Lidar-Based Estimation of Small-Scale Rainfall: Empirical Evidence

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ABSTRACT

A significant scale gap between radar and in situ measurements of rainfall using rain gauges and disdrometers indicates a pressing need for improved knowledge of rainfall variability at the spatial scales below those of today’s operational radar rainfall products, that is, 1–4 km. Lidar technology has the potential to fulfill this need, but there has been inconsistency in the literature pertaining to quantitative observations of rain using lidar. Several publications have stated that light scattering properties of raindrops could not be correlated with rain rates, while other papers have demonstrated the existence of such relationships. This note provides empirical evidence in support of the latter claim.

The authors conducted a simple experiment using a near-horizontal-pointing elastic lidar to observe rain in Iowa City, Iowa, in the fall of 2005. The lidar signal was used to estimate rainfall quantities that were subsequently compared with independent estimates of the same quantities obtained from an optical disdrometer that was placed about 370 m from the lidar, ~10 m below the lidar beam. To perform the conversion from the raw lidar signal, the authors used an optical geometry-based procedure to estimate optical extinction data. A theoretical relationship between extinction coefficients and rain rates was derived based on a theoretical drop size distribution. The parameters of the relationship were found through a best-fit procedure using lidar and disdrometer data. The results show that the lidar-derived rain rates correspond to those obtained from the optical disdrometer with a root-mean-square difference of 55%.

The authors conclude that although a great deal remains to be done to improve the inversion algorithm, lidar measurements of rain are possible and warrant further studies. Lidars deployed in conjunction with disdrometers can provide high spatial (~5 m) and temporal (~1 min disdrometer, ~1 s lidar) resolution data over a relatively long distance for rainfall measurements (1–2 km in the case of the University of Iowa lidar).

1. Introduction

Quantitative observations of rainfall made by weather services rely primarily on two types of instruments: weather radars and rain gauges. Because the sampling areas of these sensors differ by eight orders of magnitude (Ciach and Krajewski 1999a,b), we need to improve our understanding of the small-scale variability of rainfall. Statistical characterization of this variability is necessary for proper interpretation of radar and satellite remote sensing of rainfall (e.g., Krajewski and Smith 2002) and for hydrologic applications (e.g., Ogden et al. 2000). Such characterization can be achieved from the analysis of empirical data from special-purpose rain gauge and/or disdrometer networks (Krajewski et al. 2003; Ciach and Krajewski 2006), but their deployment is costly and their maintenance tedious (Steiner et al. 1999; Sieck et al. 2006). Collecting an adequate sample size takes time, considering that it rains only intermittently. Elastic lidar technology provides an opportunity to accomplish the same goal much more expeditiously. With a sampling area on the order of meters and a useful range of a few kilometers (1–2 km in the presence of rain for the lidar used in this study), lidar has the potential to bridge the space–time scale gap between radars and rain gauges.

In the 1970s and 1980s, there was some interest and success in rain measurement using the scattering of visible light over distances on the order of hundreds of meters (e.g., Rensch and Long 1970; Chimelis 1982; Stow et al. 1991). This interest has resumed in recent years with the development of techniques to calculate scattering probabilities for deformed droplets. However, some researchers have surmised that rain measurement
by lidars is impossible (Kokhanovsky and Macke 1997; Macke and Grossklaus 1998).

These papers, suggesting that lidar measurements of rain are not likely to succeed, apply a geometric-optics approximation approach in calculating backscattered intensities for nonspherical drops. The studies of Kokhanovsky and Macke (1997) and Macke and Grossklaus (1998) show no correlation between rain rates and backscattered intensities. Roy and Bissonnette (2001) measured raindrop backscatter intensities in a range of orientations at different zenith angles (0°–75°) to show the angular dependence of nonspherical meteor. These papers have been interpreted as generally implying that the measurement of rain by lidars is not possible. But the key factor for the calculations in the papers by Kokhanovsky and Macke (1997) and Macke and Grossklaus (1998) was that the raindrops had no particular orientation. In fact, rain drops maintain symmetry in the direction of the fall, which allows fixed orientation measurements of the backscattered intensities. Some 20–30 years earlier, lidar rain measurements were made by a number of investigators (e.g., Rensch and Long 1970; Chimelis 1982; Vasseur and Gibbins 1996) who showed a strong correlation between rainfall rate and lidar extinction coefficients.

To re-examine the issue, we conducted a short experiment in October 2005 in Iowa City. The elastic lidar was collocated with an independent instrument, a rain disdrometer. Here, we present our experimental methodology and results of the experiment. We close by briefly discussing the remaining questions and outlines an approach for addressing them.

2. Inversion method

Because lidar only indirectly observes such hydrologic rainfall characteristics as intensity (rate) and accumulation, one must convert the measured quantities into the quantities of interest. The inversion algorithm transforms measured quantities (the intensity of the backscattered light as a function of distance) into rainfall amounts. The inversion algorithm involves two steps: (i) determination of the extinction coefficients from raw lidar data and (ii) calculation of the corresponding rain rates. We developed an approach that uses simple geometric optics to relate lidar extinction coefficients to rain rates in a manner similar to that used for radars (e.g., Battan 1979). We determined the parameters of that relationship through a data fitting procedure.

Although our approach involves several assumptions, the impact of each has been discussed in the literature—for example, the use of a constant extinction-backscattering relation in Eq. (3) (Rocadenbosch and Comerón 1999), the use of the Marshall–Palmer drop size distribution (Willis and Tattelman 1989; Tokay and Short 1996), and the assumption of a raindrop velocity expression (Sauvageot 1994).

a. Determination of extinction coefficients

The general form for single scattering, range-corrected lidar return power can be expressed as (e.g., Kovalev and Eichinger 2004)

\[ P(r)^2 = C_0 \beta(r) \exp \left[ -2 \int_0^r \alpha(r')dr' \right], \]

(1)

where

\[ C_0 = P_0 \frac{cr}{2A} \]

(2)

is the lidar constant, \( r \) is the distance from lidar to a given sampling volume, \( A \) is the telescope aperture area, \( r \) is the laser pulse length, \( c \) is the velocity of light, and \( P_0 \) is the power transmitted by the laser. Using the lidar Eq. (1), we can estimate backscatter \( \beta(r) \) or extinction \( \alpha(r) \) coefficient. Both backscattering and extinction coefficients vary with distance and are related by a power law (Klett 1981, 1985):

\[ \beta(r) = B_0 \alpha^k(r). \]

(3)

It is customary to assume that the exponent \( k = 1 \) (Rocadenbosch and Comerón 1999). In this case, \( B_0 \) is a quantity often called the lidar ratio (or the lidar constant). With the assumptions made in Eq. (3), the inversion of the lidar data to obtain extinction or backscatter coefficient is mathematically equivalent.

With the substitution of Eq. (3) into (1), we obtain

\[ P(r)^2 = C_0 B_0 \alpha(r) \exp \left[ -2 \int_0^r \alpha(r')dr' \right]. \]

(4)

To simplify Eq. (4), we assume a constant, path-averaged value of extinction coefficient \( \alpha(r) = \alpha_{\text{avg}} \) and combine all constants \( C_1 = B_0 C_0 \). Taking the logarithm of both sides results in a simple linear form:

\[ \ln[P(r)^2] = \ln(C_1 \alpha_{\text{avg}}) - 2\alpha_{\text{avg}} r. \]

(5)

Using a standard least squares fit of the measured power, often called the slope method, we find values of \( C_1 \) and \( \alpha_{\text{avg}} \) [-2 \( \alpha_{\text{avg}} \) being the slope and \( \ln(C_1 \alpha_{\text{avg}}) \) being the value at \( r = 0 \)]. In the next step, we return to Eq. (4) and use the fact that the average extinction does
not change appreciably with distance. This assumption allows us to substitute $\alpha(r') = \alpha_{\text{avg}}$ in the integral in Eq. (4) and to perform the integration. Substituting $\alpha_{\text{avg}}$ only into the integral and omitting $\alpha(r)$ in front of the integral is a similar approach to the decomposition of extinction into an average value and its fluctuations. This leads to a range-dependent extinction coefficient (superposition of the average value and its fluctuations):

$$\alpha(r) = P(r)^2 \frac{1}{C_1} \exp(2\alpha_{\text{avg}} r).$$ (6)

The method presented above is sufficient for the purpose of this short study. Other approaches to the derivation of the extinction coefficient are discussed by Klett (1981, 1985), Kovalev and Eichinger (2004), and others (Kano 1968; Balin et al. 1987; Kovalev 1993, 1995, 2003). The uncertainty of the inversion method is discussed in, for example, Rocadenbosch and Comerón (1999).

b. Extinction versus backscattering coefficients

Elastic lidar measures the power returning to the detector as a function of time (or distance because the speed of light is fixed). The magnitude of the returning power depends on the extinction coefficient and the backscatter coefficient [the lidar Eq. (1)]. Extinction and backscatter coefficients cannot be derived independently using a single color elastic lidar. But introducing certain assumptions, such as Eq. (3), reduces the problem to only one variable, either extinction or backscattering. Therefore, estimating either extinction or backscattering from the lidar equation is mathematically equivalent. Using extinction coefficient is simpler for this method. It does not require the knowledge of $B_0$, the lidar ratio (backscatter-to-extinction ratio), for the case of rain and allows us to combine all constants in the lidar Eq. (4) into one constant, namely, $C_1$. Thus, there are only two unknowns, $C_1$ and $\alpha_{\text{avg}}$, estimated through a linear regression fit in Eq. (5).

In case of backscatter, we would not be able to separate average backscatter $\beta_{\text{avg}}$ from the fit because we would have two additional explicit constants, $B_0$ and $C_0$. The following equation shows the fit using average backscatter coefficient approach:

$$\ln[P(r)^2] = \ln(C_0\beta_{\text{avg}}) - \frac{2\beta_{\text{avg}}}{B_0} r.$$ (7)

The linear fit in Eq. (7) involves three unknown, independent constants ($C_0$, $B_0$, and $\beta_{\text{avg}}$). It is clear that estimating average backscatter $\beta_{\text{avg}}$ with this approach would require calibration (known $B_0$). Calibration of the lidar for the case of rain would be extremely difficult (horizontal beam and the presence of aerosols).

The average $\alpha_{\text{avg}}$ extinction is therefore much easier to estimate directly from the data, with the assumption of Eq. (3). Lastly, the use of extinction coefficients allows comparison to previous work.

c. Calculating rain rates

The second level of inversion involves finding a relationship between rainfall rates and the extinction coefficient. The rainfall rate $R$ is a product of the terminal velocity of each drop, the number density distribution of drops, and the volume of each drop, integrated over all drop sizes. Raindrop terminal velocity can be approximated by a power law

$$v_{\text{terminal}} = aD^b,$$ (8)

where $D$ is the raindrop diameter and $a$ and $b$ are constants (Spilhaus 1948; Atlas and Ulbrich 1977; Ulbrich 1983; Sekhon and Srivastava 1971). For the number density distribution, we assumed a Marshall–Palmer drop size distribution (M–P DSD; Marshall and Palmer 1948) in the following form:

$$N(D) = n_0 e^{-\Lambda D},$$ (9)

where $N(D)$ is the number density of drops, $n_0$ is the M–P DSD intercept constant, $D$ is the equivolumetric drop diameter, and $\Lambda$ is related to $R$. Combining Eqs. (8) and (9) with the volume of a drop $\pi D^3/6$, we obtain the rainfall rate relationship in millimeters per hour in the following form:

$$R = \frac{a\pi}{6} n_0 \int_0^\infty D^3 D^b e^{-\Lambda D} dD.$$ (10)

The integral in Eq. (9) has an analytical solution of the following form:

$$\Gamma(k) = \int_{-\infty}^{\infty} x^{k-1} e^{-cx} dx,$$ (11)

where $\Gamma(k)$ is the gamma function. As a result, we get the rainfall rate formula

$$R = \frac{C_2}{\Lambda^{3+b+1}} = C_2 \Lambda^{-(3+b+1)},$$ (12)

where
For the particular lidar wavelength (1064 nm), the total extinction coefficient is the sum of extinction from molecules (Rayleigh), particulates (aerosols), and raindrops:

$$\alpha = \alpha_{\text{Rayleigh}} + \alpha_{\text{rain}} + \alpha_{\text{aerosols}}.$$  \hspace{1cm} (14)

We neglect molecular (Rayleigh) extinction because in raining conditions it is more than three orders of magnitude smaller than the rainfall signal (Van de Hulst 1957; Bohren and Huffman 1983). The contribution from aerosols is two orders of magnitude smaller than the contribution from rain drops; it plays a role only when there is no rain.

In an electromagnetic sense, the extinction from raindrops is a product of the scattering cross-section and the raindrop size distribution integrated over all diameters (Deirmendjian 1969; Barber and Hill 1990). Assuming a Marshall–Palmer drop size distribution, geometric scattering from raindrops, and integrating over all diameters, we obtain

$$\alpha = \frac{4\pi n_0}{\lambda^3} \times 10^6 \text{ (km}^{-1}) \text{),}$$  \hspace{1cm} (15)

where the factor $10^6$ comes from the unit conversion (1 mm$^{-1}$ to 1 km$^{-1}$). Substituting $\lambda$ from Eq. (15) into Eq. (12) gives the anticipated rain rate relation

$$R = C_3 \alpha^{C_4}.$$  \hspace{1cm} (16)

Both $C_3$ and $C_4$ in Eq. (16) are constants, where $C_3 = (4\pi n_0 \times 10^6)^{-\frac{3+b+1}{3}} C_2$ and $C_4 = (3 + b + 1)/3$. Equation (16) is a general form for extinction–rainfall dependence, derived from basic definitions and using fundamental assumptions presented above. The exact numerical values of $C_3$ and $C_4$ depend on parameters in the droplet velocity spectrum and DSDs and are difficult to measure collectively. Moreover these parameters will change as the type of rainfall changes. The parameters $C_3$ and $C_4$ can be determined by the best-fit procedure using the reference disdrometer data. An analogous practice is used in the radar-rainfall community where the constants are fitted during a calibration effort in a similar formula (Z–R relationship; e.g., Battan 1979).

The best-fit procedure, leading to determining the parameters in Eq. (16), is based on minimizing $\xi$:

$$\xi = \frac{\sum [\log(R_{\text{disdrometer}}) - \log(R_{\text{lidar}})]^2}{M},$$  \hspace{1cm} (17)

that is, the sum of squared difference between the rainfall from the disdrometer and from the lidar in a log space, where $M$ is the total number of data points.

3. Instruments

a. Lidar

To test the above procedure, and thus the lidar’s ability to quantitatively estimate rainfall, we designed and conducted a simple field experiment in Iowa City, Iowa, in the fall of 2005. We used the elastic scanning lidar system of IIHR–Hydroscience & Engineering and an optical disdrometer. The IIHR’s scanning miniature lidar (SMILI) is a small, scanning lidar (Eichinger et al. 1999) that was designed to use elastic backscattering to determine the distribution and properties of atmospheric particulates. The lidar operates by emitting a pulse of infrared laser light into the atmosphere. Particulates, including rain drops in our case, interact with the pulse and scatter light back to the lidar. The term elastic refers to scattering in which no energy is lost by the photons, so that the detected light is at the same wavelength as the emitted light. The amount of returning light collected by the telescope is proportional to the cross sectional area of the water droplets in the air and the amount of atmospheric attenuation. The system is capable of both day and nighttime operation.

A Nd:YAG laser operating at 1.064 microns is the light source. The laser is attached to a Cassegrain telescope with a primary mirror diameter of 25 cm and a focal length of 2.5 m. The laser pulses at 50 Hz with ~25 mJ per pulse of power. The laser beam is emitted parallel to the telescope after going through a periscope. The system is entirely computer controlled using PC cards to control the motors, the laser, the digitizers, and other auxiliary equipment such as GPS. The lidar can be operated remotely and autonomously, using pre-programmed sequences that only require an operator to start.

The lidar was modified for use in rain. Because the amount of backscattered light from rain is much larger than that from normal particulate levels in air, several steps were necessary to reduce the normal sensitivity of the lidar. We reduced both the laser power and the gain of the avalanche photodiode and added two 10% transmission neutral density filters to the optical train to reduce by a factor of 100 the amount of light striking the detector.

b. Disdrometer

An optical disdrometer can provide data on a drop by drop basis, which allows more flexibility in analyzing time scale aspects of the evaluation and thus provides a
better interpretation of the obtained results. Furthermore, one can compare the assumed and observed extinction coefficients using disdrometer-observed DSD. Using a tipping-bucket rain gauge provides only integrated quantities of rainfall, and even those are subject to severe sampling effects at short time scales (Habib et al. 2001; Ciach 2003).

The disdrometer used was a PARSIVEL and was considered as a reference measurement to validate the lidar data. The PARSIVEL is a one-dimensional optical disdrometer, manufactured in Pfinztal, Germany by PMTech AG. An infrared laser in the disdrometer transmitter head illuminates a linear array of photodiodes housed in the device’s receiver head. Rain drops that pass through the measurement area (∼49 cm²) cause variations in the photodiode outputs, which are then digitized and processed. The magnitude of the variation is related to the drop size, whereas the duration of the variation is related to the drop fall velocity. For a description and experimental evaluation of the device, see Krajewski et al. (2006). The integration interval is adjustable; for this study, we used the instrument default in which every 30 s it outputs an estimate of the DSD.

4. Results

a. Experiment

The experimental site was located at the University of Iowa facility in the southwest outskirts of Iowa City, Iowa. We used our lidar in a nonscanning mode. The laser shot in a near-horizontal plane, with the direction of the beam fixed throughout the entire experiment. The disdrometer was located about 370 m away from the lidar. The lidar beam passed approximately 10 m above the disdrometer. In this way, the orientation of rain drops with respect to the laser beam was constant, eliminating the issue described by Roy and Bissonnette (2001) in which the extinction coefficients changed dramatically as a function of vertical angle. According to their calculations, this effect is limited when the elevation angle is greater than 75° with respect to a vertical plane (in this study the angle was ∼90°). Both instruments were synchronized to the atomic clock of the National Institute of Standards and Technology.

The graphs in Fig. 1 present the time series of lidar signal measured at the disdrometer location for the entire event (top graph) and a full spatial range lidar returns for a part of the event (color coded bottom graph). The black line in the bottom graph indicates the location of the disdrometer with respect to the distance from the lidar. The raw lidar data presented in Fig. 1 has 1-s temporal and 1.5-m spatial resolution.

The parameters C₃ and C₄ are determined by fitting the lidar data to Eq. (16). The results of the best fit for a 1064-nm lidar are presented in the following formula:

\[ R = 15.95 \alpha^{2.67}, \]  

(18)

where \( R \) is the rainfall rate in millimeters per hour and \( \alpha \) is the extinction coefficient in the units of 1 km⁻¹. The literature does not report the parameters from Eq. (18) for 1064-nm laser. The inverse of this relationship expresses the extinction coefficient as a power-law function of rainfall intensity and is of the form

\[ \alpha = 0.063 R^{0.37}. \]  

(19)

Figures 2 and 3 present a comparison between lidar-derived rain rates [calculated using Eq. (18)] and rain rates derived from the disdrometer. In most cases, the lidar tends to underestimate rain rates. At a closer look, it seems as if both the lidar and the disdrometer detect rain at the same time, but the magnitude of the signal following the detection is different (Figs. 2 and 3). This effect is not well understood yet. Both datasets in Figs. 2 and 3 have 30-s temporal resolution. Because the temporal resolution of the disdrometer data is 30 s, the lidar data had to be resampled to match the resolution of the disdrometer.

b. Uncertainties and discussion

The disdrometer estimates are subject to considerable uncertainty because the instrument has a very small sampling volume. The disdrometer estimates rainfall based on measurement of separate drops fit to a fixed DSD. When the number of drops in a time interval is small, the presence of unusually large or small drops may significantly skew the disdrometer data. Lidar also contributes toward rainfall estimate uncertainties. For eye safety considerations, the laser and lidar optical train has a wide field of view (3 mrad). This should be expected to cause complications at high rain rates at which multiple scattering becomes important. At low rain rates, the aerosol contribution to extinction becomes significant (the lidar signal never drops to zero). But at low rain rates, small droplets dominate the lidar signal. The lidar can sense extremely small raindrops suspended in the air whereas the disdrometer cannot detect droplets smaller than about 1 mm. This may be the cause of the higher relative difference between the disdrometer and the lidar for rainfall rates below 1 mm h⁻¹ (Fig. 3). The uncertainties from all these effects accumulate, and therefore a reasonable low-end detection limit for the lidar is ∼0.1 mm h⁻¹.
Using the formula from Eq. (20), we calculated the mean, fractional difference between the lidar and the disdrometer rain rates for the entire sample with values above the detection limit of 0.1 mm h\(^{-1}\):

\[
\frac{\delta R}{R} = \sqrt{\frac{\sum (R_{\text{disdrometer}} - R_{\text{lidar}})^2}{N}}. \tag{20}
\]

where \(N\) is the number of measurements in the sample. For this study we obtained the mean fractional difference of \(\frac{\delta R}{R} (\forall R_{\text{disdrometer}} > 0.1 \text{ mm h}^{-1}) \approx 55\%\).

Although the value of the difference of 55% is high, a mean error calculated for radar measurements is often of the similar order or higher (Wilson and Brandes 1979; Anagnostou et al. 1999; Ciach and Krajewski 1999b). This difference is associated with several assumptions in the inversion algorithm (e.g., extinction to backscatter, terminal velocity, DSD, etc.) Many studies have shown that DSD [and therefore Eq. (17)] varies within the

\fig. 1. Range-corrected raw lidar data. The upper graph presents the full time series of lidar back-scattered power measured directly over the disdrometer during the event on 5 Oct 2005 in Iowa City, IA. The bottom graph shows a time series of full-range lidar profiles (color coded) for the part indicated by a red time frame. The black line in the bottom shows the location of the disdrometer with respect to the lidar (370 m).
same rain event. The variability of DSD has a significant effect on the calculation of rainfall rates (Smith 1993; Smith and Krajewski 1993; Ulbrich and Atlas 2002; Steiner and Smith 2000; Tokay et al. 2008). The difference is also due in part to the limited collection area and time average of the disdrometer where a single large drop can skew the results. However, the results show the significance and the potential of lidar-based measurements as a research tool for studying rainfall.

Reconciling the conventional wisdom with the measurements is straightforward. The scattering properties of raindrops may depend strongly on the relative orientation of the drops with respect to the incoming light, so that no single inversion relationship will satisfy all possible orientations. However, there is a preferred orientation to the drops as they fall. They flatten along the plane perpendicular to their direction of motion. Thus, scanning nearly horizontally maintains the same relative orientation between droplets and illumination, whereas scanning vertically does not. Because our goal is to determine the horizontal distribution of rain, horizontal scanning parallel to the ground is the desired scanning method. Under these conditions, we are confident that horizontal spatial rainfall fields can be measured rather accurately. Arguably, lidar-based estimates of rainfall at small spatial and temporal scales might be the most accurate of all instruments available because tipping-bucket rain gauges and optical and mechanical disdrometers suffer from considerable sampling problems, (e.g., Ciach 2003; Krajewski et al. 2006) and radar cannot resolve such small spatial scales.

5. Conclusions

Our results demonstrate that lidar can provide not only qualitative observations but also quantitative estimates of rainfall. We found a theoretical formula relating rainfall intensities with extinction estimates. The parameters of the extinction–rainfall dependence were determined through a fitting procedure. The resulting estimates from both independent instruments lead to good agreement of rain quantities. Limiting the use of the lidar to horizontal scanning (fixed elevation) would allow mapping space–time rainfall fields while overcoming the issue of variable scattering properties dependent on the beam-to-droplet orientation. We also note that much research remains to be done to improve data processing, particularly in the estimation of extinction coefficients and in the extinction coefficient to rainfall rates inversion algorithm.

A concurrent application of lidar and a limited number of disdrometers should provide a powerful observation system capable of mapping horizontal distribution of rainfall with high spatial (∼5 m) and temporal (<1 min) resolution and over domains commensurate in size

![Fig. 2. A comparison of the time series between the disdrometer and the lidar-based rainfall rates.](image)

![Fig. 3. A log-log graph of lidar-derived rain rates vs disdrometer rain rates.](image)
REFERENCES


