Basin and Channel Contributions to a Model Antarctic Circumpolar Current

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(Manuscript received 1 April 2008, in final form 25 August 2008)

ABSTRACT

The idea that basinlike dynamics may play a major role in determining the Antarctic Circumpolar Current (ACC) transport is revisited. A simple analytic model is developed to describe the relationship between the wind stress and transport. At very low-wind stress, a nonzero minimum is predicted. This is followed by two distinct dynamical regimes for stronger forcing: 1) a Stommel regime in which transport increases linearly with forcing strength; and 2) a saturation regime in which the transport levels off. The baroclinic structure of the Sverdrup flux into the Drake Passage latitude band is central to the analytic model, and the geometry of characteristics, or geostrophic contours, is key to predicting the transition between the two regimes. A robustness analysis is performed using an eddy-permitting quasigeostrophic model in idealized geometries. Many simulations were carried out in large domains across a range of forcing strengths. The simulations agree qualitatively with the analytic model, with two main discrepancies being related to zonal jet structures and to a western boundary inertial recirculation. Eddy fluxes associated with zonal jets modify the baroclinic structure and lower the saturation transport value. Inertial effects increase the transport, although this effect is mainly limited to smaller domains.

1. Introduction

Most theories for what sets Antarctic circumpolar transport can be classified as either basinlike or channel-like. Basinlike theories date to Stommel (1957), who views the Antarctic Circumpolar Current (ACC) as being fed by a southward Sverdrup transport into the Drake Passage (DP) latitude band. The resulting eastward current then flows through Drake Passage and connects with the western boundary current along the eastern coast of Patagonia. The details of the dynamics in the Drake Passage latitude band are seen as nonessential, with the transport being determined by the basinlike dynamics to the north. Specifically, the transport is linearly related to the zonally integrated wind stress curl just north of Drake Passage. Computing this integral gives an estimate that is compatible with the range of values of the observed transport (Baker 1982). This idea has been refined over the years in more recent papers (Webb 1993; Warren et al. 1996; Hughes 2002); however, the fundamental argument is always that Sverdrup dynamics apply to the Southern Ocean and determine the transport.

Channel-like theories point out that Sverdrup dynamics require a western boundary layer and as such, the Sverdrup theory must fail somewhere in the Drake Passage latitude band. This has led Johnson and Bryden (1989) and others (Straub 1993; MacCready and Rhines 2001; Rintoul et al. 2001; Karsten et al. 2002; Marshall and Radko 2003; Olbers et al. 2004) to consider the ACC (or at least part of it) as a zonally reconnecting channel, with topographic form drag (e.g., Munk and Palmén 1951) acting as the momentum sink. It is widely recognized that in this geometry, transient eddies are needed to establish a vertical transmission of momentum to the topography and to close the meridional circulation (e.g., Marshall et al. 1993; MacCready and Rhines 2001). In the absence of eddies, solutions are dependent on artificially large and somewhat arbitrarily chosen viscous and diffusive coefficients.

In the paper that motivates our study, Tansley and Marshall (2001) perform a series of experiments to test the various ideas for what sets the ACC transport. They use a purely wind-driven two-layer eddy-permitting
model and consider a wide range of wind forcing in various geometries. Of particular relevance to us are their simulations in which a model Patagonia and Antarctic Peninsula are present, so that a zonally reconnecting latitude band lies adjacent to regions of blocked geostrophic contours to the north and south. In this geometry, their results show that transport does not follow any of the theories mentioned above (see their Fig. 15). For example, for forcing above a certain threshold, transport appears to saturate (i.e., vary weakly) instead of growing linearly with the wind stress as predicted by Stommel or following some other power law (e.g., Johnson and Bryden 1989; Rintoul et al. 2001). Note that attempts to verify such power laws have also been made in coarse-resolution models without success (Gnanadesikan and Hallberg 2000; Gent et al. 2001).

A posteriori, this is perhaps not surprising. The square root law predicted by Johnson and Bryden applies in a channel, whereas observations show the ACC makes a sharp northward turn just east of Drake Passage, where it joins the Falkland/Malvinas Current. This lends weight to the idea that the ACC is related to this western boundary current, as suggested by Stommel, so channel-like theories are unlikely to tell the whole story. Conversely, Stommel’s theory assumes that all of the Sverdrup transport into DP latitudes feeds the circum-polar flow, whereas this need not be the case. For example, part of the southward Sverdrup transport might occur in an abyssal layer where topography precludes circum-polar flow. Additionally, the Southern Ocean is known to be an eddy-rich environment. As such, it is not too surprising that the linear Sverdrup theory, which ignores this strong turbulence, fails.

In this article, we consider the ACC transport as the sum of a basinlike and a channel-like contribution. Our focus is primarily on the basinlike dynamics and its role in determining the total transport. The starting point is a comparison between the wind-driven circulation in closed-basin and Southern Ocean geometries. That is, we are interested in whether insight from the much-studied closed ocean basin problem with gyrelike circulations can help us to understand the mechanisms determining ACC transport. In the basin case, Sverdrup dynamics apply far from boundaries. These assume large horizontal length scales, so that relative vorticity can be neglected at least outside of the boundary layers and as a first approximation. In a two-layer flat bottom quasigeostrophic ocean, we recover the Sverdrup relation for the barotropic streamfunction \( \psi_B \)

\[
\frac{\partial \psi_B}{\partial x} = \frac{k \cdot \nabla \times \tau}{\rho_0 H}, \tag{1}
\]

where \( H \) is the total ocean depth, \( \tau \) is the horizontal wind stress at the surface, \( \rho_0 \) is the reference density, and \( \beta \) is the northward spatial derivative of the Coriolis parameter. The lower-layer streamfunction \( \psi_2 \) obeys a characteristic equation (e.g., Rhines and Young 1982; Pedlosky 1996),

\[
J[\psi_2, \Theta] = 0, \tag{2}
\]

where \( J[ \cdot ] \) is the Jacobian operator and

\[
\Theta = \psi_B + \beta L_p^2 y, \tag{3}
\]

is the characteristic function in which \( L_p \) is the baroclinic Rossby radius. Here, \( \psi_B \) and \( \Theta \) are known functions of the forcing, and it is assumed that passive boundary layers can be appended to satisfy boundary conditions. The characteristic function \( \Theta \) corresponds to a streamfunction for the characteristic velocity, and contours of \( \Theta \) are called characteristics (also called geostrophic contours).

Where \( \Theta \) contours extend back to the eastern boundary, characteristics are said to be blocked. In this case, (2) implies that the lower layer is at rest. Where characteristics do not extend back to the eastern boundary, eddy-driven gyres and inertial recirculation are free to develop in the \( \psi_2 \) field (e.g., Rhines and Schopp 1991). Thus, the Sverdrup transport across a blocked charac-teristic is top trapped, whereas this need not be the case for Sverdrup transport across a free characteristic. In our Southern Ocean geometry, this distinction should also apply to Stommel’s idea that a southward Sverdrup transport into the Drake Passage latitude band feeds the ACC. Moreover, if topography effectively prohibits circumpolar flow at depth, we should anticipate that only the upper-ocean part of the Sverdrup flux feeds the ACC.

Our main interest is thus to refine Stommel’s theory, taking into account the ideas discussed above and to compare its predictions with results from eddy-permitting numerical simulations. We simplify by considering a two-layer formulation in an idealized geometry, allowing us to carry out multiple simulations at eddy-permitting resolution in large domains. The following section presents our numerical model and compares results for a test simulation in basin and Southern Ocean-like geometries. An analytic model for the ACC transport is presented in section 3. This is compared with numerical results for a wide range of forcing strengths and different-sized domains in section 4. Various robustness tests are also presented. Finally, a brief summary ends this paper in section 5.
2. Numerical model and experimental design

The model is based on a two-layer version of the quasigeostrophic potential vorticity equations (e.g., Pedlosky 1996):

\[
D_t \left[ \nabla^2 \psi_1 - F_1 (\psi_1 - \psi_2) + \beta y - F_0 \psi_1 \right] = -A_h \nabla^3 \psi_1 + \hat{k} \cdot \nabla \times \tau + \tau_{1H1}
\]

(4)

\[
D_t \left[ \nabla^2 \psi_2 + F_2 (\psi_1 - \psi_2) + \beta y + \frac{f_0}{H_2} h_b \right] = -A_h \nabla^3 \psi_2 - r \nabla^2 \psi_2,
\]

(5)

where \( D_t \) = \( \frac{\partial}{\partial t} \) is the total time derivative, \( F_0 = \frac{\partial F}{\partial \psi} \), \( F_1 = \frac{\partial F}{\partial \psi_1} \), \( F_2 = \frac{\partial F}{\partial \psi_2} \), \( f_0 \) is the mean Coriolis parameter, \( g \) is the gravitational acceleration, \( g' \) = \( g(\nabla \theta) \) is the reduced gravity, \( (H_1 \) and \( H_2 \) are layer thicknesses, \( h_b \) is bottom topography, \( A_h \) is a lateral biharmonic viscosity coefficient, and \( r \) is a bottom drag coefficient.

a. Numerical implementation and geometry

A third-order Adams–Bashforth time-stepping scheme is used (Press et al. 1996). All the Jacobian operators are computed using the Arakawa scheme that conserves kinetic energy and enstrophy (Arakawa 1966). No normal flow and free slip conditions (i.e., \( \nabla^2 \psi = \nabla^4 \psi = 0 \)) are applied at solid walls. A multigrid method is used to do the elliptic inversion (Briggs et al. 2000). To facilitate this multigrid inversion, only rectangular-shaped domains are considered. The implementation of mass and momentum conservation laws of the quasigeostrophic equations in a multiply connected domain is similar to that of McWilliams (1977). Only wind forcing is considered, and the wind stress takes the form of

\[
\tau = \hat{r} f_0 \sin^2 (\pi y/L_y), \quad y \in [0, L_y],
\]

(6)

where \( L_y \) is the meridional extent of the domain. This is similar to other ACC (channel) studies (e.g., Treguier and McWilliams 1990) and gives the familiar double-gyre forcing typical of basin gyre studies upon taking the curl. All runs are carried out at eddy-permitting resolution and model parameters can be found in Table 1.

We wish to compare idealized basin and Southern Ocean geometries. To this end, most experiments are made both in a closed-basin and a corresponding “Southern Ocean” domain. To form the Southern Ocean domain, a segment of the meridionally oriented walls is cut and a repeating channel boundary condition is imposed. This can be thought of as a channel with two “peninsulas”: one corresponding to Patagonia and the other to the Antarctic Peninsula (see Fig. 1).

Because of our multigrid method of doing the elliptic inversion, the width of the peninsulas is constrained to be an integer multiple of \( 2^3 \), where five levels are used in our V cycle (Briggs et al. 2000). For simplicity, we take the peninsulas’ width to be zero. Although that may seem counterintuitive, its numerical implementation is straightforward, since the value of \( \psi \) is equal on both sides of the peninsula (no normal-flow boundary condition implies \( \partial \psi / \partial n = 0 \) on meridional walls).

As it is well known that topographic form drag plays a dominant role in the zonal momentum budget in the Drake Passage latitudes, we introduce topography (see Fig. 1). Following a number of studies (e.g., Wolff et al. 1991; Straub 1993; Krupitsky and Cane 1997; Tansley and Marshall 2001), we introduce a topographic ridge that is sufficiently high so as to block lower-layer potential vorticity contours. Since the lower layer is not forced directly, this effectively constrains the circumpolar flow to the upper layer. The specific topography chosen is a Gaussian ridge mimicking the Scotia Ridge and following the western boundary as a half-Gaussian that might be thought of as a continental rise. To make the comparison cleaner, an identical ridge is also introduced in the closed-basin experiments. As such, our closed-basin experiments differ somewhat from the traditional flat-bottomed double-gyre problem, since the topography introduces an asymmetry between the subtropical and subpolar gyres (see Becker and Salmon 1997). Comparison with the more standard flat-bottom basin case is given in section 4.

b. A preliminary box-to-ACC comparison

We now discuss the result of a preliminary Drake Passage opening experiment to compare solutions
our closed-basin and Southern Ocean geometries. We consider a weakly forced case in a 3840 km × 9600 km domain. The limit of weak forcing is considered, since Sverdrup dynamics might reasonably be expected to apply. The model was integrated until statistical equilibrium was reached and then long-time averages (i.e., 100 yr) were taken. Figure 2 shows the characteristic contours and time-averaged streamfunctions for each layer. Note that the characteristic function is not known a priori in the ACC case, since the barotropic streamfunction is not determined by (1) in the Drake Passage latitude band. The contours plotted for the ACC case use the model time-averaged $\psi_B$ field in (3), rather than its estimate from the linear Sverdrup theory (as plotted in the box case). Also note that the interpretation of characteristics over the bottom topography is unclear, since topography is neglected in the derivation of Eq. (3) [see also Dewar (1998) and section 4b(3)].

As expected, the Sverdrup circulation is observed for the time mean interior flow in the box case in Fig. 2. Indeed, except for a small region near the western boundary, eddy activity does not significantly affect the long-term averaged fields. Moreover, it is evident that areas of blocked characteristics (most of the domain) correspond to areas where the lower layer is at rest, consistent with Rhines and Schopp (1991). Note also that, as anticipated given our choice of topography, there is an absence of circumpolar transport in the lower layer: the meridional ridge of Fig. 1b prevents any streamlines from reconnecting zonally. Comparing $\psi_1$ for the box and ACC cases suggests that the ACC might be thought of as the sum of a basin and a channel contribution. The basin part, north of Drake Passage\(^2\), is remarkably similar to the gyrelike circulation of the box case, with a Sverdrup interior and a weak inertial recirculation to the west. The eastward jet heading toward Drake Passage includes both streamlines collected from this basin Sverdup flux and a smaller number of streamlines that remain in or near Drake Passage latitudes. The former will be referred to as the basin contribution and the latter will be referred to as the channel contribution to the circumpolar transport.

3. Analytic models

We now want to build on the close correspondence between the box and ACC solutions north of our model Drake Passage to work out an analytic prediction for the ACC transport. This correspondence may be less evident in cases where the Drake Passage opening is significantly wider. Thus, we restrict the discussion to cases where the $L_{\text{gap}}$ is relatively small compared to the basin latitudinal extent, although the effect of widening the Drake Passage is considered at the end of section 4. Moreover, we consider the maximum of the wind stress to be north of the Drake Passage, so that there is a southward Sverdrup flux into the channel region.

To a first approximation, we will consider the total ACC transport as being the sum of basinlike and channel-like contributions:

$$T = T_{\text{Channel}} + T_{\text{Basin}}.$$ (7)

Below, we argue that $T_{\text{Channel}}$ can be considered roughly as a constant background contribution, whereas $T_{\text{Basin}}$ increases linearly with the forcing strength until a threshold is reached, after which the transport saturates, becoming independent of the forcing strength. The point at which this transition occurs is linked to the geometry of the characteristics.

a. Channel contribution

One of the geometries considered by Tansley and Marshall (2001) was a channel with a blocking ridge that prevented circumpolar flow in the abyssal layer. In this geometry, they found Straub’s (1993) prediction to work in weakly forced cases. That is, marginal instability gives the interface height field slope to be approximately $-\beta H_{\text{gap}}/f_0$, so that thermal wind gives the transport to be approximately

$$T_{\text{Channel}} \approx \beta L_{\text{gap}} H L_{\text{gap}}.$$ (8)

Since this limit applies for weak forcing, we consider it to be a minimum estimate.

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\(^2\) Note that although there is a second basinlike part associated with the Antarctic Peninsula in the southernmost part of the domain, we will consider it as irrelevant to the ACC, considering its smallness.
For stronger forcing, a stronger source of eddies is required. Assuming the eddy source to be baroclinic instability in the vicinity of the current, this implies a stronger interface tilt and correspondingly stronger transport. In a flat-bottomed channel, the tilt appears to increase like $t_0^{1/2}$ as predicted by Johnson and Bryden (Tansley and Marshall 2001). In our ACC geometry, however, a basin contribution also adds to the transport—and hence to the interface tilt. Following Stommel, this should increase linearly with $t_0$ (i.e., faster than $t_0^{1/2}$), at least over a range of $t_0$. It then seems reasonable to assume that this provides the additional source of baroclinic instability needed to fuel a more vigorous eddy field. That is, it seems plausible that (8) sets a minimum transport that must be added to a basin contribution (see below).

b. Basin contribution

Using the preliminary result presented in section 2, we consider as a first approximation that Sverdrup dynamics applies north of DP. More specifically, we consider $\psi_B$ to be given by (1) and the lower layer to be at rest on blocked geostrophic contours. Since our choice of topography precludes circumpolar flow in the lower layer, we define the “basin contribution” to the ACC as only the upper-layer portion of the Sverdrup flux into DP latitudes.

1) STOMMEL REGIME

For weak forcing, characteristics are nearly zonal and blocked nearly everywhere in the domain (Fig. 2). As such, the lower layer is at rest in the DP region, so that the Sverdrup transport into the DP latitude band is top trapped. In this limit, the basin contribution is similar to that predicted by Stommel:

$$T_{\text{Stommel}} = \left[ \int_{x_{\text{west}}}^{x_{\text{east}}} \frac{\mathbf{k} \cdot \nabla \times \tau}{\rho_0 \beta} \, dx \right] \bigg|_{y = y_{\text{DP}}},$$

where $x_{\text{west}}$ is a longitude near the western boundary and $y_{\text{DP}}$ is the northern extent of the DP latitude band. As mentioned, $T_{\text{Stommel}}$ increases linearly with $t_0$.

We now argue that this Stommel regime is valid from weak to moderate forcing until the transport reaches a threshold value at which it saturates. This threshold occurs when we can no longer assume that the Sverdrup transport into the DP latitudes is top trapped everywhere along $y = y_{\text{DP}}$.

2) SATURATION REGIME

Figure 3 shows the characteristic function (3) and barotropic streamfunction (1) for three values of $t_0$. The thick lines in each of the $\psi_B$ panels show the southern branch of a separatrix in the characteristic field. We
FIG. 3. The two basic circulation regimes (Stommel and saturation) for the ACC case. Note that the streamlines shown in the DP latitude band are drawn for illustration purposes only. They are not solutions of (1). Here, \( x_0 \) is the longitude at which the separatrix \( \Theta \) contour intersects the northern limit of DP \( (y = y_{DP}) \).
expect the lower layer to be at rest on blocked geostrophic contours, for example, to the south of the separatrix and north of \( y = y_{DP} \) (shaded area in Fig. 3). For forcing stronger than a threshold value, the separatrix reaches \( y = y_{DP} \) somewhere east of the western boundary layer. At this point, we can no longer assume all the Sverdrup transport into the DP latitudes to be top trapped. Let \( x_0 \) (lower-right panel in Fig. 3) denote the longitude at which the separatrix intersects \( y = y_{DP} \). For \( x > x_0 \), the Sverdrup flux into DP is top trapped, whereas to the west it is not. Recall that in \( T_{Basin} \), we are interested in the upper-layer portion of the Sverdrup flux across \( y = y_{DP} \). Let \( T_u \) be the portion lying west of \( x_0 \) and let \( T_s \) be the portion to the east: \( T_{Basin} = T_u + T_s \).

To solve for \( T_e \), we can compute the upper-layer mass flux entering in the channel region:

\[
T_e = -H \psi_3(x, y_{DP}) \bigg|_{y = y_{DP}}^y = L_x. 
\]

Since the flow is top trapped to the east of \( x_0 \) and taking \( \psi_B \) to be zero at \( x = L_x \) on the northern peninsula, we have

\[
T_e = -H \psi_B(x, y_{DP}) \bigg|_{x = x_0}^{x = L_x} = H \psi_B(x_0, y_{DP}). 
\]

Then using (3) at \((L_x, L_y/2)\) and \((x_0, y_{DP})\),

\[
\Theta_{separatrix} = \beta L_p^2 \frac{L_y}{2} = \beta L_p^2 y_{DP} + \psi_B(x_0, y_{DP}),
\]

so that \( T_s \) is given by

\[
T_e = H \beta L_p^2 \left[ \frac{L_y}{2} - y_{DP} \right].
\]

Note that this is proportional to \( L_y^2 \) and to the meridional distance between \( y_{DP} \) and the latitude at which the separatrix characteristic intersects the eastern boundary. Note also that (13) is independent of the forcing strength. This is due to two counteracting effects: stronger wind forcing causes both an increase in the Sverdrup velocity and a shift of \( x_0 \) to the east, such that the two effects cancel.

What sets \( T_u \) is less obvious, although one possibility is that it may be small. For example, the upper-layer Sverdrup flux in this region could feed a recirculation, as illustrated schematically in Fig. 3. A plausibility argument for this comes from consideration of \( \psi_2 \). Since topography precludes a lower-layer circumpolar flow, that part of the Sverdrup flux across \( y = y_{DP} \) occurring in the lower layer must form part of a closed recirculation or gyre. Viewed from the perspective of upper-layer dynamics, this ridge in the \( \psi_2 \) field would act as an effective topography, steering the upper-layer flow so that it too ultimately returns westward as part of a recirculation. Taking this as the null hypothesis, we have that \( T_{Basin} = T_u + T_e \approx T_e = T_{saturation} \), since (13) is independent of the forcing \( \tau_0 \).

Overall, the basin contribution to the ACC is then given by \( T_{Stommel} \) from weak to moderate forcing and by \( T_{saturation} \) for stronger forcing:

\[
T_{Basin} = \begin{cases} 
T_{Stommel} = \left[ \int_{L_y}^{L_y} \frac{\mathbf{k} \cdot \nabla \times \tau}{\rho_0 \beta} \, dx \right]_{y = y_{DP}} & : \tau_0 < \tau_0[\text{threshold}] \\
T_{saturation} = H \beta L_p^2 \left( \frac{L_y}{2} - y_{DP} \right) & : \tau_0 > \tau_0[\text{threshold}] 
\end{cases}
\]

4. Numerical results

We now want to test our analytic predictions using the numerical model in section 3 with the geometry shown in Fig. 1b. We begin with a basin dimension of \( L_y \times L_x = 3840 \text{ km} \times 9600 \text{ km} \). Since this is approximately 4000 km \times 10000 km, we will refer to this using the shorthand \( 4 \times 10 \). The runs are performed at a spatial resolution of 15 km with \( L_p = 32 \text{ km} \) and all other model parameters are the same as Table 1. After a spin-up phase, the
system reaches an equilibrium state to which statistics can be applied. The distributions for 100 yr of the transport time series at equilibrium display Gaussian statistics with well-defined means and standard deviations (not shown). The average and standard deviation of the transport are shown as a function of $t_0$ in Fig. 4. The analytic prediction—the sum of Eqs. (8) and (14)—is also shown.

The general shape of the analytic prediction is basically respected by the numerical results—for example, there is a nonzero minimum at very weak wind stress, followed by a growing phase and a saturation phase. However, from Fig. 4, it is obvious that the observed saturation occurs at a smaller value than predicted by the simple theory. In the following subsection, we argue that this behavior is due to transient eddies associated with zonal jets in the ocean interior that alter the baroclinic structure of the flow.

### a. Influence of transient eddies

Figure 5 shows mean contours of $\Theta$ and $\psi_2$ in the observed saturation regime ($\tau_0 \geq 0.1 \text{ N m}^{-2}$). Note that the lower layer is not at rest over regions of blocked characteristics, in contrast to predictions of Rhines and Young (1982) and numerical simulations of Rhines and Schopp (1991). Specifically, $\psi_2 \neq 0$ to the south of the analytic separatrix (thick lines) and north of $y = y_{DP}$ (dotted lines). Figure 5 also shows the contours of the characteristic function (3) computed with the time-averaged $\psi_B$ field. In the basin part, away from the western boundary, the analytic separatrix (superimposed on the contours of $\Theta$ in Fig. 5) matches the observed division between blocked and free contours. This results because the time-average barotropic solution remains in Sverdrup balance for the large-scale interior circulation north of $y_{DP}$. The baroclinic structure, on the other hand, is significantly altered relative to the theory. Because of this, (11) no longer follows from (10), and (13) becomes an overestimate. That is, there is a reduction in $T_{\text{saturation}}$ compared to the theory (Fig. 4). Below, we show this to be related to an eddy forcing of the lower layer.

![Figure 5](http://journals.ametsoc.org/doi/pdf/10.1175/2008JPO4023.1)
To investigate the influence of transient eddies on our numerical simulations, we perform a time decomposition of (4) and (5) at statistical equilibrium. Ignoring dissipation terms,
\[
J(\psi_1, h_1) + J(\psi_1', h_1') + F_1 \left[ J(\psi_1, h_2) + J(\psi_1', h_2') \right] + \beta h_1 \psi_1 = \text{Forcing and}
\]
\[
J(\psi_2, h_2) + J(\psi_2', h_2') + F_2 \left[ J(\psi_1, h_2) + J(\psi_1', h_2') \right] + \beta h_2 \psi_2 = 0,
\]
where overbars denote time averages and primes denote transients. Note that the term involving \(J(\psi_2', h_0)\) has been omitted, since \(h_0\) is zero in the interior of the domain. Note also that (16) can be thought of as a generalization of the characteristic Eq. (2). A scale analysis suggests that eddy advection of stretching vorticity should dominate over relative vorticity advection (cf. Pedlosky 1996). Indeed, long-time averages (200 yr) of our model bear this out: \(F_2 J(\psi_1, h_2')\) clearly dominates in the region of blocked characteristics in the lower layer and the characteristic Eq. (2) becomes
\[
J(\psi_2, \Theta) = F_2 J(\psi_1', h_2').
\]
In the upper layer, the eddy activity is much stronger and longer averages are needed to smooth out the noise produced by the advection of relative vorticity. Nevertheless, here again, stretching vorticity dominates in the region of interest. This symmetry between the two layers explains why the time-averaged barotropic Sverdrup balance still applies, consistent with standard theory.

1) ZONAL JETS

The modified baroclinic structure of the flow appears related to the presence of zonal jets in the instantaneous \(\psi\) fields. For instance, Fig. 6 shows a snapshot of the potential vorticity in both layers for \(r_0 = 0.15 \text{ N m}^{-2}\) (corresponding to the upper panels of Fig. 5). A clear signal of zonal jets is visible in both layers. These jets are expected in \(\beta\)-plane turbulence (Rhines 1975) and have been found both observationally in satellite altimetry data (Maximenko et al. 2005) and numerically (e.g., Panetta 1993; Nakano and Hasumi 2005; Nadiga 2006).

In Fig. 6, we see that the jets extend beyond the separatrix and over the region of blocked contours, where \(\psi_2 \neq 0\) (cf. Fig. 5).

It is important to note that jets appear not only for the basin part of the ACC circulation but are also observed in the closed-basin configuration. Figure 7 shows the mean streamfunctions obtained with the Southern Ocean configuration and the closed-basin configuration with and without bottom topography. As before, the similarity between the ACC and the closed basin with topography geometries is evident. This indicates that the modifications to the baroclinic structure by transient eddies are
not due to the opening of Drake Passage. The greater change occurs when bottom topography is removed in the closed-basin case. Indeed, as Rhines and Schopp (1991) observed, there is essentially no flow over blocked characteristics in this case. Moreover, the strength of the jets is reduced and their distribution becomes symmetric about the center latitude. This suggests that it is the bottom topography that leads to a southward displacement and an increase in strength of the jets. We speculate that this is related to our continental rise topography causing lower-layer potential vorticity contours to bend northward near the boundary. As such, contours in the southern portion of the domain—where the jets are strong—extend back to the western boundary confluence region, where eddy activity is intense. In other words, it seems plausible that information from the confluence region is communicated into the interior along these contours. This is consistent with jets forming in the southern gyre in the case with topography and contrasts with the flat-bottom case (cf. Fig. 7), where eddy influences are limited to a narrow region near the center latitude.

Associated with the southward displacement of the jets is an increase in eddy activity, which in turn leads to a change in the baroclinic structure of the flow. As we saw above, in the ACC geometry, this leads to a reduction of $T_{\text{saturation}}$. In the box geometry, an asymmetry between the northern and southern gyres occurs. For instance, in the upper layer, the northern gyre transport is $\sim 72 \text{ Sv}$ ($1 \text{ Sv} = 10^6 \text{ m}^3 \text{ s}^{-1}$), while in the southern gyre it is only $\sim 46 \text{ Sv}$. This is consistent with the premature ACC transport saturation seen in Fig. 4.

2) TRANSPORT SPECIFICATION FROM THE MODEL RESULTS

The modified baroclinic structure of the flow east of $x_0$ implies (11) no longer follows from (10). We now ask whether (10) can nonetheless be used to compute $T_{\text{Basin}}$. Figure 8 compares $T_{\text{Basin}}$ calculated using (10) and model output to the total transport. The two curves follow one another, except for an offset approximately equal to $T_{\text{Channel}}$. This is consistent with an inspection of mean $\psi_1$ contours, which reveals that $T_{\psi_1} = 0$ for all wind stress amplitudes (not shown). Note, however, that the $x$ position separating recirculating contours from those feeding the ACC need not correspond precisely to $x_0$. Small errors in the western limit of the integral (10) do not have a large effect on the transport. This is because the southward flux across $y_{\text{DP}}$ in the upper layer is relatively small in the region where the lower layer is in motion (i.e., near $x_0$). Because of this, (10) remains useful for estimating $T_{\text{Basin}}$, even when eddy effects are prevalent.

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3 Note that for a Drake Passage opening in the north of the basin, we would expect a higher saturation phase.
b. Robustness

Below, we test the sensitivity of our numerical simulations to different parameters affecting the transport. For example, the analytic model predicts three main factors setting the basin contribution: 1) $L_x$ will affect how steep the initial Stommel growth will be; 2) the distance between the maximum of the wind forcing ($L_y/2$ in our case) and the Drake Passage northern latitude band ($y_{DP}$) will determine the value of the saturation phase; and 3) the ocean depth and Rossby radius also affect the saturation level. The influence of adding bottom topography in the interior of the basin is also considered.

We begin with the influence of the basin length. Figure 9 presents results obtained for three different basin lengths: 1) $4 \times 10$, which has the same dimensions as in Fig. 4; 2) $4 \times 5$, where the basin length has been cut by half; and 3) $4 \times 20$, where the basin length has been doubled.

Results for $4 \times 20$ are very similar to $4 \times 10$. There is a nonzero minimum at very weak wind stress followed by two distinct circulation regimes at stronger wind stress: a growing phase that fits Stommel’s prediction and a saturation phase. The latter occurs at lower $t_0$ than predicted by the theory and, again, this is due to the effect of transient eddies (not shown). On the other hand, the $4 \times 5$ case is significantly different: its growing phase is steeper than Stommel’s prediction, while its saturation phase seems to fit the analytic prediction. In the following subsection, we argue that these differences are related to the effects of an inertial recirculation at the western boundary. This effect is more significant for smaller domains and plays a lesser role when $L_x$ is increased.

1) THE INERTIAL RECIRCULATION

We first focus on the weak wind stress regime, where Stommel’s prediction underestimates the initial growing phase (Fig. 9a). Figure 10 shows a comparison between the upper-layer streamfunctions in the ACC and box configurations at weak wind stress for the $4 \times 5$ domain size. In the closed-basin case (upper panel), strong eddy activity at the confluence of the two boundary currents creates an inertial recirculation on the western side of the basin. When Drake Passage is opened (lower panel), some of the streamlines associated with this recirculation appear to unfold and add to the basin contribution. To illustrate this idea, we define the curve C (thick latitudinal line in each panel of Fig. 10) and compute the mass flux across it. In both cases, this flux corresponds approximately to $T_{Stommel}$ (cf. section 3). That the circumpolar transport is larger that $T_{Stommel}$ is then related to additional streamlines entering Drake Passage to the west of C. Further evidence that the underestimate of Stommel’s prediction is related to the inertial recirculation comes from considering the box case. There, the total upper-layer transport of the southern gyre also increases with $t_0$ at a rate faster than $T_{Stommel}$ and is similar to what is seen in Fig. 9a (not shown).

In sum, for small $L_x$ we end up with two competing effects: inertial effects increase the transport, while eddy effects tend to lower $T_{saturation}$. Therefore, the
apparent fit between the observed and predicted saturation values in the $4 \times 5$ case is most likely the result of these two compensating effects. However, the strength of the inertial recirculation does not depend strongly on the basin length (not shown), whereas Stommel’s contribution does. Therefore, as the basin length is increased, the inertial recirculation will become negligible in comparison to the Sverdrup flow entering the channel region. We emphasize that the inertial recirculation influence is probably limited in the actual Southern Ocean, considering its vast zonal extent. That this effect is weak for $4 \times 20$, which has dimensions close to the Southern Ocean, supports this claim.

2) Influence of $L_\rho$

Figure 11 shows the transport values obtained when varying the Rossby radius to test its influence on the saturation level. We chose large basin dimensions ($4 \times 10$) to avoid the superfluous effect of the inertial recirculation. The horizontal grid spacing is kept at $\Delta x \approx L_\rho/2$, so that eddies have similar resolution in each case. For a given Rossby radius, we compute the average value of four experiments performed with different wind stress amplitudes ($\tau_0$) taken in the saturation regime. Moreover, the wind stress amplitudes are chosen so that the same set of four separatrix contours is obtained for each $L_\rho$. Figure 11 shows that the relationship between the saturation value and $L_\rho$ is approximately quadratic, as predicted by Eq. (13). However, comparing results with this analytic prediction shows that the premature ACC transport saturation seen in Fig. 4 is also observed for all chosen values of $L_\rho$. This suggests that the effect of transient eddies on the transport is independent of the Rossby radius.
3) THE ADDITION OF A MIDBASIN RIDGE

Apart from the Scotia Ridge, the ACC encounters other major topographic obstacles, for example, the Kerguelen Plateau and the Pacific–Antarctic Ridge. Hughes (2002) argued that the transport depends on the relative amount of form stress falling across Scotia Ridge, that is, as opposed to across other topography. On the other hand, Tansley and Marshall (2001) tested the effect of downstream topography on the circumpolar transport with basin dimensions similar to a $2 \times 5$ domain and found it to be rather small and unpredictable. We now revisit this experiment in the context of the much larger $4 \times 20$ basin. The topography added is a 2-km-high meridionally oriented Gaussian ridge at $L_y/2$.

Figure 12 shows the transport values obtained in this experiment. Few changes are observed compared to the case without the ridge. The main difference is that for $\tau_0$ values greater than 0.2 N m$^{-2}$, saturation transport is increased slightly (a factor of $\sim 15\%$). Figure 13 shows the streamfunctions and the characteristic contours with and without the midbasin ridge for wind forcing strength $\tau_0 = 0.2$ N m$^{-2}$. Note that the introduction of topography in the basin interior makes the characteristic problem more subtle. Following Dewar (1998), we can define characteristics by assuming the lower layer to be at rest ($\Theta_{topo}$, Fig. 13g). The $\psi_1$ field then follows from the Sverdrup relation and the rest-state potential vorticity field for the lower layer defines characteristics. Where these are blocked (and ignoring eddy effects), the assumption of a motionless lower layer is consistent. Where they are closed, we expect circulation to develop in the lower layer. A comparison of Figs. 13d,g is consistent with this assessment. Interestingly, the small increase in transport seen for $\tau_0 \geq 0.2$ in Fig. 12 appears to be associated with the closed contours overlying the ridge. The lower-layer circulation associated with this feature is considerably smaller for $\tau_0 = 0.1$ and is absent altogether for weaker forcing. It then seems possible that other choices of topography (i.e., that evoke closed characteristics) may lead to larger discrepancies between simulations and theory. For our choice of topography, however, the effect of the ridge appears localized to its longitudes. This contrasts with the “continental rise” topography which, as we have seen, affects the strength and distribution of the zonal jets and leads to lower-layer motion outside of the separatrix contour.

4) DISTANCE BETWEEN $y_{DP}$ AND $L_y/2$

As mentioned in the introduction, this article focuses on basinlike dynamics. As such, we chose geometries for which Drake Passage occupies only a small range of latitudes. By this choice, we wanted to isolate the basin contribution to analyze its behavior. It can be argued, however, that the actual width of the Drake Passage is larger and that the distance between $y_{DP}$ and $L_y/2$ is smaller than what we have used until now. Coming back to the analytic model saturation regime (see section 3), the sum of the channel and basin contributions gives

$$T = HBL_{pl}^2(L_y/2 - y_{DP} + L_{gap}) = HBL_{pl}^2(L_y/2 - y_{south}),$$

where $y_{south}$ is the southern edge of Drake Passage. In other words, in the saturation regime, the relative contributions of the channel and basin are proportional to their meridional extents. Thus, the analytic theory predicts no change in the total transport saturation value as $L_{gap}$ is increased by moving $y_{DP}$ northward.

Figure 14 shows analytic predictions and numerical results obtained for a progressive widening of the gap in
FIG. 13. Contours of $\psi_1$, $\psi_2$, and $\Theta$ for the $4 \times 20$ basin case at $\tau_0 = 0.2$ N m$^{-2}$ (b), (d), (f) with and (a), (c), (e) without (left) a 2-km-high midbasin ridge. (g) Contours of $\Theta_{\text{topo}} = \beta y + (\frac{1}{2}l_z^2)\psi_h + (\frac{1}{2}l_z^2)h_0$. 
the $4 \times 10$ geometry. We open the channel from $L_{\text{gap}} = 240$ km—the value used until now—to $L_{\text{gap}} = 1680$ km, which corresponds to $y_{\text{DP}} = L_y/2$. Larger values of $y_{\text{DP}}$ lead to saturation values closer to the analytic prediction. This is consistent with our earlier results. The undersaturation seen for narrow gaps was associated with the basin contribution being reduced relative to the theory. For wide gaps, the theoretical basin contribution is small and reducing it has little effect on the total transport.

The case $L_{\text{gap}} = 1680$ km is of particular interest since $T_{\text{Basin}}$ is identically zero. Figure 14 shows that the transport overshoots the analytic prediction (8) at low $\tau_0$ and then slowly decreases until saturating near the analytic value for stronger forcing. This result differs drastically from the pure channel case (i.e., with a wall at $y = L_y/2$). Indeed, in their channel plus ridge geometry, Tansley and Marshall (2001) found the transport to increase rapidly for weak forcing and then follow a power law ($\tau_0^{1/13}$; see their Fig. 11) at stronger forcing. Thus, for forcings corresponding to our saturation regime, the pure channel gives transports well in excess of (8), whereas our results do not. This suggest that the northern gyre plays an important role even when $T_{\text{Basin}} = 0$. We speculate that the presence of the adjacent gyre serves as a source of eddies for the channel latitudes. With this source of eddies, increased interface tilt (fuelling increased baroclinic instability) is not required to close the meridional circulation, as assumed in channel-like theories of ACC transport (cf. section 3). Conversely, for weak forcing, channel-like dynamics appear to dominate so that the interface tilt (and transport) increases with the forcing strength.

The Southern Ocean likely lies between the extremes of $L_{\text{gap}} = 240$ km and $L_{\text{gap}} = 1680$ km and a behavior intermediate to the two is expected. Indeed, the cases of $L_{\text{gap}} = 720$ km and $L_{\text{gap}} = 1200$ km exhibit such behavior. For example, each curve shows a Stommel regime at weak forcing followed by a weak overshoot and undersaturation for stronger forcing.

5. Conclusions

The main objective of this paper was to revisit Stommel’s idea that basin dynamics play a significant role in determining the ACC transport. We develop an analytic model in which the total transport is composed of basin and channel contributions. The channel contribution is determined by the marginal stability argument proposed by Straub (1993) and is independent of the forcing strength. The basin contribution depends critically on the geometry of the separatrix characteristic. Two regimes result. For weak forcing, the separatrix extends across the basin and for strong forcing it crosses $y = y_{\text{DP}}$. This defines $x = x_0$, which is a point just east of the western boundary layer (on $y = y_{\text{DP}}$) in the weak forcing regime and the point at which the separatrix crosses $y = y_{\text{DP}}$ for strong forcing. The basin contribution then corresponds to the upper-layer portion of the Sverdrup flux across $y = y_{\text{DP}}$ and east of $x = x_0$. 

![FIG. 14. Mean circumpolar transport and standard deviation as a function of the wind stress amplitude in the $4 \times 10$ geometry for four different gap widths: $L_{\text{gap}} = 240, 720, 1200,$ and $1680$ km. Also shown are their respective analytic predictions. Note that the width of the Gaussian ridge has been narrowed for this experiment to facilitate the match between the tip of the peninsula and the bottom topography. This results in a slight increase in transport in the $L_{\text{gap}} = 240$ km case (cf. Fig. 4).](https://journals.ametsoc.org/content/jpo/39/6/1000.full.pdf)
For weak forcing, it increases linearly with \( \tau_0 \), following Stommel’s (1957) prediction. For strong forcing, transport saturates. That is, increasing winds causes both an increase in the total Sverdrup flux across \( \gamma_{DP} \) and an eastward migration of \( x_0 \). The two effects cancel and \( T_{\text{Basin}} \) becomes independent of wind strength. Physically, instead of adding to the circumpolar flow, stronger forcing feeds a recirculation west of \( x_0 \).

Numerical results are broadly consistent with the analytic model, though some discrepancies are observed. This is not surprising given the turbulent nature of the flow. The first main discrepancy relates to zonal jets. Transient eddies modify the characteristic Eq. (2), allowing for lower-layer flow outside of the separatrix. The barotropic transport and by extension the geometry of the characteristics are, however, not strongly affected by the eddies. Instead, some of the Sverdrup flux east of \( x_0 \) occurs in the lower layer, and \( T_{\text{Basin}} \) is reduced relative to our analytic estimate. A second discrepancy occurs in small domains, where inertial effects add to \( T_{\text{Basin}} \). Comparison between closed-basin and Southern Ocean configurations suggests that some of the streamlines associated with the inertial recirculation in the box case unfold and add to the circumpolar flow when Drake Passage is open. The overall effect is then to increase the transport in both the weak and strong wind regimes. However, this effect is negligible for large basin lengths where the Sverdrup flux is large compared to this inertial recirculation. In sum, for large basins the saturation value is lower than the analytic prediction as a result of transient eddy effects, while for small basins inertial effects are more important and transport is closer to the analytic prediction.

Various other robustness tests were also carried out. In particular, we tested robustness of our results to (i) the presence of an additional ridge far from the western boundary; (ii) variations in the strength of the stratification; and (iii) the width of the gap at Drake Passage. Generally speaking, results remained broadly consistent with the theory. Although the additional meridional ridge affected the solution locally, transport values were much as before. Simulations over a range of Rossby radii showed a remarkable consistency with the theory (which has transport scaling like the square of the Rossby radius). Finally, although the width of Drake Passage did influence the results somewhat, transports nonetheless remained close to predicted values. Disagreement was largest for weak forcing, where model transport was larger than theory but was small elsewhere.

Obviously, the Southern Ocean is far more complex than our simple two-layer quasigeostrophic model, and the many effects we neglected may be important to determining the observed transport. Even in the quasigeostrophic context, effects relating to stratification, surface density, vertical resolution, rough topography, bottom drag, and horizontal dissipation may all play a role. Additionally, realistic wind fields will certainly affect results and equations such as (13) should not be applied naively. For example, in the case of a nonsymmetric wind field \( L_y/2 \), the \( y \) position of the separatrix at \( x = L_x \) need not correspond to the latitude of maximum zonally averaged wind stress. Despite these caveats, the clear correspondence between our box and ACC geometries strongly suggests that gyrelike dynamics are fundamental to the Antarctic Circumpolar Current.

Acknowledgments. We would like to thank two anonymous reviewers for their helpful comments and FORNT, Hydro-Quebec, and NSERC for their financial support.

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