Turbulence and Wind Shear in Layers of Large Doppler Spectrum Width in Stratiform Precipitation

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(Manuscript received 8 January 2008, in final form 25 September 2008)

ABSTRACT

Weather radar observations of stratiform precipitation often reveal regions having very large measured Doppler spectrum widths, exceeding 7, and sometimes 10, m s\(^{-1}\). These widths are larger than those typically found in thunderstorms; widths larger than 4 m s\(^{-1}\) are associated with moderate or severe turbulence in thunderstorms. In this work, stratiform precipitation has been found to have layers of widths larger than 4 m s\(^{-1}\) in more than 80% of cases studied, wherein the shear of the wind on scales that are large compared to the dimensions of the radar resolution volume is the dominant contributor to spectrum width. Analyzed data show that if width \(\lesssim 7\) m s\(^{-1}\), and if the layers are not wavy or patchy, these layers have weak turbulence. On the other hand, regions having widths \(\gtrsim 4\) m s\(^{-1}\) in patches or in wavelike structures are likely to have moderate to severe turbulence with the potential to be a hazard to safe flight. To separate the contributions to spectrum width from wind shear and turbulence and to evaluate the errors in turbulence estimates, data have been collected with elevation increments much less than a beamwidth. Despite beamwidth limitations, the small elevation increments reveal impressive structures in the fields. For example, the “cat’s eye” structure associated with Kelvin–Helmholtz waves is clearly exhibited in the fields of spectrum width observed at low-elevation angles, but not in the reflectivity or velocity fields. Reflectivity fields in stratiform precipitation are featureless compared to spectrum width fields.

1. Introduction

Measurement of turbulence is of both practical and theoretical interest. The rate at which turbulent energy dissipates as heat depends on the intensity of turbulence within the inertial subrange, and the safety of flight depends on avoiding regions of intense turbulence. Weather radars have been used for these purposes for a long time (e.g., Atlas 1964; Gossard and Strauch 1983; Mahapatra 1999; Doviak and Zrnić 2006, see also the preface with links to online errata available at www.nssl.noaa.gov/papers/books). Vertical profiles of wind and turbulence in the clear atmosphere are obtained with long-wavelength (i.e., \(\lambda > 30\) cm) radar wind profilers (Woodman and Guillen 1974; Hocking 1983, 1988; Holloway et al. 1996), but the measurements strictly apply above the radar. Short-wavelength (i.e., \(\lambda \lesssim 10\) cm) weather radars typically do not have the capability to reliably measure shear and turbulence in the clear air, but can make measurements over vast regions of precipitation and clouds (Doviak and Zrnić 2006; Melnikov et al. 2007).

Turbulence in clouds and precipitation can be estimated with weather radars by measuring the spatial correlation function of the mean Doppler velocity, \(v_m\) (e.g., Mel’nickhuk 1966; Brewster and Zrnić 1986), or the velocity spectrum width, \(\sigma_v\) (Doviak and Zrnić 2006, section 10). The latter approach is used in our study. Accurate radar measurement of turbulence depends on knowing the shear of the radial component \(v_r(r)\) of the mean or steady wind, principally at points \(r\) within the radar’s resolution volume, \(V_6\) [i.e., the volume of space enclosed by the 6-dB surface of the radar’s two-way weighting function; Doviak and Zrnić (2006, section 4.4.4)]. Within the context of this paper, \(v_r(r)\) is associated...
with the temporal or horizontal average of wind over domains much larger than the overturning time or scales of turbulence that contribute to spectrum width measurements. Because of $v_s(r)$ shear, procedures are needed to separate the shear and turbulent contributions to the measured spectrum width $\sigma_r$; the diacritical indicates an estimated value. In addition, $\hat{\sigma}_r$ includes contributions from spectral broadening mechanisms other than shear and turbulence (Doviak and Zrnić 2006, section 5.3); all these mechanisms need to be taken into account. Børresen (1971), conducting radar observations in stratiform precipitation at slant soundings, concluded that vertical shear could not be neglected in determining turbulence. He used data collected with $2^\circ$ elevation increments (i.e., twice the beamwidth) at relatively close ranges (i.e., $12$–$17.5$ km), and expressed concern that such relatively large elevation increments (i.e., about $500$ m in height) were too coarse for accurate measurement of shear and turbulence. As shown by Chapman and Browning (2001), the correct separation of the shear and turbulent contributions to $\sigma_r$ is a necessary step in measuring turbulence, and is one of the aims of this paper.

We show that the $v_s(r)$ shear contribution to $\sigma_r$ can dominate the turbulent one in layers of large $\sigma_r$, and that an overestimation of turbulence is likely due to the underestimation of the vertical (i.e., elevation) shear. Underestimates of shear are caused by a too large beamwidth, and/or too coarse of an elevation angle step and corresponding height increments; both underestimate the $v_s(r)$ shear. Local shear can be determined using $\hat{v}_m$ (i.e., the first moment of the estimated Doppler spectra calculated from data collected in a dwell-time $T_D$ typically less $0.1$ s). But $\hat{v}_m$ includes, along with $v_s(r)$, a turbulent component. The impact of using $\hat{v}_m$ to estimate $v_s(r)$ shear is discussed in section 3. Fang (2003) estimated $\hat{v}_m$, measured at contiguous radar resolution volumes (i.e., $V_6$, locations spaced at the $1^\circ$ beamwidth, $\theta_1$, the one-way half-power width), and linear interpolation to estimate the shear contributions to $\sigma_r$, and found $v_s(r)$ shear often cannot be accurately estimated. Alternatively, one could apply higher-order interpolation techniques to better estimate $v_s(r)$ within $V_6$ as shown by Hocking (2003). But we chose to collect data with small elevation increments to minimize the effects of coarse-elevation sampling. Whether interpolation or dense sampling is used, if the shear is not uniform across $V_6$, numerical techniques will likely be required to calculate the shear contributions to $\sigma_r$ (see the appendix). We demonstrate that angular spacing much smaller than a beamwidth reduces appreciably the bias errors. In so doing, we were able to observe the “finer” structure of the $v_s(r)$ field, and also reveal interesting features of wind and turbulence. Beamwidth, however, still limits the observations of fine details and the accurate assessment of shear (see the appendix). If horizontal homogeneity of the wind can be assured, $v_s(r)$ can be calculated using velocity azimuth display (VAD) techniques (Lhermitte and Atlas 1961) at close ranges where vertical resolution is fine, and thus the $v_s(r)$ shear can be estimated more accurately (Fang 2003). Chapman and Browning (2001) implemented such an approach using a $60$-km-diameter circle centered on the radar to determine $v_s(r)$, and calculated the $v_s(r)$ shear using a fixed $200$-m vertical separation. Fang (2008) used Weather Surveillance Radar-1988 Doppler (WSR-88D) data in stratiform precipitation and ranges of less than $20$ km to obtain vertical steps of $v_s(r)$ as fine as $10$ m in a shear layer.

In stratiform precipitation, the median of the $\sigma_r$ data lies in an interval between $1$ and $3$ m s$^{-1}$ (Fang et al. 2004) in agreement with measurements made by Rogers and Tripp (1964) and Børresen (1971). Our observations, of $\sigma_r$ fields, also exhibit median widths of $1$–$3$ m s$^{-1}$, but often vast areas (i.e., hundreds of square kilometers) of exceptionally large $\sigma_r$ (i.e., larger than $6$ and even $10$ m s$^{-1}$) are found; these are larger than those median values found in squall lines (Fang et al. 2004).

In aviation meteorology, $\sigma_r$ data $\geq 4$ m s$^{-1}$ are used to indicate the potential for turbulence considered a hazard to aircraft and/or its crew and passengers (Lee 1977). A threshold of $4$ m s$^{-1}$ is used because it is accepted as an indicator of turbulence possibly hazardous to aircraft and/or its crew (Lee 1977; Evans 1985). Because $\sigma_r$ is a function of range and radar parameters, even if turbulence is homogeneous and has an outer scale larger than the dimensions of $V_6$, a better metric to assess the potential hazard to safe flight is the turbulent energy dissipation rate; this metric can be derived from $\sigma_r$ under some circumstances (section 3). Under these circumstances, the $4$ m s$^{-1}$ threshold corresponds to a turbulent energy dissipation rate of about $1.6 \times 10^{-2}$ m$^2$ s$^{-3}$, corresponding to moderate turbulence (Hocking and Mu 1997, their Table 2). To convert the turbulent dissipation rate, $\epsilon$, to aircraft shocks, information about aircraft parameters is required, including weight, wing area, airspeed, and other factors.

Airplane response to turbulence is mostly affected by the along-track gradients of the vertical wind (Proctor et al. 2002), a component typically not measured with airborne or ground-based weather radars. Nevertheless, good correlation between the variance of vertical and along-track wind components has been observed in strong convection (Hamilton and Proctor 2006a,b). Lee (1977), Bohne (1981), Meischner et al. (2001), and Cornman et al. (2003) found, in thunderstorm environments, strong correlation between aircraft shocks and...
large $\sigma_v$ measured by airborne and/or ground-based weather radars. We call $\sigma_v$ “large” if it equals or exceeds $4 \text{ m s}^{-1}$. But we show that turbulence hazards to safe flight can often be nonexistent in stratiform precipitation, although $\sigma_v$ s are large. On the other hand, stratiform weather can also harbor regions of large $\sigma_v$ that could be a hazard to safe flight, and we provide examples of such cases.

Patterns of large $\sigma_v$ are shown in section 2, the procedure to estimate shear and turbulence is described in section 3, and section 4 contains the results.

2. Patterns of large spectrum width in stratiform precipitation

Our measurements were made with National Severe Storm Laboratory’s (NSSSL) research and development WSR-88D 11.1-cm wavelength radar (i.e., KOUN) located in Norman, Oklahoma. The $\sigma_v$ images are presented in two formats: 1) a conical section called a plan position indicator (PPI) and 2) a vertical cross section called a range–height indicator (RHI). All RHIs herein are obtained from elevation scans. The $\sigma_v$ data are displayed if the signal-to-noise ratio (SNR) is $10 \text{ dB}$ or larger. An example of a PPI with large $\sigma_v$ values is shown in Fig. 1a. Two major features of such fields are that 1) maximal $\sigma_v$ s are extremely large (e.g., exceeding $9.0 \text{ m s}^{-1}$ in Fig. 1a) and 2) regions of large widths often look like two facing semicircles, each having large $\sigma_v$ values about $180^\circ$ apart. Such patterns led to the suggestion that these large $\sigma_v$ regions are associated with a layer of $v_\rho(r)$ shear (Melnikov and Doviak 2002). The measured Doppler velocity field (not shown here) also shows the presence of $v_\rho(r)$ shear in the layer of large $\sigma_v$ s at heights between 2.5 and 3 km.

The RHI (Fig. 1b), at an azimuth of $35^\circ$ for the $\sigma_v$ field in Fig. 1a, clearly shows the layer of large $\sigma_v$ at a height of about 2.8 km. Results presented in the appendix show that the reduction of large $\sigma_v$ s at shorter ranges is principally due to the decreasing size of $V_o$. The decrease of the elevation shear due to smaller projections of the horizontal wind onto the radials at higher elevation angles also contributes to the decrease. The apparent increase in layer thickness at long ranges is due to the beam broadening with range.

To reveal the fine structure of strong $\sigma_v$ layers, RHI data were acquired at small elevation increments through the regions of large $\sigma_v$ observed in PPI displays. In constructing the RHI displays, data were collected with an elevation increment of $0.125^\circ$, about an eighth of the beamwidth, as the beam scanned elevation angles at a rate of $0.5^\circ \text{ s}^{-1}$. RHI displays clearly show layers (e.g., Figs. 1b, 1d, 1f, and 1g; the RHI in Fig. 1d is only displayed to $50 \text{ km}$ to better distinguish the wavy structure of the layer), whereas $\sigma_v$ s in PPI displays may look patchy (e.g., Fig. 1e). Vertical cross sections can also be constructed from consecutive PPI scans, but PPI data are typically collected with elevation increments equal to or greater than a beamwidth. In such-constructed RHIs, details of the layers are often difficult to recognize, but can be revealed by data collected with 0.125° increments (e.g., the wavy structure of the layer in Figs. 1d and 1f). Furthermore, coarse elevation increments generate significant bias errors in the estimates of turbulence intensity (see the appendix). The solid black lines in Fig. 1f show the three lowest elevation angles of the PPI scans used for the volume coverage pattern (VCP 11) for the network of WSR-88D weather radars. One can see that the region of large $\sigma_v$ lies below $1.5^\circ$ where only the scan at $0.5^\circ$ cuts through this layer.

Data from $\sigma_v$ fields in stratiform precipitation were collected with the KOUN radar during the cold seasons of 2001–06 (i.e., October–March inclusively). Surveillance was performed in the PPI scan mode followed by RHI scans through zones of large $\sigma_v$ observed in the PPI. Table 1 lists the properties of the $\sigma_v$ fields on 87 days. Regions of enhanced $\sigma_v$ appearing as horizontally stretched areas are defined as layers if their horizontal length is longer than $50 \text{ km}$ and $\sigma_v$ s are $\gtrsim 4 \text{ m s}^{-1}$. The horizontally quasi-uniform layer of large $\sigma_v$ at the height of about $3 \text{ km}$ in Fig. 1b is classified as a layer (“L” in Table 1), whereas isolated regions of $\sigma_v$ s $\gtrsim 4 \text{ m s}^{-1}$ (Fig. 1c) having horizontal dimensions $\lesssim 50 \text{ km}$ are classified as “patches” (P in the table). The maximal $\sigma_v$ s in Table 1 (i.e., $\sigma_v,\text{max}$) are ensured to be a representative one for the layer, and not associated with artifacts (e.g., aircraft echoes) or outliers, by stipulating that at least five occurrences of the tabulated $\sigma_v,\text{max}$ be present within $5 \text{ km}$.

Although there can be multiple layers of enhanced $\sigma_v$ fields (e.g., Fig. 1b at heights below $1 \text{ km}$, and at about 2.8 and $5.5 \text{ km}$; Fig. 1g at heights of 0.5 and $4.5 \text{ km}$), a $\sigma_v$ field is classified as a two-layer field (e.g., 2L in Table 1) if both layers have $\sigma_v \gtrsim 4 \text{ m s}^{-1}$ (e.g., Fig. 1g). If all regions and layers have $\sigma_v$ less than $4 \text{ m s}^{-1}$, the $\sigma_v$ field entry in the table is labeled as “uniform” (e.g., Fig. 1h).

For the majority of days (i.e., $\approx 70\%$), layers (labeled L or WL) of large $\sigma_v$ were found. A wavy layer (i.e., WL in Table 1; e.g., Figs. 1d and 1f) is defined as such if a wavelike structure can be resolved; these are to be distinguished from uniform layers (i.e., L in Table 1; e.g., Fig. 1b) of large $\sigma_v$. If there are multiple layers (e.g., 2L, 3WL), their parameters are separated by a solidus in Table 1. Sometimes layers of large $\sigma_v$ exhibit nearly periodic patches of very strong widths (e.g., Fig. 3c) suggestive of breaking waves. Such patterns are also
classified as wavy layers (i.e., WL) because of their spatial periodicity. In 12 cases (labeled U), the $\sigma_r$ fields were relatively uniform (i.e., $\sigma_r < 4 \text{ m s}^{-1}$ everywhere even though layers of enhanced $\sigma_r$ could be seen). Layers are not necessarily constant in height (e.g., Figs. 1f and 1g); $H$ in Table 1 is the mean layer height over the range that the layer was observed. No correlation was found between the maximal $\sigma_r$ and $H$. 

FIG. 1. Various patterns of $\sigma_r$ fields in (a),(e) PPIs and (b),(c),(d),(f),(g),(h) RHIs obtained from KOUN.
Table 1 contains data collected with the pulse repetition frequency (PRF) equal to 1013 Hz, corresponding to an unambiguous range of 148 km. The pulse-pair processing based on the zero- and one-lag correlation functions was used to estimate \( v_m \) and \( \sigma_v \). Surveillance PPI scans with PRF = 320 Hz have been used to ensure that no second trip echoes have contaminated the measured spectrum widths for heights above 2 km (\( \sigma_v \) was not measured in the surveillance scan). At lower heights, some second trip contaminations could have occurred that did not obviously alter the layer structure of the \( \sigma_v \) fields but that could have affected the number of maximal spectrum widths listed in Table 1; thus, these data should be taken with caution. The separation techniques outlined in section 3 below have been applied only for layers with heights above 3 km where there were no second-trip echoes.

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Values of $\hat{c}_v \geq 4 \text{ m s}^{-1}$ that correlate well with moderate or stronger turbulence experienced by aircraft in flights through thunderstorms (Lee 1977) also appear consistently in layers of stratiform precipitation. Thus, these layers could be classified as having the potential for being hazardous for safe flight. But in the cases of wavy and uniform $\hat{c}_v$, layers with $\hat{c}_{v_{\text{max}}} < 7 \text{ m s}^{-1}$, it is shown that the shear contributes mostly to $\hat{c}_v$. That is, the turbulent contributions are significantly less, so the flight danger is also less.

A general rule of thumb in preventing turbulence encounters is to avoid areas of reflectivity factors larger than 40 dBZ (Hamilton and Proctor 2006b). Measurements (Lee and Carpenter 1979) of turbulence with aircraft penetrations in and around thunderstorms suggest that pilots can avoid moderate or stronger turbulence by staying more than 15 km away from regions of large $\hat{c}_v$. Thus, these layers could be classified as having the potential for being hazardous for safe flight.

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The combined shear contribution $\overline{\sigma_s^2}$ can be estimated using $\overline{v_m}$. That is, $\overline{v_m} \approx \overline{v_r} + \overline{v_{\text{lat}}} \equiv \overline{v_r}$, where $\overline{v_{\text{lat}}}$ is the true radial velocity component of one member of the ensemble of large-scale turbulence. The approximation sign is used because $\overline{v_m}$ has variance associated with changes in the scatterer configuration, whereas $\overline{v_r} + \overline{v_{\text{lat}}}$ are the true weighted mean radial velocity and the true weighted velocity associated with turbulence on scales that are large compared to the dimensions of $V_o$. The $\overline{v_{\text{lat}}}$ is considered frozen and uniform across $V_o$ during $T_D$, and if the $v_r$ shear is also uniform, $\overline{\sigma_s^2}$ can be expressed as the sum of three shear addends (Doviak and Zrnić 2006, section 5.3):

$$
\overline{\sigma_s^2} = \overline{\sigma_{s,h}^2} + \overline{\sigma_{s,o}^2} + \overline{\sigma_{s,t}^2} = (r_o \sigma_o \overline{k_o})^2 + (r_o \sigma_o \overline{k_o} \cos \theta_e)^2 + (\sigma_r \overline{k_r})^2,
$$

where $\overline{\sigma_{s,h}^2}$, $\overline{\sigma_{s,o}^2}$ are the two-way second central moments of the power pattern in elevation and azimuth. Although shear is rarely uniform, we make this simplifying assumption to ease the calculations, and because for shear layers with thicknesses that are large compared to the beamwidth, the shear is practically uniform. The $\overline{\sigma_{s,h}^2}$ errors due to this assumption are evaluated in the appendix. For circularly symmetric beams, $\sigma_o = \theta/(4\pi /2) = 0.28^\circ (4.9 \times 10^{-3} \text{ rad})$ for the KOUN radar, and $\sigma_o = \sigma_o / \cos \theta_e$. Here, $\overline{k_o}$, $\overline{k_o}$, and $\overline{k_r}$ are the $\overline{v_m}$ shears in elevation ($\theta_e$), azimuth ($\phi$), and range ($r$). According to Eq. (5.76) of Doviak and Zrnić (2006), $\sigma_r = 0.35 \text{ c} / \text{r}$/2, where $c$ is the speed of light and $\tau$ is the transmitted pulse duration. For KOUN, $\sigma_r = 82 \text{ m}$. Radial shear, $\overline{k_r}$, is computed using $\overline{v_m}$ along a radial. The azimuth (elevation) shear is calculated by least squares fitting $\overline{v_m}$ at contiguous azimuths (elevations). In stratiform weather, horizontal shear is typically negligibly small, and thus the contribution to $\overline{k_r}$ from azimuth shear likewise should be negligibly small. On the other hand, vertical shear can be very strong, and thus its accurate estimation is a main focus of this work. Our approach of dense elevation sampling is similar to the dense radial sampling used by Hocking (2003) for the Mesoscale–Stratosphere–Troposphere Radar (MST). Sampling at elevation increments much less than a beamwidth provides a better representation of the $\overline{v_r}$ field and, consequently, a better estimate of $\overline{\sigma_{s,o}^2}$, as illustrated in the appendix.

To accurately separate the mean wind and large-scale turbulence shear from the small-scale turbulence contributions to $\overline{k_r}$ requires a detailed knowledge of $\overline{v_r}$, principally across $V_o$. Shear is typically underestimated using $\overline{v_m} = \overline{v_r}$. There are at least three causes for the underestimation: 1) coarse elevation steps lead to undersampling $\overline{v_r}$, 2) a finite beamwidth smoothes the estimates $\overline{v_m}$ of $\overline{v_r}$, and 3) the assumption of uniform shear in using Eq. (4). In the appendix, we illustrate how significant the underestimation of shear is due both to coarse elevation increments and to shear calculations based on $\overline{v_m}$.

To mitigate the effects of undersampling, elevation profiles of the Doppler velocity were executed using elevation increments (i.e., $0.125^\circ$) that are small compared to the beamwidth, $\theta_e$. Such fine increments were achieved using an elevation scan rate of $0.5^\circ \text{ s}^{-1}$ and a $T_D = 253 \text{ ms}$. With the pulse repetition frequency (PRF) of $1013 \text{ Hz}$ used in these experiments, $256$ weather signal samples were obtained for each estimate of the spectral moments. This large number of samples was chosen to reduce the statistical uncertainties of the $\overline{v_m}$ and $\overline{k_r}$ measurements. Although this large number of samples reduces measurement errors associated with rapid weather signal fluctuations (order of tens of milliseconds), the still short $T_D$ (i.e., $0.25 \text{ s}$) does not remove the variations of the $\overline{\sigma_s^2}$ estimates due to $2\overline{\theta_v} \overline{\delta v}$. To calculate the azimuthal shear, two RHIs, with a $2^\circ$ azimuthal separation, are used. This separation is obtained from an analysis of azimuthal shears calculated from data; the $2^\circ$ separation reduces significantly the false azimuthal shears associated with statistical uncertainties in $\overline{v_m}$ to levels below the observed mean azimuth shears, but yet preserves a reasonably high azimuth resolution. The two RHI scans, spaced $2^\circ$ apart in the azimuth and covering elevations from $0^\circ$ to $20^\circ$, were completed in $80$ s. It is assumed that the Doppler velocity and spectrum width fields do not change significantly during the $1.5\text{-min}$ period; observations show that the fields are stationary for much longer periods.

The RHIs for some Doppler velocity fields showed velocity aliasing (i.e., $|\overline{v_m}| > v_o = 27.9 \text{ m s}^{-1}$). Thus, velocity data were dealiased before calculating the shear. The azimuthal, elevation, and radial shears of $\overline{v_m}$ are calculated as follows. Let $A$ and $B$ denote the two RHIs, $l$ numerates the elevation angles in an RHI, and $n$ numerates the $V_o$ locations along a radial. Then, for $\text{RHI}_A$, the estimated azimuthal shear $\overline{k_o}$ (i.e., $\overline{\Delta \tilde{v}_m}$ per differential arc length, $r_o \cos \theta \Delta \phi$) for the $l$th radial and $n$th range is $k_o(l,n) = [\overline{\tilde{v}_m}(A,l,n) - \overline{\tilde{v}_m}(B,l,n)] / 2 \Delta \phi r_o \cos \theta$ where $\beta = 0.035$ rad is the azimuth increment between $\text{RHI}_A$ and $\text{RHI}_B$. The elevation shear is $\overline{k_o}(l,n) = [\overline{\tilde{v}_m}(A,l+1,n) - \overline{\tilde{v}_m}(A,l,n)] / 2 \Delta \theta$ where $\Delta \theta$ is the elevation increment; in our measurements, $\Delta \theta = 0.125^\circ (2.18 \times 10^{-3} \text{ rad})$. The radial shear is $\overline{k_r}(l,n) = [(\overline{\tilde{v}_m}(A,l,n+1) - \overline{\tilde{v}_m}(A,l,n))] / 2 \Delta r / c \tau$, where $c \tau / 2 = 250 \text{ m}$ is the radial separation of the $V_o$. Once $\overline{\sigma_s^2}$ is determined using (4), the small-scale turbulent contribution of $\overline{\sigma_s^2}$ to $\overline{k_r}$ is calculated using (3), assuming contributions from the coupled term are negligible.
Having calculated $\bar{\sigma}_{\text{sst}}$, an estimate, $\hat{\varepsilon}$, of the turbulent energy dissipation rate can be made using the formulas of Labitt (1981):

$$\bar{\sigma}_{\text{sst}} \approx A \Gamma(2/3)(\hat{\varepsilon} \sigma r_o)^{2/3}(1 - \gamma/15), \quad \text{if} \quad \sigma_s(r_o, \sigma_e) - 1 < 1, \quad \text{or}$$

$$\bar{\sigma}_{\text{sst}} \approx A \Gamma(2/3)(\hat{\varepsilon} \sigma r_o)^{2/3}(1 - 4\zeta/15), \quad \text{otherwise}, \quad (5a)$$

to first order in $\gamma$ and $\zeta$, where $A$ is the universal constant ($A \approx 1.6$), $\Gamma(x)$ is the gamma function, $\gamma = 1 - \sigma_{/}/(r_o \sigma_e)^2$, and $\zeta = 1 - (r_o \sigma_e/\sigma_r)^2$. For the WSR-88D, the range where $r_o \sigma_e$ equals $\sigma_e$ occurs at $r_o = 17$ km. So for $r_o > 17$ km, $(5a)$ is used and for shorter distances, $(5b)$ is applied. These formulas have been derived assuming that the outer scale of the inertial subrange is 2 or more times larger than the $V_s$ dimensions (Bohne 1981), and the $v_1$ shear is negligible. If the $V_s$ dimensions are larger than about one-half of the outer-scale $L_B$ of the inertial subrange of the turbulence scales, an estimate of the dissipation rate can be made using an estimate of $L_B$, and an assumption of the spectral form for scales larger than $L_B$ (Hocking 1999).

By using $\check{v}_m$ to calculate shear, we might have removed shear associated with large-scale isotropic turbulence, and thus we would underestimate $\hat{\varepsilon}$. There is no guarantee that scales of isotropic turbulence extend to 2 or more times the maximum dimensions of $V_s$ as required for the use of (5). Moreover, turbulence, on scales that are large compared to $V_s$, in stratiform weather could be anisotropic and we are thus justified in removing its contribution to $\sigma_s$ to have at least an estimate of $\sigma_{\text{sst}}$. If a reasonable guess of the outer scale of turbulence can be made, a good estimate of $\hat{\varepsilon}$ can be obtained (Bohne 1982).

4. Results

Measured $\check{v}_m$ and $\check{\sigma}_v$, and calculated $\check{\sigma}_{\text{sst}}$, and $\check{\sigma}_{\text{sst}}$ due to the elevation and range shear at $r_o = 40$ km, are presented as a function of elevation angle in Fig. 2. The azimuthal shear contribution is not shown in Fig. 2 because it is about two orders of magnitude less than the elevation shear contribution. In the region of large $\check{\sigma}_v$, $\check{\sigma}_{\text{sst}}$ follows well $\check{\sigma}_v$, demonstrating that the main contributor to $\check{\sigma}_v$ is the elevation shear of $\check{v}_e$. For distances beyond about 40 km, the procedure, described in section 3, frequently underestimates $\check{\sigma}_{\text{sst}}$, and produces larger $\check{\sigma}_{\text{sst}}$. As discussed in the appendix, $\check{\sigma}_{\text{sst}}$ is overestimated if the shear layer is narrower than the width of the beam.

The separation procedure in section 3 has been applied to all locations, and the resultant fields of $\check{\sigma}_{\text{sst}}$ are shown in Fig. 3 for four cases. The color scales were selected to highlight features; thus, specified values, obtained from digital data, might not be apparent in the figures. Figure 3a shows $\check{\sigma}_{\text{sst}}$ in which the maximum $\check{\sigma}_v$ (i.e., $\check{\sigma}_{\text{sst}}$,max), in the 1-km-thick layer centered at 2.7 km, was 11 m s$^{-1}$ (Table 1). The calculated $\check{\sigma}_{\text{sst}}$ in this layer shows a median value of about 1.7 m s$^{-1}$ and large spatial fluctuations. These spatial fluctuations can be explained, for the most part, by $\check{\sigma}_v$ variance due to the short $T_D$ (i.e., 0.25 s, which is short compared to the lifetime of the turbulent eddies that contribute to $\check{\sigma}_v$) used to make a measurement. In this layer, azimuthal shears were less than $0.7 \times 10^{-3}$ s$^{-1}$ and had a median value of $0.2 \times 10^{-3}$ s$^{-1}$, but the elevation wind shear had a maximal value of $51 \times 10^{-3}$ s$^{-1}$ with the mean value near $25 \times 10^{-3}$ s$^{-1}$. Thus, the main contribution to $\check{\sigma}_v$ comes from the elevation shear of the radial wind.

We have applied the procedure described in section 3 to five other cases (7, 12, 13, and 30 December 2001 and 26 April 2002) having $\check{\sigma}_v$ patterns similar to those seen in Fig. 1 but with maximal spectrum widths less than 7 m s$^{-1}$ (e.g., Fig. 1d). In all five cases, there was no substantial contribution from turbulence; that is, the main contributor to $\check{\sigma}_v$ was the vertical shear of the wind. Furthermore, we calculated the median $\check{\sigma}_{\text{sst}}$ for regions outside the layers of large $\check{\sigma}_v$ and found a median value of about 1.5 m s$^{-1}$ for all the cases presented. Using this value in (5a), a median range of about 50 km, and assuming that the outer scale of the isotropic turbulence is more than a couple of hundred meters (i.e., larger than the characteristic beamwidth $r_o \sigma_e$), $\hat{\varepsilon}$ is calculated to be about $3.7 \times 10^{-3}$ m$^2$ s$^{-3}$, which is light turbulence according to Hocking and Mu (1997). This value of $\hat{\varepsilon}$ compares reasonably well with the $\varepsilon$
measurements made in clear air using data from a 6-m-
wavelength wind profiler; at the lowest altitude of 5 km
that the profiler was capable of making measurements,
the median value of $e$ for winter months was about $10^{-2}$
$m^2 s^{-2}$ (Nastrom and Eaton 1997), but the profiler data
exhibit a monotonic increase for decreasing heights
below 10 km. Thus, it is expected that there would even
be closer agreement with the median values found in
stratiform precipitation at altitudes typically below 5
km. The closeness of the median $e$ in stratiform pre-
cipitation and that in clear air could be coincidental, but
it is also likely due to the fact that convection is weak
and does not add appreciably to the ambient $e$.

Because median $\sigma_{\text{st}}$ values within the layers were
substantially less than $4 \text{ m s}^{-1}$, there should be negli-
gible turbulent effects on the smoothness of flight of an
aircraft (Lee 1977). Although we have not analyzed all
the cases, we conclude that uniform layers of large $\sigma_v$,
but not exceeding $7 \text{ m s}^{-1}$, are likely no more turbulent
than the surrounding areas.

Fig. 3. (a) Small-scale turbulent contributions ($\sigma_{\text{st}}$) to the $\sigma_v$ field in Fig. 1b. (b) The $\sigma_{\text{st}}$ for the $\sigma_v$ field in Fig. 1d. The fields of (c) $\sigma_v$, (d) $\sigma_{\text{st}}$, (e) $\nu_m$, and (f) $Z$ on 28 Nov 2001. The fields of (g) $\sigma_v$ and (h) $\sigma_{\text{st}}$ on 15 Feb 2002.
The vertical cross section of the $\sigma_v$ field for data collected on 28 November 2001 is presented in Fig. 3c. The layer of large $\sigma_v$, at a height of about 3.5 km, exhibits periodic patches of $\sigma_v$ with maximal values of about 9 m s$^{-1}$. It is likely that the periodic structure is due to breaking Kelvin–Helmholtz (KH) waves associated with shearing instabilities (Gossard and Hooke 1975). Periodic disturbances are also seen in the RHI of the Doppler velocity field ($\hat{v}_m$, Fig. 3e), but not in the reflectivity factor $Z$ field (Fig. 3f). The $Z$ field exhibits nonuniformities, but these can hardly be related to KH waves without the $\sigma_v$ or $\hat{v}_m$ images.

Because the thickness of the $\sigma_v$ layer in Fig. 3c is about 1–2 km, it is assumed that the outer-scale $\Lambda_o$ of the isotropic turbulence is about the thickness of the layer and thus more than 2 times the beamwidth $\theta_1$ at most ranges. Thus, (5) should provide reasonable estimates of $\varepsilon$. Because there is a monotonic relation between $\sigma_{sst}$ and $\varepsilon$, patches of large $\sigma_{sst}$ in Fig. 3d correspond to patches of large $\varepsilon$. Maximal $\varepsilon$ is about 0.11 m$^2$ s$^{-3}$ and the median value is about 0.07 m$^2$ s$^{-3}$, but no corrections have been applied to eliminate the possible negative bias due to the filtering of the larger scales of isotropic turbulence. Nevertheless, these values compare reasonably well with the maximal value of 0.08 m$^2$ s$^{-3}$ and a median value of 0.05 m$^2$ s$^{-3}$ measured for a thunderstorm using two different radar techniques (Brewster and Zrnić 1986). This level of $\varepsilon$ corresponds to the upper end of the moderate turbulence in its effect on aircraft (Hocking and Mu 1997) and to severe turbulence according to measurements made by Trout and Panofsky (1969). But it is noted that the level of hazard to an aircraft is a function both of the level of $\varepsilon$ as well as the aircraft’s parameters (i.e., size, weight, speed, etc.).

Data collected on 15 February 2002 (Fig. 3g) present a pattern of $\sigma_v$ in a form of a “cat’s eye” (Gossard and Hooke 1975) that can be seen at heights between 3.5 and 5 km at distances of 50–80 km. Reinking (2004), using a 8.7-mm-wavelength radar, also observed these patterns in the $\sigma_v$ fields of mountain-influenced airflows. Similar patterns have been reported for fields of Z and $\hat{v}_m$, but not in $\sigma_v$ (Gossard and Hooke 1975; Doviak and Zrnić 2006, section 11.7; Chapman and Browning 1999). Such patterns are attributed to Kelvin–Helmholtz waves. We have found that the $Z$ fields of stratiform precipitation are practically featureless compared to the variety of patterns seen in the $\sigma_v$ fields.

Maximal measured $\sigma_v$ in Fig. 3g is 9.4 m s$^{-1}$ with a median $\sigma_v$ of about 8.7 m s$^{-1}$ for the upper layer of large $\sigma_v$. The $\varepsilon$ field corresponding to this layer shows maximal value of 0.29 m$^2$ s$^{-3}$ with a median $\varepsilon$ of about 0.15 m$^2$ s$^{-3}$, about the same as that calculated for the data on 28 November 2001. The $\varepsilon = 0.15$ m$^2$ s$^{-3}$ suggests extreme turbulence.

5. Summary and conclusions

Radar observations of spectrum width ($\sigma_v$) fields in stratiform precipitation frequently exhibit the presence of layers with widths of 4 m s$^{-1}$ or larger (about 80% of the analyzed cases), which according to research findings on thunderstorms (Lee 1977) corresponds to moderate or strong turbulence (i.e., derived gust velocities exceed 6.1 m s$^{-1}$). The eight cases analyzed using the procedures described in section 3 suggest that if the $\sigma_v$ in the layer does not exceed 7 m s$^{-1}$, and if the layer does not exhibit a wavy or patchy structure, the main contributor to $\sigma_v$ is the vertical shear of the horizontal wind (i.e., the turbulent component of $\sigma_v$ is small and about the same as in areas surrounding the layer). On the other hand, there are three cases for which $\sigma_v$ has values exceeding 7 m s$^{-1}$ and yet turbulence appears to be insignificant. For example, the data on 27 November 2001 showed a layer (Fig. 1b) in which the maximal value of $\sigma_v$ was 10 m s$^{-1}$, corresponding to severe turbulence if under convective conditions. But, after removing the shear contribution to $\sigma_v$, the intensity, $\sigma_{sst}$ (Fig. 3a), of the turbulence, on scales that are small compared to the $V_6$ dimensions, was mostly about 1.5 m s$^{-1}$ (corresponding to weak turbulence). But in the two other analyzed cases (Figs. 3d and 3h), the analysis procedure reveals a patchy turbulent field with maximal $\sigma_{sst}$ exceeding 9 m s$^{-1}$. To better relate these radar estimates of turbulence to actual levels of turbulence, there is a need for in situ measurements from instrumented aircraft.

At flight altitudes far above the ascent or descent paths from or into airports, shear is not hazardous to aircraft and/or its crew and passengers. Thus, $\sigma_v$ data should be corrected for shear before they can be reliably used for warnings of turbulence hazards. Perhaps some of the positive bias, noted by Sharman et al. (2006) when comparing in situ measurements of turbulent energy dissipation rates with those obtained from WSR-88D spectrum widths, might be associated with shear. Although turbulence in stratiform weather rarely does damage or causes destruction of aircraft, it can cause in-flight injuries to passengers or crew not safely secured to their seats. Strong shear and turbulence near the ground, during the time the aircraft is ascending from or descending into airports, can cause accidents because pilots have less time to respond to unexpected changes in altitude. For example, pilots reported severe turbulence below 1 km in light rain into the Dallas–Forth Worth (DFW), Texas, airport, and in another case a loss
of altitude from about 500 to 120 m was reported due to turbulence during a descent into John F. Kennedy International Airport (JFK; Bieringer et al. 2004). According to Table 1, about 40% of large \( \bar{v}_s \)'s occur near the ground (i.e., \( H \leq 1 \) km), and the knowledge of their presence could be useful to pilots.

For WSR-88D network radars, the elevation step is 1° (i.e., the beamwidth) and larger, and these coarse increments cause significant underestimates of the vertical shear. These underestimates can lead to a significant overestimation of the turbulent contribution of \( \sigma_{sst} \) to \( \bar{v}_s \) if vertical shear is computed from velocity changes versus elevation angles at a constant range.

Comparisons of median turbulent energy dissipation rates \( (\bar{\varepsilon}) \) outside layers of large spectrum widths are in reasonable accord with those found in clear air at comparable heights. This closeness between the median \( \bar{\varepsilon} \) in stratiform precipitation and that in clear air could be coincidental, but it is also likely due to the fact that convection is weak, and thus it does not add appreciably to the ambient \( \bar{\varepsilon} \).

Sometimes, layers of large \( \bar{v}_s \) exhibit periodic or wavy patterns (Figs. 1f,d and 3c,g). Similar patterns presented in the literature have been observed in fields of reflectivity in clear air. Reflectivity fields of stratiform precipitation are practically featureless compared to the presence could be useful to pilots. The true second central moment \( \sigma_{sst}^2 \) (thin solid lines in Figs. A1b and A1c), calculated from the same numerically evaluated spectra used to calculate \( \nu_m \), are displayed as a function of the elevation angle at closely spaced points (i.e., every 0.125° in Fig. A1b and every 0.1° for the \( DE = 1.0^\circ \) step in Fig. A1c). Because \( \sigma_{sst}^2 = 0, \sigma_{sst}^2 \) is also the true \( \sigma_{sst}^2 \). Measurements are spaced at \( DE \), and there are at least two approaches to estimating \( \sigma_{sst}^2 \) from \( \nu_m \). In section 3, we have adapted the approach of Istok and Doviak (1986) to least squares fit a linear radial velocity profile to \( \nu_m \) data at contiguous measurement points, and used Eq. (4) to estimate \( \sigma_{sst}^2 \) [this estimate is indicated in the legends of Fig. A1 with (4)] appended; only elevation shear is significant and three contiguous data points are used in the fitting]. The dashed lines in Figs. A1b and A1c depict the estimated \( \sigma_{sst}^2 \) using this approach. By subtracting the \( \sigma_{sst}^2 \) estimates from the \( \sigma_{sst}^2 \) measurements, estimates of \( \sigma_{sst}^2 \) can be obtained. In Fig. A1c it is evident that this approach can produce negative \( \sigma_{sst}^2 \)'s. Comparing Figs. A1b and A1c, one can conclude that the 1° elevation increment is too large to accurately estimate \( \sigma_{sst}^2 \). Even if the data are at the fine measurement increment of 0.125° and are linearly least squares fitted, there is significant error in estimating \( \sigma_{sst}^2 \) at long ranges. An alternative approach is to numerically integrate a higher-order polynomial fit to the \( \nu_m \) measurements (Hocking and, in particular, Dr. W. K. Hocking for his constructive comments that helped us to improve the manuscript. We thank our reviewers for their encouragement for this study. Michael Schmidt and Richard Wahkinney maintained the WSR-88D KOUN in impeccable condition. We thank our reviewers for their comments that helped us to improve the manuscript and, in particular, Dr. W. K. Hocking for his constructive comments and suggestions. Funding for this study was provided by the U.S. Department of Commerce/NOAA/Office of Oceanic and Atmospheric Research under NOAA–University of Oklahoma Cooperative Agreement NA17RJ1227.

## APPENDIX

### Turbulence Estimate Errors due to Elevation Undersampling and Beamwidth

Herein, the error in estimating turbulence intensity (i.e., \( \sigma_{sst}^2 \); we omit diacriticals because the calculated variables are the expected ones) is calculated if the shear contributions of \( \sigma_{sst}^2 \) to \( \sigma_{sst}^2 \) are based on the spatial distribution of \( \nu_m \) at fine elevation increments 1) at \( DE = \Delta \theta_r = 0.125^\circ \) or 2) at \( DE = 1^\circ \), about equal to the one-way 3-dB beamwidth (i.e., \( \theta_1 = 0.94^\circ \)) of the KOUN radar. A \( \nu_s \) model (dashed lines in Fig. A1a) is assumed wherein \( \nu_s \) is constant at \(-2 \) m s\(^{-1}\) below 2 km, constant at \( 22 \) m s\(^{-1}\) above 3.5 km, and has three linear intervals between 2.0 and 3.5 km \( \nu_s = -0.5 \) m s\(^{-1}\) at \( h = 2.25 \) km; \( \nu_s = 20.5 \) m s\(^{-1}\) at \( h = 3.25 \) km). \( Z \) is uniform, and \( \sigma_{sst}^2 = 0 \). The model \( \nu_s \) profile is similar to observations presented in Fig. 2. The \( \nu_m \) (solid lines) is calculated from spectra evaluated using a two-dimensional numerical integration of the antenna-pattern-weighted \( \nu_s \). The beam pattern is assumed to be circularly symmetric and Gaussian shaped (a good approximation for the KOUN beam), but truncated at \( \pm 3^\circ \) about the beam center (the contribution from beyond this angular interval has been found to be negligible).

The true second central moment \( \sigma_{sst}^2 \) (thin solid lines in Figs. A1b and A1c), calculated from the same numerically evaluated spectra used to calculate \( \nu_m \), are displayed as a function of the elevation angle at closely spaced points (i.e., every 0.125° in Fig. A1b and every 0.1° for the \( DE = 1.0^\circ \) step in Fig. A1c). Because \( \sigma_{sst}^2 = 0, \sigma_{sst}^2 \) is also the true \( \sigma_{sst}^2 \). Measurements are spaced at \( DE \), and there are at least two approaches to estimating \( \sigma_{sst}^2 \) from \( \nu_m \). In section 3, we have adapted the approach of Istok and Doviak (1986) to least squares fit a linear radial velocity profile to \( \nu_m \) data at contiguous measurement points, and used Eq. (4) to estimate \( \sigma_{sst}^2 \) [this estimate is indicated in the legends of Fig. A1 with (4)] appended; only elevation shear is significant and three contiguous data points are used in the fitting]. The dashed lines in Figs. A1b and A1c depict the estimated \( \sigma_{sst}^2 \) using this approach. By subtracting the \( \sigma_{sst}^2 \) estimates from the \( \sigma_{sst}^2 \) measurements, estimates of \( \sigma_{sst}^2 \) can be obtained. In Fig. A1c it is evident that this approach can produce negative \( \sigma_{sst}^2 \)'s. Comparing Figs. A1b and A1c, one can conclude that the 1° elevation increment is too large to accurately estimate \( \sigma_{sst}^2 \). Even if the data are at the fine measurement increment of 0.125° and are linearly least squares fitted, there is significant error in estimating \( \sigma_{sst}^2 \) at long ranges. An alternative approach is to numerically integrate a higher-order polynomial fit to the \( \nu_m \) measurements (Hocking and, in particular, Dr. W. K. Hocking for his constructive comments that helped us to improve the manuscript. We thank our reviewers for their comments that helped us to improve the manuscript and, in particular, Dr. W. K. Hocking for his constructive comments and suggestions. Funding for this study was provided by the U.S. Department of Commerce/NOAA/Office of Oceanic and Atmospheric Research under NOAA–University of Oklahoma Cooperative Agreement NA17RJ1227.)
(2003) to estimate $\sigma^2_{s, \theta}$ [e.g., computing the Doppler spectrum using the fitted $v_m$ instead of using Eq. (4) as is done in computing $\sigma^2_{s, \theta}$]. We have not evaluated $\sigma^2_{sst}$ errors using this alternative approach.

Figure A1b shows that both the true $\sigma^2_{s, \theta}$ and estimated $\sigma^2_{s, \theta}$ depend on the distance to $V_6$, as they should. At short ranges the beamwidth is much smaller than the layer’s 1-km thickness, and measured Doppler velocities of $v_m$ follow well the actual profile (e.g., Fig. A1a; $r_o = 25$ km). This is why the $\sigma^2_{v}$ and $\sigma^2_{s, \theta}$ curves at 25 km in Fig. A1b have flat tops (i.e., are independent of elevation angle). At longer ranges, the flatness length decreases and disappears at distances beyond 40 km. The $\sigma^2_{v}$ curves increase with distance from the radar but remain small (less than 1 m$^2$ s$^{-2}$) within 40 km. The $\sigma^2_{sst}$ estimates depend on range and on the thickness of the layer. Beyond 40 km, $\sigma^2_{sst}$ exceeds 1 m$^2$ s$^{-2}$ and become large (i.e., up to 9 m$^2$ s$^{-2}$ at $r_o = 100$ km). Beamwidth smearing leads to a heavily smoothed Doppler velocity $v_m$ profile and, thus, mostly an underestimate of $\sigma^2_{s, \theta}$. The underestimated shear leads to the appearance of a “turbulent contribution” even though turbulence is not present. We also note at the edges of the shear layer that if $\sigma^2_{s, \theta}$ is calculated assuming the wind is linear, $\sigma^2_{s, \theta}$ can be overestimated, resulting in negative $\sigma^2_{sst}$. We conclude that, by applying the procedure described in section 3, we can expect some incorrect “turbulent” contributions whenever the thickness of the uniform shear layer is narrower than the beamwidth.

![Figure A1.](image-url)
Although we considered herein an illustrative example of the velocity height profile, we conclude that sampling of the wind field with elevation increments much finer than a beamwidth is required to significantly reduce the bias in the estimates of turbulence. Under the assumption that wind is horizontally uniform, a dense vertical sampling of the wind field can be obtained by combining data collected along the range with that collected at various elevation angles (Melnikov and Doviak 2008).

REFERENCES


