On the Parameterization of Evaporation of Raindrops as Simulated by a One-Dimensional Rainshaft Model

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ABSTRACT

The process of evaporation of raindrops below cloud base is investigated by numerical simulations using a one-dimensional rainshaft model with bin microphysics. The simulations reveal a high variability of the shape of the raindrop size distributions, which has important implications for the efficiency of evaporation below cloud base.

A new parameterization of the shape of the raindrop size distribution as a function of the mean volume diameter is suggested and applied in a two-moment microphysical scheme. In addition, the effect of evaporation on the number concentration of raindrops is parameterized. A comparison of results of the revised two-moment scheme and the bin microphysics rainshaft model shows that the two-moment scheme is able to reproduce the results of the reference model in a wide parameter range.

1. Introduction

Evaporation of raindrops can lead to a significant reduction of the surface precipitation compared to the precipitation flux at cloud base. A precise parameterization of this process is therefore an important issue in quantitative precipitation forecasts, and even for the estimation of precipitation by radar, because a significant amount of rain can evaporate below the lowest radar elevation. The evaporation of raindrops also provides an important link between cloud microphysics and cloud dynamics. In mesoscale convective systems, the evaporation of raindrops determines the strength of the cold pool and subsequently the organization and lifetime of a convective system. For boundary layer clouds (e.g., marine stratocumulus), recent observations show that often more than 80% of the drizzle drops evaporate below cloud base, and the associated cooling of the boundary layer has an important impact on the macroscopic cloud structure (vanZanten et al. 2005).

In cloud-resolving numerical models, the evaporation of raindrops has received surprisingly little attention up to now. Usually the parameterizations follow Kessler (1969) and assume an exponential drop size distribution combined with a power law relation for the fall speed (Lin et al. 1983; Reisner et al. 1998; and many others). In the following, the applicability of these assumptions, especially of the first one, will be investigated in the context of a two-moment parameterization.

A crucial step for all parameterizations of evaporation of raindrops or other microphysical processes is the choice of an appropriate drop size distribution (DSD). More general than the simple exponential distribution is a gamma distribution given by

\[ n(D) = N_0 D^\mu \exp(-\lambda D) \]  

(e.g., Ulbrich 1983, among others). Here, \( n(D) \) is the drop size distribution in m\(^{-4}\), \( D \) is the drop diameter in m, \( N_0 \) the intercept parameter with units in m\(^{-\mu+4}\), \( \lambda \) the slope in m\(^{-1}\), and \( \mu \) the dimensionless shape parameter. Figure 1 shows gamma distributions for different values of the shape parameter \( \mu \), with the same liquid water content and mean volume diameter.

The variability of the shape parameter \( \mu \) and its parameterization has been the focus of many investigations, especially within radar meteorology (Ulbrich 1983; Testud et al. 2001; and others). Recently Zhang et al. (2001) suggested the empirical relationship

\[ \lambda = 0.0365\mu^2 + 0.735\mu + 1.935 \]
between the shape parameter \( \mu \) and the slope \( \lambda \) based on disdrometer measurements in Florida (see also Brandes et al. 2003; Zhang et al. 2003; Brandes et al. 2007). Using a very simple rainshaft model, Seifert (2005, hereafter S05) showed that the \( \mu-\lambda \) relation of Zhang et al. (2001) is probably a result of gravitational sorting (differential particle sedimentation), collision–coalescence, and collisional breakup.

Independently, Milbrandt and Yau (2005a, hereafter MY05) developed a parameterization of \( \mu \) as a function of the mean volume diameter \( D_m \) for application in cloud-resolving models, especially in two-moment microphysical schemes. Their parameterization is motivated by the result that the description of the sedimentation process can be improved by using a diagnostic \( \mu-D_m \) relation, and their relation is based on a rainshaft model of pure sedimentation, neglecting all other processes. Nevertheless, in a subsequent paper Milbrandt and Yau (2006, hereafter MY06) were able to show that a two-moment microphysical parameterization with diagnostic \( \mu-D_m \) relations for all particle classes in a mixed-phase model performs better in simulating a hail storm than a scheme that assumes constant shape parameters.

Here, a one-dimensional rainshaft model will be used to investigate the variability of the shape parameter \( \mu \). As shown (e.g., by Hu and Srivastava 1995) rainshaft models that include the processes of collision–coalescence, collisional breakup, evaporation, and sedimentation are idealized but very useful tools to analyze the behavior and evolution of the raindrop size distribution below cloud base. Using such a model that explicitly predicts the evolution of the raindrop size distribution, Hu and Srivastava (1995) were, for example, able to show how collisional breakup affects the evaporation rate. Being one-dimensional, these rainshaft models are computationally cheap, and therefore it is possible to investigate a wide parameter range of initial conditions or rain events. In the following, such a set of one-dimensional simulations will be analyzed, and a parameterization of the shape parameter \( \mu \) and the evaporation rate will be suggested and tested.

2. Governing equations

For horizontally homogenous conditions, the time evolution of the drop size distribution below cloud base is determined by particle sedimentation, collision–coalescence, collisional breakup, and evaporation. For simplicity the air velocity will be neglected throughout this paper. Using the drop size distribution \( f(x) \) as a function of particle mass \( x \), which is related to \( n(D) \) by

\[
    f(x) = n(D) \frac{D}{dx},
\]

we can formulate the budget equation for \( f(x, z, t) \) as

\[
    \frac{\partial f}{\partial t} + \frac{\partial}{\partial z} [v(x) f(x, z, t)] + \frac{\partial}{\partial x} \left[ \frac{dx}{dt} f(x, z, t) \right] = \sigma_1 + \sigma_2 - \sigma_3,
\]

For particle sedimentation, which is described by the second term, the terminal fall velocity \( v(x) \) of Beard (1976) is used. The third term of Eq. (4) describes the effect of evaporation, which leads to a differential mass change of individual raindrops given by

\[
    \frac{dx}{dt} = 2 \pi D G(T, p) f_s(D) S, \quad \text{where} \quad S = \frac{e}{e_{sat}(T)} - 1 \quad \text{and} \quad G(T, p) = \left[ \frac{R_v T}{D_v e_{sat}(T)} + \frac{I_v^2}{K_T R_v T^2} \right]^{-1},
\]

where \( D_v \) = the diffusivity of water vapor in air, \( p \) = air pressure, \( e \) = vapor pressure, \( e_{sat} \) = saturation vapor pressure, \( \pi \) = \( \rho_i \), \( R_v \) = the gas constant of water vapor, \( L_v \) = the latent heat of evaporation of water, and \( K_T \) = the heat conductivity of air. The ventilation factor \( f_s(D) \) is given by

\[
    f_s = a_v + b_v N_s^{1/3} N_e^{1/2},
\]
with \( a_v = 0.78, b_v = 0.308, N_{sc} = 0.71, \) and \( N_{ke} = n_0 D/n_v, \) where \( \nu_v = \) the kinematic viscosity of air (Pruppacher and Klett 1997).

The rhs of Eq. (4) includes the collision–coalescence and collisional breakup terms given by

\[
\sigma_1 = \frac{1}{2} \int_0^x f(x, x', z, t) f(x', z, t) C(x - x', x') \, dx',
\]

\[
\sigma_2 = \frac{1}{2} \int_0^x \int_0^x f(x', z, t) f(x', z, t) B(x', x') P(x, x', x) \, dx' \, dx'', \quad \text{and}
\]

\[
\sigma_3 = \frac{1}{2} \int_0^x f(x, z, t) f(x', z, t) K(x, x') \, dx'.
\]  

(9)

(10)

(11)

Here \( C \) is the coalescence kernel, \( K \) the collection kernel, \( B \) the breakup kernel, and \( P \) the breakup or fragment distribution function (see, e.g., Hu and Srivastava 1995; Pruppacher and Klett 1997). To specify these functions, the collision efficiencies of Pinsky et al. (2001) are used as well as several parameterizations of coalescence and breakup (Low and List 1982; Beard and Ochs 1995; Brown 1997). A detailed description of the applied breakup scheme is given in Seifert et al. (2005).

Many properties and solutions of Eq. (4), which can also be interpreted as a 1D rainshaft model, have been discussed by Hu and Srivastava (1995). For example, they demonstrated that collisional breakup accelerates the depletion of rainwater below cloud base by evaporation (see their Fig. 8) because breakup produces small raindrops that evaporate more efficiently than larger ones.

In the following, we will use Eq. (4) as a reference model to discuss the effects of the individual processes on the raindrop size distribution and especially the gamma shape parameter \( \mu. \) We will also use this spectral model as a guidance and benchmark to formulate an improved two-moment bulk parameterization.

### 3. Some results using simplified models

#### a. Effects of pure evaporation on \( \mu \)

Considering evaporation only, Eq. (4) reduces to

\[
\frac{\partial f}{\partial t} + \frac{\partial}{\partial x} \left[ \frac{dx}{dt} f(x, z, t) \right] = 0.
\]  

(12)

For evaporation (i.e., \( dx/dt < 0 \)), this equation describes a shift of the DSD toward smaller and smaller sizes; formally, Eq. (12) can be interpreted as a nonlinear advection equation along the mass coordinate \( x. \) During this process the total number of drops is conserved, and only the introduction of a minimum drop diameter or mass would result in a depletion of the number of drops. As a matter of fact, raindrops are usually defined as drops larger than \( D^* = 80 \, \mu m \) diameter; smaller drops would be called cloud droplets. Using this definition, evaporation can reduce the number concentration of raindrops by making them smaller than \( D^* \).

More precisely, the raindrop size distribution (RSD) is therefore defined as

\[
f_r(x) = \begin{cases} 
  f(x), & \text{for } D(x) > D^* \\
  0, & \text{else.}
\end{cases}
\]  

(13)

The moments of the RSD are given by

\[
\mathcal{M}_k = \int_{D^*}^\infty D^k f_r(x) \, dx
\]

\[
= \int_0^{D^*} D^k f_r(x) \, dx
\]

\[
= \int_0^{D^*} D^k n_r(D) \, dD.
\]  

(14)

To calculate the shape parameter \( \mu, \) we assume that \( n_r(D) \) can be reasonably well described by a gamma distribution (which is not 0 for \( D < D^* \)): that is, truncation effects are neglected. Overall this assumption is often quite good because the mean diameter is almost always much larger than \( D^*. \) Assuming that \( n_r(D) \) is a gamma distribution, \( \mu \) can be calculated from

\[
\mu = \frac{(1 - \xi)n + 1}{\xi - 1}, \quad \text{where}
\]

\[
\xi = \frac{\mathcal{M}_{n-1} \mathcal{M}_{n+1}}{\mathcal{M}_n^2}
\]  

(15)

(16)

for any \( n \geq 1. \) Solving Eq. (12) for different initial conditions in the form of gamma distributions with different initial shape parameters \( \mu_0 \) allows us to investigate the time evolution of \( \mu \) and \( D_m \) during the evaporation process. For simplicity, a constant temperature of 20°C and a constant relative humidity of 90% are assumed. The time series of \( \mu \) for different initial spectra shown in Figs. 2 and 3 reveal that evaporation will almost always lead to a decrease of \( \mu \) and \( D_m; \) however, for small mean volume diameter and small initial \( \mu \) (i.e., drizzle with an almost exponential RSD), both the shape parameter \( \mu \) and the mean volume diameter \( D_m \) remain more or less constant during evaporation. This behavior shows that for large \( D_m, \) the overall shift of the RSD toward smaller sizes dominates the time evolution of \( \mu \) and \( D_m. \) Only if the RSD contains many
drizzle-sized drops do the removal of drops and the shift of the RSD seem to cancel each other out, leading to almost constant $\mu$ and $D_m$ during evaporation.

b. Effects of breakup–coalescence on $\mu$

For pure breakup–coalescence, that is, for the system approaches an equilibrium solution in which coalescence and breakup are exactly balanced for all drop sizes (Valdez and Young 1985; Brown 1986; List et al. 1987; Hu and Srivastava 1995; and others). Because of fundamental scaling relationships, these asymptotic equilibrium RSDs have the same shape, independent of the rainwater content. For the breakup parameterizations used here, the equilibrium RSD has a gamma shape parameter of $\mu_{eq} \approx 1$ and an equilibrium mean volume diameter of $D_{eq} = 1.1$ mm. The time evolution of $\mu$ for different initial conditions is shown in Fig. 4 for a rainwater content of 1 g m$^{-3}$. Note that the higher the rainwater content, the faster the system approaches the equilibrium RSD because the time evolution of the breakup–coalescence equation scales with $1/L_r$ (Srivastava 1988).

4. Numerical experiments using a 1D rainshaft model

To investigate the variability of the shape of the RSD and its importance for a two-moment bulk parameterization, a simplified model of a nonstationary precipitation event is used. A time-dependent model is used because gravitational sorting can only show its full effect in a nonstationary model. In a stationary model, like “model 2” of Hu and Srivastava (1995), for example, all drop sizes have sufficient time to reach all height levels, and the RSD is very much constrained by the boundary conditions and the coalescence–breakup
processes. Especially in convective precipitation, gravitational sorting is an important process, and the simple experiments described here are trying to mimic the time evolution of a convective precipitation event.

As in Seifert and Beheng (2001) and S05, a homogeneous initial cloud is assumed between a cloud base height $z_{\text{base}}$ and a cloud top $z_{\text{top}}$ with an initial DSD given by

$$f(x, z, 0) = \begin{cases} Ae^{-Bx}, & z \geq z_{\text{top}} \geq z_{\text{base}} \geq 0, \\ 0, & \text{else}. \end{cases}$$

(18)

The parameters $A$ and $B$ are calculated from the initial liquid water content $L_0$ and the initial mean volume radius $r_0$. Note that in the following $r_0$ will be chosen in the range of 10 to 15 $\mu$m. To calculate the evaporation rate below cloud base, a constant temperature $T_{\text{pbl}}$ and relative humidity RH$_{\text{pbl}}$ are assumed. In the following, all simulations assume $T_{\text{pbl}} = 20^\circ C$. Holding these parameters constant is another simplification. The advantage of this setup is that the time evolution is completely determined by Eq. (4) and therefore this simple system can be used to investigate the microphysics of an idealized rain event without complication by dynamical feedbacks. This setup is maybe somewhat artificial, especially for strong precipitation, but at the leading edge of a convective system, for example, the precipitation will actually fall into an undisturbed boundary layer.

This 1D rainshaft model is numerically solved by using the schemes of Bott (1998) and Bleck (1970) with 130 spectral bins (i.e., mass doubling every fourth bin), a vertical grid spacing of $\Delta z = 50$ m, and a time step of 1 s.

a. Diagnostic relations for $\mu$

From an explicit bin microphysics simulation of this idealized rainfall event, the shape of the RSD can be derived. Figure 5 shows a scatterplot of the shape parameter $\mu$ for various initial conditions defined by $L_0$, $r_0$, $z_{\text{base}}$, $z_{\text{top}}$, and RH$_{\text{pbl}}$. This result is similar to Fig. 2 of S05, but because of the evaporation, which is now taken into account, the scatter is much larger and especially smaller values of $\mu$ occur more often. The same data are also shown in Fig. 6 by plotting $\mu$ as a function of the mean volume diameter $D_m$. The mean volume diameter—which is related to $\lambda$ by

$$\lambda = [(\mu + 3)(\mu + 2)(\mu + 1)]^{1/3}D_m^{-1}$$

(19)

—has several advantages. First, it makes it easier to identify the breakup equilibrium regime around $D_m = D_{\text{eq}}$, and second, it makes it possible to distinguish RSDs with smaller mean diameters from RSDs with larger mean diameters that have the same $\lambda$. For an individual “convective” rain event, the precipitation at the ground usually starts with large drops and high $\mu$, then $\mu$ reaches a minimum during the precipitation maximum, maybe coming close to equilibrium in strong...
precipitation (see Fig. 1 of S05), and then the raindrops become smaller and $\mu$ might become larger again or not depending on the relative humidity and evaporation and the rainwater content. This behavior can be roughly seen in Fig. 6. Especially when evaporation is taken into account, the scatter in the $\mu$–$\lambda$ or $\mu$–$D_m$ relation becomes very large. Therefore, any diagnostic parameterization of $\mu$ as a function of $\lambda$ or $D_m$ can only be a very crude approximation of the complicated time evolution of the RSD. A formulation that proved to be useful is

$$
\mu = \begin{cases} 
6 \tanh[c_1(D_m - D_{eq})]^2 + 1, & D_m \leq D_{eq} \\
30 \tanh[c_2(D_m - D_{eq})]^2 + 1, & D_m > D_{eq}
\end{cases}
$$

(20)

where $c_1 = 4 \, \text{mm}^{-1}$ and $c_2 = 1 \, \text{mm}^{-1}$. For $D_m \approx D_{eq} = 1.1 \, \text{mm}$, this parameterization will give low values of $\mu$, assuming that the RSD is close to the equilibrium distribution. For small mean diameters, larger values of $\mu$ will occur but are limited to an arbitrarily chosen intermediate value of 7. For large mean volume diameters, gravitational sorting dominates, which can produce very narrow size distribution and therefore very high values of $\mu$.

In contrast to the monotonically increasing $\mu$–$D_m$ relation of MY05, which is based on a pure sedimentation model, Eq. (20) also considers other effects like collision–coalescence, collisional breakup, and evaporation. Although formulated as a $\mu$–$D_m$ relation, the parameterization suggested here is similar to the $\mu$–$\lambda$ relation of Zhang et al. (2001), as can be seen in Fig. 5. There are two main differences. First, the $\mu$–$D_m$ relation [Eq. (20)] is steeper for large mean volume diameters compared to the $\mu$–$\lambda$ relation. Second, the lower branch that describes RSDs with small mean volume diameter gives somewhat lower values of $\mu$ for large $\lambda$. The latter difference might be due to the fact that the data of Zhang et al. (2001) contain mainly heavy convective events whereas the simulated data from the 1D bin model used here also includes light convective rain (drizzle), evaporating rain, or even stratiform situations. Using a larger observational dataset than Zhang et al. (2001) (e.g., by including more stratiform-like precipitation), measurements also show smaller $\mu$ values for large $\lambda$ (E. A. Brandes 2006, personal communication).

b. Parameterization of evaporation in a two-moment bulk model

The bulk evaporation rate of the rainwater content can be parameterized by integration of Eq. (5):

$$
\frac{\Delta L_r}{\Delta t} = \int_0^\infty \frac{dx_r}{\Delta t} n(D) dD = 2\pi N_0 G(T, p) \tilde{J}_e S
$$

(21)

($\tilde{J}_e$ is defined in the appendix). For the number concentration, the parameterization is not as straightforward as for rainwater content. As described in section 3, the change of raindrop number is defined by the number of drops becoming smaller than 80 $\mu$m in diameter. A direct calculation would therefore strongly depend on the assumed size distribution in this size range. Taking into account the high variability of the shape parameter $\mu$, this parameterization is obviously a difficult problem. A pragmatic approach to the problem is

$$
\frac{\Delta N_r}{\Delta t} = \gamma \frac{N_r \Delta L_r}{\Delta t}.
$$

(22)

where the coefficient $\gamma$ nicely hides all unknown details. Khairoutdinov and Kogan (2000), Milbrandt and Yau (2005b), and Morrison and Grabowski (2007) assume that $\gamma = 1$; that is, that the mean volume diameter does not change during evaporation. Khairoutdinov and Kogan (2000) show some results that support this choice for drizzling stratocumulus based on their bin microphysics model (their Fig. 2). As shown in section 3, this is consistent with the fact that they simulated a drizzle event. In general, a compelling physical explanation of $\gamma \approx 1$ cannot easily be found. For $D_m \gg 80 \mu$m and $\mu \gg 1$, on the one hand, one would expect $\gamma = 0$ because within a small time interval evaporation would only make the raindrops smaller without evaporating any of them completely. On the other hand, if the
RSD contains many small drops, those will evaporate much more quickly than large ones, resulting in $\gamma \geq 1$. The question of which one of these two competing processes actually dominates depends on the given drop size distribution. The assumption of $\gamma = 1$, which is equivalent to a constant $D_m$ during evaporation, is probably a good one for broad RSDs and/or drizzle, but maybe not for strong convective rain. Figures 7 and 8 show scatterplots of $\gamma$ based on the 1D rainshaft bin microphysics model for many different initial conditions and relative humidities and, as expected, $\gamma$ decreases with increasing $\mu$. Using Eq. (20) for the shape parameter, a possible parameterization for $\gamma$ is

$$\gamma = \frac{D_{eq}}{D_m} \exp(-0.2\mu),$$

with $D_{eq} = 1.1$ mm (see Fig. 8). This parameterization is only a crude attempt to model the complicated behavior of the raindrop size distribution.

c. Results of the two-moment bulk model

The parameterizations of the shape parameter $\mu$ and the evaporation coefficient $\gamma$, which have been introduced in the previous sections, can be combined with the warm rain scheme of Seifert and Beheng (2001, 2006) to build a complete description of all relevant warm rain processes (see the appendix).

This scheme can now be evaluated by comparison with the results of the bin microphysical scheme for the idealized rain event as simulated by the 1D rainshaft model. Figure 9 shows the time evolution of the surface rain rate, the mean volume diameter, and the shape parameter for an initial cloud with $L_0 = 7$ g m$^{-3}$, $r_0 = 13$ µm, $z_{\text{base}} = 3$ km, and $z_{\text{top}} = 8$ km (this case differs from Fig. 1 of S05 only in the cloud base height).

Again, we can see the time evolution of a typical strong convective rain event in three stages. During the first stage only the largest drops arrive at the surface; the second stage is characterized by strong precipitation with the RSD being close to breakup equilibrium (i.e., a broad size distribution); and during the last stage the smaller drops dominate as in the stratiform region of a convective system. These three stages can be distinguished by the mean volume diameter, with $D_m > D_{eq}$ during stage one, $D_m \approx D_{eq}$ during the equilibrium stage, and $D_m < D_{eq}$ during the final stratiform-like period. Over the complete event the mean volume diameter decreases monotonically, although during the equilibrium stage $D_m$ is almost constant. The shape parameter $\mu$ reaches its minimum in the equilibrium stage and decreases (increases) during the first (last) stage. The two-moment parameterization is able to reproduce this behavior qualitatively, and for this individual event the quantitative agreement is also very good, except for the facts that (i) $\mu$ starts to increase again too early and (ii) because of the assumptions made by using Eq. (20), $\mu$ is constrained to an upper limit of 7, whereas the bin model simulates much larger values at the end of this event. In the evaporating case (Fig. 9b) with a relative humidity below cloud base of 70%, the maximum rain rate is reduced from about 150 to 80 mm h$^{-1}$, and overall about 55% of the precipitation evaporates before reaching the ground. Compared to the simulation without evaporation, the mean volume diameter drops off more rapidly during the decaying third stage of the event, and the shape parameter does not increase to
high values but remains low, reaching only a value of 3 at the end of the event. The two-moment scheme captures these differences to the nonevaporating case quite well, although the increase of $\mu$ in the final stage of the event is now too strong. This could only be improved by making $\mu$ a function of relative humidity. As it is now, the $\mu$-$D_m$ relation [Eq. (20)] tries to make a compromise between nonevaporating and heavily evaporating situations.

Figure 10 shows similar time series for a weaker precipitation event with an initial cloud water content of $L_0 = 1$ g m$^{-3}$. The time evolution is now much slower (e.g., the maximum rain rate is reached after 45–50 min compared to only 20 min for $L_0 = 7$ g m$^{-3}$ in Fig. 9) and the maximum rain rate is about an order in magnitude smaller, reaching only 13 mm h$^{-1}$ in the bin model. An interesting difference to the strong precipitation event is that the event now starts with relatively small drops and the largest mean diameters occur during the period of the highest rain rates. In some sense this is no longer a convective rain event; rather, it has more of the character of stratiform precipitation. The two-moment scheme overestimates the maximum precipitation rate, which also occurs a bit too early (3 min to be precise). The correspondence in mean diameter is very good (e.g., with a maximum mean diameter of about 0.85 mm in both models), but significant differences exist for the shape parameter. Without evaporation being active, the bin model simulates quite narrow size distributions even in weak precipitation. In the two-moment scheme, the shape parameter $\mu$ cannot exceed a value of 7 because for this event the mean diameter is always smaller than $D_{eq}$. Assuming a constant relative humidity of 90%, about 70% of the precipitation evaporates and

![Figure 9](https://example.com/fig9.png)

**Fig. 9.** Time evolution of the rain rate $R$ (blue), the shape parameter $\mu$ (red; plotted is $0.1\mu$), and the mean volume diameter $D_m$ (green) for a strong rain event with $L_0 = 7$ g m$^{-3}$, $r_c = 13$ $\mu$m, $z_{base} = 3$ km, $z_{top} = 8$ km, and (left) RH$_{pbl}$ = 100% (i.e., no evaporation) and (right) RH$_{pbl}$ = 70%. Solid lines are the results of the bin model, dashed lines represent the bulk model.

![Figure 10](https://example.com/fig10.png)

**Fig. 10.** As in Fig. 9, but for $L_0 = 1$ g m$^{-3}$ (i.e., a weaker rain event) with (left) RH$_{pbl}$ = 100% (i.e., no evaporation) and (right) RH$_{pbl}$ = 90%. Note the different scaling of the x and y axes compared to Fig. 9.
the maximum rain rate is reduced to 5 mm h⁻¹. The drops reaching the surface are significantly smaller, and also the shape parameter is reduced and decreases during the precipitation event. The two-moment scheme is able to simulate the reduction of the drop sizes but assumes a \( \mu \)-value of 7 all the time because this is the limit in Eq. (20) for small mean diameters.

These two examples of a strong and a weak precipitation event show that the two-moment scheme is able to describe the time evolution of the various parameters and achieves a sufficiently good agreement with the reference bin model.

In a numerical weather prediction or regional climate model, the most important parameter that depends on raindrop evaporation is the accumulated surface precipitation over a certain time period. Figure 11 shows the accumulated surface precipitation for various cases as a function of the initial cloud water content and the assumed (constant) relative humidity below cloud base. The parameterization achieves a very good agreement with the bin model over a wide parameter space, but in cases of strong evaporation of heavy rain the two-moment scheme underestimates the surface precipitation slightly (i.e., it overestimates evaporation).

**d. Comparison with other parameterization assumptions**

In recent years, almost all of the published two-moment schemes assumed a constant and usually small shape parameter \( \mu \) and neglected the size effect of evaporation (i.e., they assumed that \( \gamma = 1 \)). For example, MY06 used \( \mu = 2 \) and \( \gamma = 1 \) as one configuration in their sensitivity study (FIX03 in Table 1 of MY06). Figure 12 shows that in the idealized 1D rainshaft model this choice leads to an overestimation of evaporation compared to the bin model for the strong precipitation case with \( L_0 = 7 \text{ g m}^{-3} \) and an underestimation for the weak, more stratiform-like case with \( L_0 = 1 \text{ g m}^{-3} \). Combining the \( \mu-D_m \) relation of MY05 with an assumption of \( \gamma = 1 \) (this choice corresponds to DIAG_A of MY06) leads to similar results, but the precipitation rates in the weak case with \( L_0 = 1 \text{ g m}^{-3} \) are even higher.

The underestimation of evaporation for the weak precipitation case is mainly due to the lack of the size effect of evaporation (i.e., the assumption of \( \gamma = 1 \)). For weak rain rates this size effect introduces a positive feedback because the smaller drops will evaporate even more quickly, which makes it an important process in light to medium rain. Using the \( \mu-D_m \) relation of MY05, which is always larger than 2 in the relevant size range, results in an additional overestimation of the rain rates due to the higher bulk sedimentation velocity.

For stronger precipitation (i.e., the case in which \( L_0 = 7 \text{ g m}^{-3} \)), the mean size of the distribution is strongly constrained by the coalescence–breakup processes, and the size effect of evaporation becomes less important. In this case, both the value of \( \mu = 2 \) and the \( \mu-D_m \) relation of MY05 lead to an overestimation of the bulk evaporation because the assumed shape parameters are already too large for this precipitation event.

**5. Summary and conclusions**

An improved two-moment parameterization of raindrop evaporation below cloud base has been suggested
and tested against a spectral bin reference model. It has been shown that an accurate parameterization of evaporation is not a trivial problem and the suggested relations can only be a first step toward a better understanding of this complicated process. The complications arise from the high variability of the raindrop size distribution, especially of the shape parameter \( \gamma \), and from the nonlinear feedbacks between evaporation and breakup – coalescence, as already shown by Hu and Srivastava (1995). The suggested diagnostic relations for the shape parameter \( \gamma \) and the evaporation parameter \( \lambda \) are still very uncertain for several reasons:

- The idealized rainfall simulation is probably not realistic enough, although it reproduces the observations of Zhang et al. (2001) in a statistical sense.
- Both the \( \mu-D_m \) relation and the parameterization of the evaporation coefficient \( \lambda \) rely heavily on the ability of the spectral bin microphysics model to simulate the coalescence–breakup process in a realistic way. Especially for collisional breakup, the uncertainties in the kernels are still significant. It is quite possible that this leads to an overestimation of evaporation in the spectral bin model, especially in moderate to heavy rain.
- In light precipitation, the bin model allows high \( \mu \) values at \( D_m = D_{eq} \) (i.e., the system is not in coalescence–breakup equilibrium), although the mean volume diameter is identical to the equilibrium diameter. The parameterization always predicts \( \mu = 1 \) for \( D_m = D_{eq} \) because it would assume coalescence–breakup equilibrium.
- The dependency of the RSD on relative humidity is poorly understood and has, to the author’s knowledge, not yet been investigated based on observations.

Overall, this study shows once more that the key to an improved understanding and parameterization of the warm rain processes is reliable measurement of the drop size distribution, which would be necessary to validate—or falsify—the theoretical models.

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APPENDIX

**Bulk Parameterization of Evaporation, Sedimentation, Self-Collection, and Breakup**

The change of mass \( x \) by evaporation of a single raindrop of diameter \( D \) is given by

\[
\frac{\partial x}{\partial t}_\text{eva} = 2\pi DG(T,p)f_v(D)S, \quad (A1)
\]

\[
G(T,p) = \left[ \frac{R_T}{D_v c_{sat}(T)} + \frac{L^2_u}{K_T R_T^2} \right]^{-1}, \quad (A2)
\]

and the ventilation factor

\[
f_v = a_v + b_v N_{Sc}^{1/3} N_{Re}^{1/2}, \quad (A3)
\]
For the terminal fall velocity of raindrops, we use the approximation

$$v(D) = a - be^{-cD},$$  \hspace{1cm} (A4)

with $a = 9.65 \text{ m s}^{-1}$, $b = 9.8 \text{ m s}^{-1}$ and $c = 600 \text{ m}^{-1}$ which is a simplification of the Rogers et al. (1993) formula but is more accurate than the usual power law approximations. Assuming a gamma distribution for raindrops, we can now integrate for the bulk evaporation rate:

$$\frac{\partial L_r}{\partial t}_{\text{eva}} = \int_0^\infty \frac{\partial x_r}{\partial t}_{\text{eva}} n(D) \, dD = 2\pi N_r G(T, p) \overline{\psi} S,$$

where

$$\overline{\psi} = \int_0^\infty [a_v D^{\mu+1} + b_v N_r^{1/3} v_r^{-1/2} D^{\mu+3/2}(a - be^{-cD})^{1/2}] e^{-\lambda D} \, dD.$$  \hspace{1cm} (A5)

Using power series expansion for $(1 - \xi e^{-cD})^{1/2}$, we find the approximation

$$\overline{\psi} = a_v \Gamma(\mu + 2)\lambda^{-(\mu+2)} + b_v N_r^{1/3}/v_r^{1/2} \Gamma(\mu + 5/2)\lambda^{-(\mu+5/2)} \left[ 1 - \frac{b}{2a} \left( \frac{\lambda}{c + \lambda} \right)^{(\mu+5/2)} - \frac{1}{8} \left( \frac{b}{a} \right)^2 \left( \frac{\lambda}{2c + \lambda} \right)^{(\mu+5/2)} - \cdots \right].$$  \hspace{1cm} (A6)

Compared to the commonly used approximation of $v(D)$ by a power law, this formula is slightly more accurate (e.g., for very large mean volume diameters), but in general the difference is small (and probably negligible).

In a two-moment scheme, the prognostic variables are $N_r$ and $L_r$. The slope $\lambda$ is a function of $\bar{x}_r = L_r/N_r$ and $\mu$, given by

$$\lambda = \left[ \frac{\pi p_w}{6} (\mu + 3)(\mu + 2)(\mu + 1) \bar{x}_r^{-1} \right]^{1/3},$$  \hspace{1cm} (A7)

and the intercept parameter is proportional to the number concentration

$$N_0 = \frac{N_r}{\Gamma(\mu + 1)} \lambda^{\mu+1}.$$  \hspace{1cm} (A8)

For the sedimentation velocities of number and mass concentration in a two-moment scheme, we find

$$v_k = a - b \left( 1 + \frac{c}{\lambda} \right)^{-(\mu+k+1)}$$  \hspace{1cm} (A9)

with $k = 0$ for the number concentration and $k = 3$ for mass. And here the difference from Kessler’s assumption of a power law approximation of $v(D)$ is not negligible. Accretion and self-collection of raindrops is parameterized by

$$\frac{\partial L_r}{\partial t}_{\text{ac}} = k_{cr} L_r L_r \Phi_{ac},$$  \hspace{1cm} (A10)

$$\frac{\partial N_r}{\partial t}_{\text{sc}} = k_{cr} N_r L_r,$$  \hspace{1cm} (A11)

with $k_{cr} = 5.8 \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-1}$, $k_{rr} = 4.33 \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-1}$, and $\Phi_{ac}$ as given by Seifert and Beheng (2001). As in Seifert and Beheng (2006), collisional breakup is taken into account by a simple relaxation scheme toward the equilibrium mean volume diameter $D_{eq} = 1.1 \text{ mm}$:

$$\frac{\partial N_r}{\partial t}_{\text{br}} = \begin{cases} \left[ k_{br} (D_m - D_{eq}) \right] + 1 & \text{if } D_m > 0.3 \text{ mm} \\ 0 & \text{else} \end{cases},$$  \hspace{1cm} (A12)

with $k_{br} = 1 \text{ mm}^{-1}$.

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