in Parol et al. (1991). The kink in the Baum curve arises from the discrete variation in habit mixture as particle size varies. As this figures illustrates, $\beta$ can be used as a surrogate for cloud particle size, though $\beta$ loses sensitivity to size for particles larger than 30 μm in radius. A fundamental radiometric measure of cloud microphysics is provided by $\beta$ that is available day and night and does not require any a priori assumptions on particle size, shape, or distribution.

Figure 2 demonstrates the relationship between $T_c, e_c$, and $\beta$ on the split-window observations. This figure shows the variation of $T_{11}$ and $T_{11}-T_{12}$ for three sets of values of $T_c$ and $\beta$ characteristic of ice clouds. The curves are computed by varying $e_c$ from near zero to unity holding $\beta$ and $T_c$ constant. As Fig. 2 shows, measurements of $T_{11}$ and $T_{11}-T_{12}$ are not sufficient for a unique estimate of $T_c, e_c$, and $\beta$. However, $T_c, e_c$, and $\beta$ do not constitute three independent pieces of information. Given a value of $T_c, e_c$ and $\beta$ are determined directly from the 11- and 12-μm radiative transfer equations. Therefore the information content is fundamentally from $T_c$ and the spectral variation of $e_c$; $\beta$ is a suitable metric of this spectral variation of $e_c$ because it offers a direct link to the microphysics as shown by Fig. 1 and can be derived from the two split-window channels. Uniqueness is not the main limitation of the split-window approach. The main weakness is the inherent lack of sensitivity of the split-window observations to $T_c$ for optically thin clouds; this is discussed in sections 7 and 8.

It is important to note that the observations used here have a spatial resolution of 1–4 km and therefore may be composed of pixels that are not completely cloudy. In infrared remote sensing, the observed emissivities are actually the product of the true emissivities and subpixel cloud fraction. In infrared remote sensing literature, this product is often referred to as the effective cloud amount (Menzel et al. 2008). Figure 3 demonstrates the sensitivity of estimating $\beta$ in the presence of partially cloudy observations when the pixels are assumed to be completely cloud covered, as done in our retrieval approach. The curves in Fig. 3 were generated by multiplying each emissivity in Eq. (1) by the subpixel cloud
fraction. Each curve was computed assuming a fixed value of \( \epsilon_{c,11} \) (cloud emissivity) and a true value of \( \beta = 1.0 \). As this figure shows, the estimated \( \beta \) always decreases as the cloud fraction falls below 1.0. The effect is larger for larger emissivities. Therefore, this effect is largest for optically thick cloud with small subpixel cloud amounts. Because this paper deals with cirrus that are optically thin and are typically spatially extensive, we are confident that the impact of subpixel cloudiness on our retrievals is not due to subpixel cloud fraction variations.

For optically thicker cloud, the radiative level for larger emissivities. Therefore, this effect is largest for optically thick cloud with small subpixel cloud amounts. Because this paper deals with cirrus that are optically thin and are typically spatially extensive, we are confident that the impact of subpixel cloudiness on our retrieved \( \beta \) values is small. While this effect may be significant when using sounder data that typically have much larger pixel sizes, the pixel sizes employed here should be adequate to allow us to assume fully cloud-filled pixels. In addition, the AVHRR processing employed here screens all pixels that occur on cloud edges, which should also mitigate this effect. For the remainder of this paper, we will assume all pixels are completely cloud covered and that the cloud emissivity variations are not due to subpixel cloud fraction variations.

3. Generation of CALIPSO, MODIS, and AVHRR comparisons

The main analysis approach used in this paper is the generation of simultaneous collocated comparisons between the AVHRR on NOAA-18 and MODIS and CALIPSO data from the EOS A-train. CALIPSO will provide vertical profiles of cloud extinction and cloud phase. At the date of the submission of this paper, the only available CALIPSO products were profiles of cloud layers and midlayer cloud temperatures (Vaughan et al. 2004). In addition to the midlayer temperatures, the cloud-top temperatures \( T_c \) and pressures \( P_c \) were estimated from the cloud-top altitudes and the atmospheric profiles in the level-1 CALIPSO data. At the time this study was done, the opacity and phase information was not yet available and the CALIPSO products used here were currently listed as provisional. Even with these limitations, we contend that the CALIPSO data provide the best source of validation information for these AVHRR algorithms.

The cloud-top information was provided by the midlayer temperatures in the CALIPSO data. For optically thin cloud, the geometric midlayer level should approximate well the effective radiative level of the cloud (the fundamental parameter measured by the AVHRR and MODIS). For optically thicker cloud, the radiative level begins to approach the cloud top. Therefore, for clouds with optical depths much greater than one, we expect the values of \( \epsilon_c \) from CALIPSO to be slightly underestimated because of the overestimation of the effective radiative temperature. Once the CALIPSO extinction data are available, we will refine this analysis by utilizing methods to adjust the AVHRR and MODIS results to the physical cloud-top levels.

To compute the 11-\( \mu \)m cloud emissivity, \( \epsilon_c \), from CALIPSO, Eq. (A1) is used with the values of \( T_c \) being provided by CALIPSO to determine the 11-\( \mu \)m cloud emission. In Eq. (A1), the observed 11-\( \mu \)m radiance comes directly from the AVHRR and the clear-sky radiance is computed as described in the appendix. The above cloud emission and transmission are ignored, as this introduces only a small error in \( \epsilon_c \) for high clouds. In this sense, the \( \epsilon_c \) value is really derived from CALIPSO and AVHRR but throughout the rest of this manuscript, this \( \epsilon_c \) value will be denoted as being from CALIPSO. In addition, the same procedure is used to compute the 12-micron cloud emissivity that is used to estimate \( \beta \) from the combined CALIPSO and AVHRR observations.

In addition to the CALIPSO cloud products, this paper also compares the split-window results to the Collection-5 NASA Aqua/MODIS cloud products (MYD06). The standard MODIS products provide estimates of \( T_c \) and \( \epsilon_c \) derived from the CO2 slicing technique (Menzel et al. 2008). The cloud mask and infrared cloud phase products within the MYD06 files are used when compiling water and ice cloud properties (Platnick et al. 2003). While the MYD06 products provide the values of \( T_c \) and \( \epsilon_c \) required for these comparisons, the MYD06 data do not provide \( \beta \). To compute \( \beta \) from MODIS, we employed a similar procedure to that used to generate \( \beta \) values from the combined CALIPSO and AVHRR measurements. We used the \( T_c \) values from MYD06 and the observed AVHRR 11- and 12-\( \mu \)m radiances to compute the 11- and 12-micron cloud emissivities and were then able to compute \( \beta \). While the MODIS 11- and 12-\( \mu \)m radiances
could also be used, only clear-sky estimates for the AVHRR 11- and 12-µm radiances were readily available. Also, the spectral differences between the MODIS and AVHRR channels do cause β to vary, which would complicate this analysis.

In summary, the CALIPSO and MODIS values of β are actually the values of β computed from the AVHRR measurements assuming the cloud temperature provided by CALIPSO or MODIS. The same is true for the CALIPSO εc value. However, the MODIS εc values shown later are the actual values provided by the MODIS (MYD06) data. Of the six parameters used to characterize the performance of the AVHRR parameters in section 8, three are totally independent of the AVHRR measurements (Tc from CALIPSO and Tc and εc from MODIS) and three are derived from AVHRR radiances using non-AVHRR cloud temperatures (εc and β from CALIPSO and β from MODIS). Even though three of the parameters were derived from AVHRR radiances, they were not a product of the 1DVAR retrieval approach used to generate the actual AVHRR products.

To enable comparisons to the AVHRR results, the MODIS and CALIPSO products were mapped to the same 0.5° grid used in PATMOS-x. The MODIS IR cloud products are provided at a resolution of 5 km. The CALIPSO products included the 1-km cloud layer products and the atmospheric profiles from the level-1b data. The standard level-3 MODIS products could not be used for this analysis because of the lack of gridcell time information in the data.

The analysis shown later is composed of those grid cells that have nearly simultaneous data from NOAA-18/AVHRR, MODIS/Aqua, and CALIPSO. The goal of this analysis was to focus on single-layer ice clouds. To accomplish this, the following conditions were used to filter the data: the maximum time difference between the AVHRR and CALIPSO observations was limited to 10 min. To reduce the errors caused by the different sampling provided by AVHRR, MODIS, and CALIPSO, just the grid cells where CALIPSO saw only high cloud as the topmost cloud layer and where the standard deviation of the cloud-top temperature was less than 10 K were included. Grid cells were also required to be covered by at least 10% high cloud as determined by CALIPSO. Then, to avoid very large differences in viewing angle between AVHRR and CALIPSO, which only views nadir, the maximum AVHRR viewing angle was limited to 30°. Last, because of limitations of our ability to model the clear-sky radiances in polar regions, the analysis excluded grid cells with central latitudes poleward of 60°. The application of the above filtering results in approximately 10 000 points (0.5° grid cells) for analysis for August 2006. Figure 4 shows the coverage of the cells that met the above temporal and viewing-angle criteria. Most days in the month contributed data to this analysis. As Fig. 4 shows, the data are spread uniformly over most of the globe for both orbital nodes (day and night). Because of a lack of CALIPSO data, two days late in the month were not included in this analysis. While this number represents a small fraction of the original data (the globe is covered by 165 018 0.5° cells), it represents orders of magnitudemore comparisons than provided by collocating AVHRR with any particular surface site for a month.

4. Retrieval methodology

The retrieval methodology employed in this study is the method of optimal estimation as described by Rodgers (1976). This method has been most often applied to the problem of temperature and moisture sounding. Its application to similar cloud remote sensing applications has been described in Heidinger and Stephens (2000), Miller et al. (2000), Heidinger (2003), and Cooper et al. (2003). What distinguishes optimal estimation approaches to the direct minimization techniques is the required use of a priori estimates of the retrieved parameters and estimates of the error covariance matrices of the measurements, the forward model, and the a priori estimates of the retrieved parameters. In the formulation used here, the observation vector v comprises T11, T11−T12, and the parameter vector x comprises Tc, εc, and β. The Jacobian or kernel matrix K for this method is therefore as follows:

\[
K = \begin{pmatrix}
\frac{\partial T_{11}}{\partial T_{c}} & \frac{\partial T_{11}}{\partial \epsilon_c} & \frac{\partial T_{11}}{\partial \beta} \\
\frac{\partial T_{11} - T_{12}}{\partial T_{c}} & \frac{\partial T_{11} - T_{12}}{\partial \epsilon_c} & \frac{\partial T_{11} - T_{12}}{\partial \beta}
\end{pmatrix}
\]  (2)

The details of the formulation of the forward model and the above kernel matrix are described in the appendix.

The optimal estimation approach is run until the following convergence criterion is met [which is taken out of Marks and Rodgers (1993)]:

\[
\sum |\delta_\times (S_\times)^{-1} \delta_\times | < 3,
\]  (3)

where \(\delta_\times\) is the change in x and \(S_\times\) is the error covariance matrix associated with x. Once the retrieval converges, \(S_\times\) can be used to gauge the success of the retrieval. For our analysis, the uncertainty estimate for each element of x is computed as the square root of the associated diagonal element of \(S_\times\). Convergence is typically achieved in 2–4 iterations. To facilitate the proper use of these retrieval results, we generate quality flags for each
AVHRR pixel based on the values of the estimated error for each parameter relative to the assumed uncertainty of the a priori value. We assign a quality flag of 1 to cases when the uncertainty estimate is greater than two-thirds of the a priori uncertainty. If the estimated uncertainty is less than one-third of the assumed error of the a priori estimate, the highest quality flag of 3 is assigned. Errors that are between one- and two-thirds of the a priori uncertainty are assigned a quality flag of 2. Values that did not converge are assigned a quality flag of 0.

5. Generation of a priori $T_c$ and $\epsilon_c$ distributions from CALIPSO data

One of the requirements of running a retrieval cast into the optimal estimation framework is to specify a priori values of the retrieved parameters and their uncertainty values. These a priori values are generated for this work from the CALIPSO observations used to verify the performance of the AVHRR results in section 8. The steps used to generate the CALIPSO data were described in section 3 and their spatial coverage is shown in Fig. 4. We analyzed the CALIPSO data separately for each AVHRR cloud type as determined by the AVHRR cloud typing algorithm. For example, Fig. 5 shows the distributions of the CALIPSO-derived values of $T_c$ and $\epsilon_c$ for the clouds designated as cirrus from the AVHRR cloud typing algorithm. Note, for cirrus and for multilayer cloud, we expressed the a priori value of $T_c$ as being relative to the tropopause temperature $T_{\text{trop}}$ because the tropopause does offer a physical limit to the vertical extent of most clouds. The tropopause data were taken from the National Centers for Environmental Prediction (NCEP) reanalysis (Kalnay et al. 1996). Table 1 gives the values used for each cloud type. For clouds designated as opaque ice, water, fog, and supercooled water, we use $T_{11}$ as the a priori value of $T_c$.

Note that these values are derived from gridded data. Because of the nonlinear variation of $\beta$, this analysis should not be used for estimating the a priori values for the pixel-resolution estimation of $\beta$. Instead, we based our values on the range of $\beta$ shown for ice and water clouds in Giraud et al. (1997). Because the averaging associated with making mean values over 55-km grid cells reduces the variability with respect to that seen in pixel-level data, the uncertainties in $T_c$ and $\epsilon_c$ used in retrieval are twice the interquartile distances of the distributions derived from the CALIPSO gridcell data. In general, we have tried to overestimate the uncertainty measurements for all parameters to prevent the retrieval from being overconstrained.

6. Estimate of the forward model error

Another requirement of the optimal estimation approach is the prescription of the uncertainty in the forward model. The section above described the analysis used to estimate the values, $\mathbf{x}_a$ and their uncertainties, $\mathbf{S}_a$. The estimation of the forward model errors is more difficult. The contributions to this error include any instrumental issues such as those due to calibration and noise effects. In addition, the forward model uncertainty should include the effects of errors in the surface temperature, surface emissivity, and atmospheric profiles.

The largest source of error in the forward model for split-window measurements is in the clear-sky radiative transfer and specifically in the specification of the surface temperature. To determine the accuracy of the clear-sky radiative transfer, an analysis of the distribution of the biases between the observed and modeled $T_{11}$ and $T_{11} - T_{12}$ was determined for August 2006. The results are
shown in Fig. 6 and are computed for NOAA-18/AVHRR and NOAA-15/AVHRR from 1 August 2006. The observed values of $T_{11}$ and $T_{11}-T_{12}$ are the mean clear-sky values over each grid cell observed by AVHRR and the modeled values are generated using atmospheric profiles and a fast radiative model as described in the appendix. The biases are shown as a function of orbital node and are separated by land and ocean. For NOAA-18, ascending corresponds to roughly 1330 local time (LT) and descending corresponds to roughly 0130 LT. For NOAA-15, the equator crossing times are roughly 0730 and 1930 LT. The error bars represent the standard deviations and the curves show a simple sinusoidal approximation to the data. The values over the ocean show little mean bias. The values over land, however, show much larger biases in both $T_{11}$ and $T_{11}-T_{12}$. The cause of the biases over land is the lack of accurate surface temperature fields in the NWP data. The NWP data used here are those from the NCEP reanalysis (Kalnay et al. 1996). Similar biases are also seen when using NCEP’s Global Forecast System (GFS) forecasts. The $T_{11}$ curves show a cold bias over land during the day and a warm bias at night. Based on these curves, the uncertainties in $T_{11}$ are assumed to be 2 K over ocean and 4 K over land. The uncertainties in $T_{11}-T_{12}$ are assumed to be 1 K. This error source is denoted as $\delta_{\text{clear}}$ below. The clear-sky radiances are adjusted using the bias curves in Fig. 6. These globally based metrics have been derived for other months and show little variation. As described in the appendix, the surface emissivity over land was provided by the databases of Seemann et al. (2008). Without knowledge of the surface emissivity, the biases over land would be much larger.

Because the retrieval is cast in terms of the fundamental parameters $T_c$, $e_c$, and $\beta$, the actual forward model uncertainty due to errors in cloudy radiative transfer is small. For example, casting the retrieval in terms of $e_c$ eliminates the uncertainty due to the estimation of optical depth from the ice water path. One of the largest sources of error in simulating the cloudy radiances with a simple model is due to spatial heterogeneity. We approximate this error by assigning it the value of the standard deviation in $T_{11}$ and $T_{11}-T_{12}$ computed for a $3 \times 3$ box centered on each pixel. While the uncertainties are also larger in the presence of multilayer clouds, currently no increase in the forward model uncertainty occurs for known multilayer situations. This error is denoted as $\delta_{2d}$.

### Table 1. A priori values and their uncertainty used in constructing $S_i$ in the optimal estimation retrieval. Values are a function of cloud type and are based on analysis of MYD06 Collection-5 results.

<table>
<thead>
<tr>
<th>Cloud type</th>
<th>$T_c$ Value</th>
<th>$T_c$ Uncertainty</th>
<th>$e_c$ Value</th>
<th>$e_c$ Uncertainty</th>
<th>$\beta$ Value</th>
<th>$\beta$ Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fog</td>
<td>$T_{11}$</td>
<td>10</td>
<td>0.7</td>
<td>0.5</td>
<td>1.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Water</td>
<td>$T_{11}$</td>
<td>10</td>
<td>0.9</td>
<td>0.2</td>
<td>1.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Supercooled water</td>
<td>$T_{11}$</td>
<td>10</td>
<td>0.9</td>
<td>0.2</td>
<td>1.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Opaque ice</td>
<td>$T_{11}$</td>
<td>10</td>
<td>0.9</td>
<td>0.2</td>
<td>1.1</td>
<td>0.4</td>
</tr>
<tr>
<td>Single-layer cirrus</td>
<td>$T_{trop} + 15$</td>
<td>20</td>
<td>0.5</td>
<td>0.5</td>
<td>1.1</td>
<td>0.4</td>
</tr>
<tr>
<td>Multilayer cirrus</td>
<td>$T_{trop} + 15$</td>
<td>20</td>
<td>0.7</td>
<td>0.5</td>
<td>1.1</td>
<td>0.4</td>
</tr>
</tbody>
</table>
The last and probably least significant forward model error term is that due to instrumental effect, $d_{\text{instr}}$. This term includes noise, calibration, and spectral response errors. Our knowledge of the AVHRR indicates that the uncertainty in $T_{11}$ is 1.0 K and the uncertainty in $T_{11} - T_{12}$ is 0.5 K (Schwalb 1982).

The actual computation of the uncertainty of the forward model is given by the following relation:

$$
\delta = \delta_{\text{instr}} + (1 - e_c)\delta_{\text{clear}} + \delta_{2d}.
$$

While the contributions of $\delta_{\text{instr}}$ and $\delta_{2d}$ remain fixed throughout the retrieval, the clear-sky contribution decreases as the cloud emissivity $e_c$ increases.

7. Example application of the split-window approach

To illustrate the strengths and weaknesses of the split-window approach, pixel-level images of the results for a scene from 17 April 1987 from NOAA-9 are shown in Fig. 7. This scene was analyzed by Travis et al. (1997) and consists predominately of two regions of ice cloud. The ice clouds over Iowa are predominately meteorological while those over Missouri are from contrails. One benefit of this scene is that contrails are known to be composed of small particles and should therefore result in large values of $\beta$. Another benefit of showing this scene is that it demonstrates the ability of this technique to extract useful information on cloudiness from the pre-EOS era.

Figures 7a–c show a standard 0.63-µm reflectance and 11-µm brightness temperature image. The false color image shown in Fig. 7a was constructed from the 0.63-, 0.86-, and 11-µm observations. An inverse scaling was applied to the 11-µm observations. The ice/high clouds in Fig. 7a appear as blue–white clouds.

The resulting cloud properties produced from the split-window approach are shown in Figs. 7d–f. The results for any non–ice clouds are masked. Note that cloud mask and cloud typing algorithms do prevent some apparent ice cloud from being processed but their overall performance is adequate. The results are consistent with the expected properties of ice cloud. In addition, the contrail clouds show the expected larger $\beta$ values than the non–contrail ice clouds. The $e_c$ and therefore the optical depth values in the contrails are less than 0.5. While most solar reflectance–based retrievals are known to have a large uncertainty for optically thin ice cloud, the comparisons with CALIPSO shown later demonstrate that the split-window approach has skill for $e_c$ values in this range.

While the images of the results show a realistic depiction of ice cloud properties, they do not reveal the limitations of the split-window approach. To show these limitations, Figs. 7g–i show the ratio of the estimated uncertainties from the optimal estimation approach and the uncertainty of the a priori estimates. A value of 1.0 means the uncertainties are the same and the retrieval added no value to the a priori values. Values much less than unity show regions where the retrieval was able to
FIG. 7. Example cloud products from the split-window technique applied to clouds classified as being ice phase for 17 Apr 1987 using NOAA-9 AVHRR data. Scene consists of meteorological ice clouds over Iowa and contrails over Missouri. The error ratio is the ratio of error estimates of the final retrieval to those of the a priori (first guess). Values of the error ratio much less than unity indicate a skillful retrieval. This scene is described in Travis et al. (1997).
greatly improve upon the a priori values. These figures show that the split-window approach is unable to improve on a priori values of $T_c$ for optically thin cirrus. The retrievals of $\epsilon_c$ are shown to improve upon a priori estimate for all regions. It is important to note that the a priori uncertainty is a function of cloud type. For ice clouds typed as opaque ice (known to have $\epsilon_c \approx 1$), the a priori uncertainty for $\epsilon_c$ is much less that for clouds typed as cirrus. This analysis shows that there is skill in estimating $\beta$ for most of the ice cloud pixels except in the optically thick regions. This too is expected since $\beta$ is a ratio of absorption optical depths, which becomes difficult to infer in opaque regions. While these results are qualitative and derived from one scene, the quantitative global analysis shown in the next section indicates that the results are consistent with these. In summary, Fig. 7 demonstrates the information content of the split-window observations and its limitations. While the VIS/IR technique would be able to estimate $\epsilon_c$ and $T_c$, it would provide no information on the microphysics, which as Fig. 7 shows, can provide insight into the cloud physics. The IR-only approach would grossly overestimate the values of $T_c$ and provide no information on the opacity or microphysics. Figure 7 also demonstrates the ability of the optimal estimation approach to self-diagnose the performance and value of the retrieval.

8. Characteristics of the AVHRR split-window retrievals revealed by using collocated cloud temperature information from CALIPSO and MODIS

As described in section 3, we can use the CALIPSO cloud temperatures along with the collocated AVHRR radiances to directly estimate new values of $\epsilon_c$ and $\beta$. These values can be thought of as the AVHRR values that would be retrieved given improved knowledge of the cloud location. In the same way, the MODIS cloud temperatures can be used with the AVHRR radiances to demonstrate the impact of the improvement in cloud-height estimation offered by MODIS on the $\epsilon_c$ and $\beta$ values. The goal of this section is to use the direct measurements of $T_c$ provided by CALIPSO and MODIS and the inferred estimates of $\epsilon_c$ and $\beta$ provided through the convolution of AVHRR radiances with CALIPSO and MODIS cloud temperature to reveal characteristics of the split-window retrievals from the AVHRR data alone. As stated in section 3, of the three CALIPSO parameters shown here, two ($\epsilon_c$ and $\beta$) are derived in combination with AVHRR radiances. Of the MODIS parameters shown here, one ($\beta$) is derived in combination with AVHRR radiances. The MODIS $\epsilon_c$ values are taken from MYD06. Therefore, half of the non-AVHRR products shown here are in fact influenced by the collocated AVHRR radiances. For clarity, the AVHRR + MODIS and AVHRR + CALIPSO products will be simply referred to as MODIS and CALIPSO products. The reliance on the AVHRR radiances is a critical aspect of this analysis. Without it, for example, we would be unable to characterize the performance of the AVHRR products as a function of $\epsilon_c$, which is the dominant predictor of performance. Even for the CALIPSO and MODIS parameters that are generated with AVHRR radiances, we contend that the parameters are independent of the methodologies employed in the 1DVAR retrieval and therefore offer a meaningful basis for characterization of the retrieval results. When CALIPSO measurements of $\epsilon_c$ and $\beta$ become available, the need to couple with AVHRR radiances will vanish.

This section provides an analysis of the simultaneous collocated AVHRR, MODIS, and CALIPSO data for August 2006 described above. As the title of this paper states, the goal of this analysis is to characterize the performance of the AVHRR split-window retrievals in the presence of cirrus cloud. As stated previously, the performance of the split-window approach for lower-level cloud approaches that of a single-channel IR approach. The performance of a single-channel IR approach for cloud-height estimation is discussed by Nieman et al. (1993). As stated in section 3, the MODIS and AVHRR parameters represent the mean values of all ice cloud pixels in each cell. Unfortunately, the CALIPSO phase products are not yet included in the standard CALIOP products. Therefore, the CALIPSO high-cloud values ($P_c < 440$ hPa) are compared to the ice cloud values from MODIS and AVHRR. Given the constraint that the grid cells included in the analysis were those where CALIPSO observed only high cloud, these comparisons should be valid.

The following three figures illustrate the quantitative performance of the AVHRR, MODIS, and CALIPSO results relative to each other. For example, Fig. 8 presents the results for $T_c$. The figures follow the same pattern. The panel on the left shows a scatterplot of the AVHRR or MODIS values versus the CALIPSO values. The center panel shows the differences between AVHRR or MODIS values versus the CALIPSO $\epsilon_c$ values. The values were plotted as a function of $\epsilon_c$ because it is the dominant modulator of the split-window performance for high cloud. The right panel shows the probability and the cumulative distribution functions for the entire dataset. The symbols in the left and center panels attempt to convey some of the major properties of the distributions. Each scatterplot is broken into eight
FIG. 8. Comparison of NOAA-18/AVHRR, Aqua/MODIS, and CALIPSO/CALIOP $T_c$ for August 2006. (left) Scatterplots of the AVHRR or MODIS values vs the CALIPSO or MODIS values. (middle) Differences between AVHRR or MODIS values vs the CALIPSO $e_c$ values. (right) Probability and the cumulative distribution functions for the entire dataset. The diamond symbols in the left and middle panels denote the modal values in each $x$-variable bin. The horizontal lines give the $x$-variable range of each bin. The vertical lines provide the interquartile distance of the $y$ variable in each bin. The lines cross at the mean values of the $x$ and $y$ variables. The bins were chosen to split the number of points equally among eight bins.
bins in terms of the \(x\)-axis variable. The bins are chosen so that they contain the same number of points. The vertical position of the horizontal line is the mean value of the \(y\) variable in each bin. The width of the horizontal line is the width of the \(x\)-variable bin. The length of the vertical line is the interquartile distance (IQD) of the distribution of the \(y\) variable in the bin. The horizontal placement of the vertical line represents the mean value of the \(x\) variable within the bin. The vertical placement of the diamond shape is the mode value of the \(y\) variable within the bin. Its horizontal position is the mean value of the \(x\) variable. The dots represent the scatter of the individual values. To avoid confusion, only every fifth point is plotted. Tables 2, 3, and 4 provide the statistics of the comparisons for all ice clouds.

As stated above, Fig. 8 shows the results of the AVHRR, MODIS, and CALIPSO \(T_c\) comparisons. In terms of the total distribution, the AVHRR \(T_c\) values are 1.4 K warmer than CALIPSO. Most of the bias occurs at \(\epsilon_c < 0.6\). For \(\epsilon_c < 0.4\), the bias is roughly 8 K. At very cold values of \(T_c\), AVHRR is biased warm by roughly 10 K. The MODIS versus CALIPSO results are very similar to those from the AVHRR. They show the same positive bias when \(\epsilon_c \approx 0.4\) and the same positive bias when the CALIPSO \(T_c \leq 210\) K. The MODIS and AVHRR comparisons do not show this disagreement for \(T_c < 210\) K. This may indicate a systematic difference in the radiative temperature measured by AVHRR and MODIS and the midlayer temperature measured by CALIPSO for cold values of \(T_c\). In general, AVHRR \(T_c\) values are colder than MODIS by 3.2 K although the mode difference is less than 1 K.

As expected from the predictions of uncertainty in Fig. 7, the emissivity results presented in Fig. 9 demonstrate that the AVHRR (and MODIS) estimates of \(\epsilon_c\) are well correlated with those from CALIPSO, even for small values of \(\epsilon_c\) (optically thin cirrus). As is consistent with the radiative transfer equation, the overestimation of \(T_c\) for small values of \(\epsilon_c\) is consistent with the overestimation of \(\epsilon_c\) seen in the AVHRR versus CALIPSO comparisons. The total distribution shows mean and mode differences near zero with an IQD of 0.08. The MODIS versus CALIPSO results are similar though the \(\epsilon_c\) biases with CALIPSO are larger at small values of \(\epsilon_c\) than seen with AVHRR. The AVHRR versus MODIS \(\epsilon_c\) comparisons show that AVHRR values were smaller than MODIS for almost all values of \(\epsilon_c\).

Figure 10 shows the comparisons of the microphysical parameter \(\beta\). As discussed above, these values of \(\beta\) are derived from the gridcell mean radiances and the mean values of \(T_c\). The mean and mode of the total distribution all show values near zero. In addition, the means and modes as functions of \(\epsilon_c\) also show values near zero expect for MODIS when \(\epsilon_c\) exceeds 0.7. Again, since the previous results show good performance of \(\epsilon_c\) from AVHRR and MODIS, it is not surprising to see good performance on the estimation of \(\beta\), given that \(\beta\) is derived from spectral emissivity values.

9. Accuracy of high-cloud amounts

In addition to \(T_c\), \(\epsilon_c\), and \(\beta\), other important climatological parameters are the layered cloud amounts. In this section, the zonal distributions of the high cloud are analyzed where high-cloud amount is defined as the fraction of pixels over a 0.5° grid cell that were determined to be high cloud. PATMOS-x has adopted the ISCCP convention of defining high clouds as those clouds with pressures less than 440 hPa and low clouds as those clouds with pressures greater than 680 hPa. Midlevel clouds span the pressures between. In PATMOS-x, cloud-top pressure is derived from interpolating within the NWP temperature profile with \(T_c\). Layered cloud amounts have proven to be a useful metric for climate variability (Eleftheratos et al. 2007) and for comparisons with climate models (Zhang et al. 2005).

As shown in the last section, the split-window estimates of \(T_c\) are known to be heavily reliant on the a priori values for optically thin cirrus. In this section, the impact

<table>
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<th>Parameter</th>
<th>(N)</th>
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<th>Mode (K)</th>
<th>Std dev (K)</th>
<th>IQD (K)</th>
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<td>6 K</td>
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<td>-0.02</td>
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<tr>
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<tr>
<td>(P_c)</td>
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<td>7.5 hPa</td>
<td>7.5 hPa</td>
<td>91 hPa</td>
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<th>Mode (K)</th>
<th>Std dev (K)</th>
<th>IQD (K)</th>
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<td>14 K</td>
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of this weakness on the derived high-cloud amounts is explored. Figure 11 shows a comparison of the zonally averaged high-cloud amounts derived from August 2006 for NOAA-18/AVHRR, Aqua/MODIS, and CALIPSO. Only data from the 0.5° cells that have simultaneous data are included. These are the same data as used previously except that the requirement of a minimum amount of high cloud observed by CALIPSO is absent. Because

Fig. 9. As in Fig. 8 but for the comparison of $\varepsilon_c$. 
CALIPSO is the most sensitive of the instruments to the presence of optically thin high cloud, the CALIPSO results in Fig. 11 are shown as a function of the minimum allowed $\epsilon_c$ value, denoted as $\epsilon_{c,\text{min}}$. For example, the CALIPSO curve for $\epsilon_c = 0.4$ is derived from all cells where $\epsilon_c \leq 0.4$ is ignored. The curves for $\epsilon_c = 0.0$ correspond to no filtering. The CALIPSO results are shown for $\epsilon_{c,\text{min}} = 0.0$, 0.2, 0.4, 0.6, and 0.8. The CALIPSO

Fig. 10. As in Fig. 8 but for the comparison of $\beta$. 

(a) AVHRR vs CALIPSO $\beta$

(b) MODIS vs CALIPSO $\beta$

(c) AVHRR vs MODIS $\beta$
Methods applied to imager data to derive long-term cloud climatologies (VIS/IR and IR). The analysis shown here clearly demonstrates that the application of the split-window approach to the AVHRR data could form the basis for a new cloud climatology that could span from 1981 to the present, adding much to the information currently available over this time period. A comparison of the split-window results relative to those from CALIPSO and Aqua/MODIS demonstrates and quantifies the strengths and weaknesses of the approach in a manner never possible before.

Based on the analysis presented here, the split-window method was shown to estimate accurately the values of \( e_c \) and \( \beta \) for high clouds with emissivities less than 1 (i.e., cirrus). For opaque ice clouds, the values \( T_c \) agreed well with those from MODIS and CALIPSO as expected. For semitransparent cirrus, the estimation of \( T_c \) was shown to be highly sensitive to the first guess. The accuracy of \( T_c \) from the split-window method determined by comparison to CALIPSO was shown to be comparable than that from MODIS. The above weakness in estimating \( T_c \) does not prevent the AVHRR from producing global high-cloud distributions that agree favorably with both MODIS and CALIPSO. Because the split-window method applied to the AVHRR can perform these retrievals during the day and night consistently, the resulting climatology of these parameters should add to the information currently available from existing satellite climatologies.

Another conclusion from this paper is that the MYD06 estimates of \( T_c \) and \( e_c \) also perform well relative to CALIPSO retrievals of \( T_c \) and to the CALIPSO/AVHRR-based estimates of \( e_c \). MYD06 currently does not derive any microphysical information from its infrared channels. As done here, a simple computation of the cloud emissivity at 12 \( \mu \)m would allow for an inclusion of \( \beta \) values within MYD06. Estimation of the cloud emissivities at other MODIS wavelengths such as 8.5 \( \mu \)m would allow for a more refined estimate of cloud microphysics. Optimal use of 8.5, 11, and 12 \( \mu \)m for cloud microphysics studies, such as provided by the Visible and Infrared Imager Radiometer Suite (VIIRS), continues to be a goal of this research.

Several steps remain before the goal of a 30-yr climatology from the split-window approach is completely realized. First, the split-window method described here will need to be applied to the entire AVHRR/2 and AVHRR/3 data record through the PATMOS-x project. This will result in a new climatology of \( T_c \), \( e_c \), and \( \beta \) and layered cloud amounts that, as described above, should complement the existing information. In addition, the uncertainty estimates from the optimal approach will be used to place error bars on the long-term time series.
derived from PATMOS-x. In the future, the resulting PATMOS-x time series will be analyzed to explore the variability in cirrus properties over the last three decades. Last, methods to ensure physical consistency of the cirrus cloud time series between the AVHRR record and that of its successor, VIIRS, will be explored.

Acknowledgments. The MODIS products were provided by the NASA Level 1 and Atmosphere Archive and Distribution System (LAADS). The clear-sky radiative transfer model used here was provide by Mr. Hal Woolf of UW/SSEC; Dr. Bryan Baum and Mr. Amato Evan provided very significant reviews of this manuscript. The views, opinions, and findings contained in this report are those of the author(s) and should not be construed as an official National Oceanic and Atmospheric Administration or U.S. government position, policy, or decision.

APPENDIX

Infrared Forward Model and Derivation of the Kernel Matrix

The forward model for infrared radiances used here is the following:

\[
I = \epsilon_c I_{ac} + T_{ac} \epsilon_c B(T_c) + I_{clr}(1 - \epsilon_c), \quad (A1)
\]

where \( I \) is the top-of-atmosphere radiance, \( I_{ac} \) is the radiance contribution from the region above the cloud, and \( I_{clr} \) is the clear-sky radiance. Here \( T_{ac} \) represents the above-cloud transmission and the \( B \) operator is the Planck function. The cloud is represented by its cloud-top temperature \( T_c \) and its emissivity \( \epsilon_c \).

The clear-sky transmission profiles are computed using the Pressure Layer Fast Algorithm for Atmospheric Transmittances (PFAAST) radiative transfer model Li et al. (1998). The atmospheric profiles are provided by NCEP reanalysis data (Kalnay et al. 1996). The surface emissivity is provided by the surface emissivity database of Seemann et al. (2008).

Using the forward model given in Eq. (A1), the terms that compose the kernel matrix shown in Eq. (2) can be analytically derived. One term that is needed is the derivative of the 12-\( \mu \)m cloud emissivity with respect to the 11-\( \mu \)m emissivity given by

\[
\frac{\partial \epsilon_{c,12}}{\partial \epsilon_{c,11}} = \beta(1 - \epsilon_{c,11})^{\beta - 1}. \quad (A2)
\]

For notational convenience, the term \( \alpha \) that is repeated in several equations is defined as

\[
\alpha = I_{ac} + T_{ac} B(T_c) - I_{clr}. \quad (A3)
\]

Using the definitions of Eqs. (A1), (A2), and (A3), the elements of Eq. (2) are determined to be as follows:

\[
\frac{\partial T_{11}}{\partial T_c} = T_{ac,11} \epsilon_c \frac{\partial B_{11}}{\partial T_c} \left( \frac{\partial B_{11}}{\partial T_{11}} \right)^{-1}, \quad (A4)
\]

\[
\frac{\partial T_{11}}{\partial \epsilon_c} = (\alpha_{11}) \left( \frac{\partial B_{11}}{\partial T_{11}} \right)^{-1}, \quad (A5)
\]

\[
\frac{\partial T_{11}}{\partial \beta} = 0, \quad (A6)
\]

\[
\frac{\partial (T_{11} - T_{12})}{\partial T_c} = \frac{\partial T_{11}}{\partial T_c} - \left( T_{ac} \epsilon_{c,12} \frac{\partial B_{12}}{T_c} \right) \left( \frac{\partial B_{12}}{T_{12}} \right)^{-1}, \quad (A7)
\]

\[
\frac{\partial (T_{11} - T_{12})}{\partial \epsilon_c} = \frac{\partial T_{11}}{\partial \epsilon_{c,11}} - (\alpha_{12}) \left( \frac{\partial B_{12}}{\partial T_{12}} \right)^{-1} \ln(1 - \epsilon_{c,11})(1 - \epsilon_{c,12}). \quad (A9)
\]

REFERENCES


