Thermodynamic Coupled Modes in the Tropical Atmosphere–Ocean: An Analytical Solution

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ABSTRACT

The present study provides a consistent and unified solution for the two types of thermodynamical coupled modes in the atmosphere–ocean climate system: the tropical meridional mode and the subtropical dipole mode. The solution is derived analytically from a linear model that couples a simple atmosphere to a slab ocean via the wind–evaporation–SST (WES) feedback. For a mean zonal wind, the results show that the wind (hence latent heat flux) anomaly and the SST anomaly differ in phase such that the tropical mode propagates downwind and the subtropical modes propagate upwind, with both modes being damped by the SST-driven component of latent heat flux. Despite the existence of positive WES feedback, the large-scale subtropical modes are always stable, while the tropical mode can become unstable only when the air–sea coupling is strong and the mean wind is easterly. Furthermore, the mean meridional winds break the equatorial symmetry and enable the coupled modes to intensify in the Southern (Northern) Hemisphere for a southerly (northerly) component. For realistic parameter values, these thermodynamical coupled modes have periods and damping time scales in years; hence, they may play important roles in the tropical interannual-to-decadal climate variability.

1. Introduction

In addition to the dynamical coupled mode, like El Niño–Southern Oscillation (ENSO), several thermodynamical coupled modes have also been identified in the tropical atmosphere–ocean system. In the tropical Atlantic, for example, the interhemispheric sea surface temperature (SST) gradient and the cross-equatorial surface wind display significant coherence on interannual–decadal time scales (Servain 1991; Nobre and Shukla 1996; Chang et al. 1997), which is known as the Atlantic meridional mode, causing strong anomalies in precipitation and hurricane activities (Kossin and Vimont 2007). A similar meridional mode of coupled atmosphere–ocean variability is also shown to exist in the eastern tropical Pacific (Chiang and Vimont 2004) and in the tropical Indian Ocean (Wu et al. 2008). In the subtropics of the southern Atlantic and the southern Indian Ocean, however, the SST anomalies are dominated by a zonal dipole pattern on interannual and longer time scales, which is known as the subtropical dipole mode, affecting rainfall over the adjacent continents through changes in the subtropical high (Venegas et al. 1997; Behera and Yamagata 2001; Fauchereau et al. 2003).

A common feature of these modes is the covariability among the wind, latent heat flux, and SST anomalies on interannual/decadal time scales, which leads to an explanation in terms of the wind–evaporation–SST (WES) feedback (Chang et al. 1997; Xie 1999; Sterl and Hazeleger 2003; Trzaska et al. 2007; Lee and Wang 2008). Schematically, the WES feedback, thermodynamical in nature, works in the following way for SST dipoles under an easterly mean wind [see Xie and Philander (1994), Liu (1996), and Lee and Wang (2008) for a complete explanation of the WES feedback]: assuming that an anomalous SST dipole induces westerly wind anomalies over the warm pole and easterly wind anomalies over the cold pole, we would see weaker winds, less evaporation, and less latent heat loss over the warm SST and stronger winds, more evaporation, and more latent heat loss over the cold SST, which in turn would enhance the initial dipole SST anomaly and create a positive air–sea feedback. By contrast, if the induced wind anomalies are easterlies over warm SST and westerlies over cold SST, then we get a negative feedback. Correctly determining the wind
response to SST anomalies in the tropical and subtropical oceans is challenging and proves to be difficult.

Linear stability analysis of various simple models reveals that the WES feedback, indeed, supports an unstable meridional mode for the tropical Atlantic basin (Xie 1999; Kossin and Vimont 2007; Wang and Chang 2008a). When the damping effects of other atmospheric and oceanic processes are taken into account, however, such a mode cannot be self-sustained for realistic parameter regime (Wang and Chang 2008a,b). As a result, the tropical meridional modes and subtropical dipole modes observed in practice are all forced responses with their characteristics largely determined by the atmospheric forcing. For example, Enfield et al. (1999) showed that the meridional dipole mode occurs only infrequently in the tropical Atlantic and no more so than expected from random stochastic forcing, which brings into question whether those modes are real physical modes or just statistical artifacts by definition (Dommenget and Latif 2000).

To shed light on this controversy, we start with a conceptual model of the WES feedback and solve it analytically for the coupled modes and then try to argue that the analytical modes bear some similarities to observations. The rest of the paper is organized as follows. The simple coupled model is first introduced in section 2, then solved analytically for mean zonal wind in section 3 and mean meridional wind in section 4, with the results summarized and discussed in section 5.

2. A simple atmosphere–slab ocean coupled model

The model used in this study is very similar to that of Xie (1996, 1999), which is made of a Gill-type atmosphere model coupled to a slab ocean model.

a. The model

In the tropics, given a heating source, the atmospheric response may be roughly described by the Gill model (Gill 1980):

\[
\begin{align*}
ru' - f\nu' &= -\frac{\partial \Psi'}{\partial x} \\
ru' + fu' &= -\frac{\partial \Psi'}{\partial y} \\
r^2 \Psi' + \epsilon^2 \left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) &= -\epsilon \frac{Q'}{\rho_a H_o},
\end{align*}
\]

where \( u' \) and \( v' \) are the zonal and meridional wind anomaly, \( \Psi' \) is the geopotential anomaly, \( r \) is a linear damping, \( f \) is the local Coriolis parameter, \( \epsilon \) is the reduced gravity wave speed, \( \rho_a \) is the air density, \( H_o \) is a depth scale for the lower troposphere, \( \epsilon \) is an SST-heating efficiency, and \( Q' \) is the heat flux anomaly across the air–sea interface (positive upward).\(^1\)

We further ignore all the ocean dynamics and use a slab model to describe the SST anomalies:

\[
\frac{\partial T'}{\partial t} = -\frac{Q'}{\rho_o H_o C_o},
\]

where \( T' \) is the SST anomaly, \( \rho_o \) is the water density, \( H_o \) is the oceanic mixed layer depth, and \( C_o \) is the specific heat of water.

Over tropical oceans, a major component of the surface heat flux is the latent heat:

\[
Q_{\text{LH}} = \rho_o L_E C_E U(q_s - q_a),
\]

where \( L_E \) is the latent heat of vaporization, \( C_E \) is the bulk exchange coefficient, \( U \) is the surface wind speed, \( q_s \) is the saturated specific humidity, and \( q_a \) is the specific humidity at reference height. Since only the WES feedback is of interest here, we also ignore the effects of other heat flux components in the model.\(^2\) Then, linearizing (3) around a mean climate state \((\bar{\Pi}, \bar{v}, \bar{q})\) yields the heat flux anomaly:

\[
Q' = \rho_o L_E C_E [(\bar{q}_s - \bar{q}_a)U' + \bar{U}(q'_s - q'_a)].
\]

From the Clausius–Clapeyron relationship, we know that the specific humidity anomaly depends on the SST anomaly (after linearization):

\[
\frac{q'}{q} = \left(\frac{L_E}{R_v T''}\right) T',
\]

where \( R_v \) is the gas constant for water vapor. Further noting that

\[
U' = \frac{\bar{v} U' + v U'}{U},
\]

we may express the heat flux anomaly in terms of the wind-induced surface heat exchange (WISHE) (Emanuel 1987; Neelin et al. 1987; Zhou and Carton 1998) and the SST-induced surface heat exchange:

\(^1\)Assuming the surface heat flux locally realized as diabatic heating in the troposphere is probably an oversimplification. For example, the surface latent heat flux is minimum over the western Pacific warm pool, but the tropospheric heating rate is maximum there. Such a phase shift, however, can be easily incorporated into (1) by replacing \( \epsilon \) with \( \epsilon e^{i \theta} \), where \( \theta \) is the zonal shift between tropospheric heating and surface heat flux (An 2000). Noting that the modal structure generally holds, we only present the case \( \theta = 0 \) here.

\(^2\)As along as a linear approximation \( a U'' + b T'' \) applies, the results may be generalized to include contributions from other surface heat fluxes (e.g., sensible heat).
Substituting (5) into Eqs. (1) and (2), we now clearly see that the Gill atmosphere model and the slab ocean model form a coupled system:

\[
\begin{align*}
ru' - fu' &= -\frac{\partial \Psi'}{\partial x}, \\
rv' + fu' &= -\frac{\partial \Psi'}{\partial y}, \\
rv' + c^2\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) &= -\gamma \left[ a \left(\frac{\mathbf{u}}{U}u' + \frac{\mathbf{v}}{U}v'\right) + bT'\right], \\
\frac{\partial T'}{\partial t} &= -a\left(\frac{\mathbf{u}}{U}u' + \frac{\mathbf{v}}{U}v'\right) - bT',
\end{align*}
\]

(6)

with the coupling parameter given by

\[
\gamma = \epsilon \frac{\rho_o H_o C_o}{\rho_a H_a},
\]

the WES feedback parameter due to WISHE,

\[
a = \frac{\rho_a L_E C_E}{\rho_o H_o C_o} (\bar{\eta}_s - \bar{\eta}_a),
\]

and the damping parameter due to SST-driven component of surface heat flux change,

\[
b = a \frac{L_E \overline{U}}{R_e T'},
\]

b. The parameters

For the atmosphere model, we choose a strong damping \( r = 1 \text{ day}^{-1} \), a reduced gravity wave speed \( c = 20 \text{ m s}^{-1} \), a shallow troposphere \( H_a = 1 \text{ km} \), and a moderate heating efficiency \( \epsilon = 10\% \) (see Table 1). Along with a 50-m slab ocean, this essentially gives a coupling strength \( \gamma = 1.7 \times 10^4 \text{ J kg}^{-1} \text{ K}^{-1} \), a wind–evaporation feedback \( a = 6 \times 10^{-8} \text{ K m}^{-1} \), and a heat flux damping \( b = 0.002 \text{ day}^{-1} \), which are consistent with the parameter values used by Xie (1996, 1999) and Battisti et al. (1999).

In model (6), two additional key parameters are the mean zonal wind and the mean meridional wind. Figure 1 shows the latitudinal distribution of the zonal average of Comprehensive Ocean–Atmosphere Dataset (COADS) climatological winds (Worley et al. 2005). As we can see, easterly trade winds prevail in the tropics, with southerly winds in the Southern Hemisphere and northerlies in the Northern Hemisphere converging at the intertropical convergence zone (ITCZ). To obtain an analytical solution, we take a linear approximation such that \( \overline{\mathbf{u}}/U = \text{const} \) and \( \overline{\mathbf{v}}/U = \text{constant} - \delta y \), with \( |\delta y| \ll 1 \). As will be clear in sections 3 and 4, the distribution of mean winds has a large impact on the dispersion and structure of coupled modes.

3. Thermodynamical coupled modes under mean zonal wind

The free wave solution of the coupled system (1) and (2) has already been identified by Liu and Xie (1994) and Xie (1996) and is characterized by westward and equatorward propagation as a result of the WES feedback mechanism. Owing to the \( \beta \) effect, however, the tropical zone acts as a waveguide such that a disturbance, once generated, tends to be trapped in the vicinity of the equator (Matsuno 1966). Therefore, in this paper we seek solutions subject to the boundary condition that the disturbance fields vanish as \( |y| \rightarrow \infty \). This boundary condition is necessary as the Gill model (1) is only valid within the tropical atmosphere.

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**Table 1. Parameter values.**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>Atmospheric damping rate</td>
<td>1 day(^{-1} )</td>
</tr>
<tr>
<td>( c )</td>
<td>Reduced gravity wave speed</td>
<td>20 m s(^{-1} )</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Efficiency of SST heating</td>
<td>1% - 10%</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Equatorial beta plane</td>
<td>( 2.3 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1} )</td>
</tr>
<tr>
<td>( \rho_o )</td>
<td>Air density</td>
<td>1.2 kg m(^{-3} )</td>
</tr>
<tr>
<td>( \rho_a )</td>
<td>Water density</td>
<td>( 1 \times 10^3 \text{ kg m}^{-3} )</td>
</tr>
<tr>
<td>( H_a )</td>
<td>Lower troposphere depth</td>
<td>1 km</td>
</tr>
<tr>
<td>( H_o )</td>
<td>Oceanic mixed layer depth</td>
<td>50 m</td>
</tr>
<tr>
<td>( C_o )</td>
<td>Specific heat of water</td>
<td>( 4.2 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1} )</td>
</tr>
<tr>
<td>( C_E )</td>
<td>Bulk exchange coefficient</td>
<td>( 1.1 \times 10^{-3} )</td>
</tr>
<tr>
<td>( L_E )</td>
<td>Latent heat of vaporization</td>
<td>( 2.4 \times 10^6 \text{ J kg}^{-1} )</td>
</tr>
<tr>
<td>( R_w )</td>
<td>Gas constant for water vapor</td>
<td>( 460 \text{ J kg}^{-1} \text{ K}^{-1} )</td>
</tr>
<tr>
<td>( \mathcal{T} )</td>
<td>Mean tropical SST</td>
<td>300 K</td>
</tr>
<tr>
<td>( U )</td>
<td>Mean speed of trade winds</td>
<td>7 m s(^{-1} )</td>
</tr>
<tr>
<td>( \bar{\eta}_a )</td>
<td>Mean specific humidity</td>
<td>18 g kg(^{-1} )</td>
</tr>
<tr>
<td>( \bar{\eta}_s )</td>
<td>Mean specific humidity at saturation</td>
<td>22 g kg(^{-1} )</td>
</tr>
</tbody>
</table>

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**Fig. 1.** Zonally averaged mean winds based on 1948–2007 COADS climatology. Within 30°S–30°N, the normalized zonal wind \( \overline{\mathbf{u}}/U \) is approximately a constant, and the normalized meridional wind \( \overline{\mathbf{v}}/U \) is approximately a linear function of latitude (the dashed line is the least squares fit).
a. Equatorial trapped modes

For a zonally infinite domain, \((u', v', \Psi', T')\) takes the wave form

\[
\begin{pmatrix}
  u' \\
  v' \\
  \Psi' \\
  T'
\end{pmatrix} = \begin{pmatrix}
  \tilde{U}(y) \\
  \tilde{V}(y) \\
  \Psi(y) \\
  \tilde{T}(y)
\end{pmatrix} e^{i(kx-\omega t)}.
\]

Then the Gill atmosphere–slab ocean model (6) becomes

\[
\begin{aligned}
    r\tilde{U} - f\tilde{V} &= -ik\Psi \\
    r\tilde{V} + f\tilde{U} &= -\frac{d\Psi}{dy} \\
    r\Psi + ic^2k\tilde{U} + c^2\frac{d\tilde{V}}{dy} &= -\gamma \left[ a\left( \frac{\Pi}{U} \tilde{U} + \frac{\nu}{U} \tilde{V} \right) + b\tilde{T} \right] \\
    i\omega\tilde{T} &= a\left( \frac{\Pi}{U} \tilde{U} + \frac{\nu}{U} \tilde{V} \right) + b\tilde{T}.
\end{aligned}
\]

Eliminating \(\tilde{T}, \Psi,\) and \(\tilde{U}\) from (7), we obtain a single equation in \(\tilde{V}\):

\[
\begin{aligned}
    c^2 \frac{d^2\tilde{V}}{dy^2} - i\sigma \left( \frac{f}{r} + \tilde{v} \right) \frac{d\tilde{V}}{dy} + \left[ \frac{i\beta c^2 k}{r} + k\sigma - i\sigma \left( \frac{\beta}{r} + \frac{dv}{dy} \right) \right] \tilde{V} &= 0, \\
    -r^2 - c^2k^2 - f^2 - k\sigma \frac{f}{\nu} \tilde{V} &= 0,
\end{aligned}
\]

where parameters \(s = \gamma a\Pi/U\) and \(\tilde{v} = \nu/\pi\), frequency \(\sigma = \omega\Pi/(b-i\omega)\), assumption \(\Pi/U = \text{const.}\), and the \(\beta\)-plane approximation \(df/dy = \beta\) are applied.\(^3\)

Then, we change the dependent variable

\[
\tilde{V} = Z(y) \exp\left( \frac{\sigma a}{2c^2} \Phi \right)
\]

with

\[
\frac{d\Phi}{dy} = \frac{f}{r} + \tilde{v}
\]

and obtain a second-order differential equation in a new variable \(Z\):

\[
\begin{aligned}
    c^2 \frac{d^2Z}{dy^2} + \left[ \frac{i\beta c^2 k}{r} + k\sigma - \frac{im a}{2} \left( \frac{\beta}{r} + \frac{dv}{dy} \right) \right] - c^2k^2 - f^2 - k\sigma \frac{f}{\nu} + \frac{m^2 \sigma^2}{4c^2} \left( \frac{f}{r} + \tilde{v} \right)^2 \right] Z &= 0.
\end{aligned}
\]

One observation of (8) is that its solution, describing the meridional structure of disturbance \((u', v', \Psi', T')\), tightly depends on the Coriolis parameter \(f = f_0 + \beta y\) and the mean wind parameter \(\tilde{v}\), both functions of \(y\).

Let us first consider the case that the mean winds are predominately zonal; that is, \(\tilde{v} = 0\). Changing the independent variable

\[
\xi = \left( \frac{4r^2c^2 - \sigma^2}{\beta^2r^2c^4} \right)^{1/4} f,
\]

we transform Eq. (8) to the well-known Weber differential equation:

\[
\frac{d^2Z}{d\xi^2} + \left[ \frac{(rk - i\beta/2)\sigma + i\beta c^2k - rc^2k^2 - r^3}{\beta\sqrt{4r^2c^2 - \sigma^2}} - \frac{1}{4} \xi^2 \right] Z = 0
\]

with the parabolic cylinder function \(D_n(\xi)\) as the general solution (see Miller 1972; Zhang and Jin 1996). When Eq. (9) is further subject to boundary condition \(Z \rightarrow 0\) as \(|\xi| \rightarrow \infty\), nonzero solutions exist if and only if the dispersion relation

\[
\frac{(rk - i\beta/2)\sigma + i\beta c^2k - rc^2k^2 - r^3}{\beta\sqrt{4r^2c^2 - \sigma^2}} = n + \frac{1}{4}
\]

holds for nonnegative integer \(n\). The solution \(D_n\) is given by the Hermite polynomial \(H_n^*\):

\[
D_n(\xi) = 2^{-n/2}e^{-c^2/4} H_n\left( \frac{\xi}{\sqrt{2}} \right).
\]

Obviously, Eq. (10) is equivalent to a quadratic polynomial in \(m\), which generally has two roots. However, a close examination reveals that \(\tilde{V}\) corresponding to one root does not converge to zero as \(|\xi| \rightarrow \infty\), implying a spurious solution. Henceforth, we shall focus only on the other root that describes a set of coupled modes converging to zero at high latitudes, with the stability and meridional structure specified by Eqs. (10) and (11), respectively.

For an easterly mean wind and strong air–sea coupling, Figs. 2a,b and 3a,b show the dispersion relation and spatial structure of the leading modes. As can be seen, these are equatorial trapped modes, whose existence is maintained by the thermodynamical feedbacks between the winds, latent heat flux, and SST anomalies. The zeroth mode \((n = 0)\), antisymmetric about the equator, propagates westward with large SST loading in the tropics and strong cross-equatorial winds, hereafter referred to as the “tropical modes.” Meanwhile, the first and higher modes \((n = 1, 2, \ldots)\), symmetric for old
mode and antisymmetric for even mode, propagates eastward with large SST and wind loading in the subtropics and little signal at the equator (except for high zonal wavenumbers, which are not of interest here)—hence, the “subtropical modes.” From Figs. 3a,b we know that such propagations result from the phase difference between the wind and SST anomalies of the coupled modes, accompanied by a positive WES feedback. Because of the heat flux damping, however, the coupled modes become unstable only when the WES feedback is strong enough (Figs. 2a,b). This suggests a tight dependency of mode stability on the air–sea coupling strength.

\[ b - i\omega = \begin{cases} \frac{\beta rsb}{\beta^2 c^2 - r^4 + \beta rs} & \text{for mode } n = 0, \\ \frac{2n(n + 1)\beta s b [2n(n + 1)\beta s + r^3 \pm i(2n + 1)r \sqrt{4n(n + 1)\beta^2 c^2 - r^4}]}{[2n(n + 1)\beta s + r^3]^2 + (2n + 1)^2r^2[4n(n + 1)\beta^2 c^2 - r^4]} & \text{for mode } n = 1, 2, \ldots \end{cases} \]

**FIG. 2.** Dispersion relation of the thermodynamical coupled modes for mean easterly and (top) strong coupling ($\epsilon = 10\%$), (middle) weak coupling ($\epsilon = 3\%$), and (bottom) westerly: (left) growth rate $\Im[\omega]$ and (right) frequency $\Re[\omega]$. The vertical axis is in units of $b$ and the horizontal axis is the zonal wavenumber in units of $\sqrt{\beta/c}/2$. For parameter values in Table 1, wavenumber 1 is equivalent to a zonal scale of 110° longitude, growth rate 1 to an $\epsilon$-folding time scale of 16 months, and frequency 1 to a period of 8 years. Positive (negative) frequency for positive wavenumber means that the mode is propagating eastward (westward). Meanwhile, positive (negative) growth rate means that the mode is unstable (stable). Note that a growth rate exceeding $-1$ (not zero) means the WES feedback is positive, as the SST-driven heat flux exchange induces an uniform damping at rate $b$ in the coupled system, which is independent of the WES feedback.

**b. Sensitivity to coupling strength**

As we are only interested in the large-scale air–sea interactions, a useful approximation can be obtained here by assuming $k = 0$ in Eq. (10), which leads to an analytical solution:
where coupling parameter $s = \gamma a$ for westerlies and $s = -\gamma a$ for easterlies. Generally speaking, the direction of the WES feedback is determined by the real part of complex frequency $b - io$, such that positive means the induced wind response reinforcing the initial SST anomaly and negative means the wind weakening the SST anomaly. Meanwhile, stability of the coupled modes is determined by the real part of $-io$, such that positive (negative) means unstable (stable).

For realistic parameters values, we usually have $r < \sqrt{\beta c}$. Then, under a mean easterly wind, the tropical mode ($n = 0$) becomes unstable only when the air–sea coupling is strong enough to induce a positive WES feedback, which requires...
\[\gamma_a > \frac{\beta^2 c^2 - r^4}{\beta r}.\]

For subtropical modes \((n = 1, 2, \ldots)\), the situation is a little bit subtle. First, the WES feedback become positive when coupling strength passes the first threshold:
\[\gamma_a > \frac{r^3}{2n(n + 1)\beta}.\]

At this stage, however, the positive WES feedback is unable to offset the heat flux damping; therefore, a stable mode is observed. Then, if the coupling is sufficiently strong to pass the second threshold,
\[\gamma_a > \frac{2[(2n + 1)^2\beta^2 c^2 - r^4]}{\beta r},\]

mode \(n\) becomes unstable. For reasonable parameter values, the coupling strength probably lies within the range bound by these two thresholds.

In the Gill atmosphere–slab ocean model (6), the coupling strength is mainly adjusted by the SST-heating efficiency \(\epsilon\). For a weak heating case \((\epsilon = 3\%)\), Figs. 2c,d and 3c,d show a stable tropical mode damped by negative WES feedback and stable subtropical modes rectified by positive WES feedback, just as expected from the above analyses. Comparing with the strong heating case (Figs. 2a,b and 3a,b), we notice that, as the thermodynamical coupling strengthens, the WES feedback from the SST anomalies in the tropical (subtropical) modes. The total effect of such phase changes is to intensify the WES feedback—from negative to positive for the tropical mode and slightly strengthening for the subtropical modes.

For mean westerlies, however, one can easily show that the WES feedback is always positive while the coupled modes are always stable (also see Figs. 2e,f).

For a constant meridional wind
\[\overline{\theta} \text{ and } \overline{v} \text{ (hence } \overline{v} \text{)},\]

one can easily show that Eq. (8) is equivalent to the Weber differential equation,
\[\frac{d^2 Z}{d\xi^2} + \left(\frac{A}{\beta c\sqrt{B}} - \frac{1}{4} \xi^2\right) Z = 0, \tag{13}\]

in a new independent variable:
\[\xi = \left(\frac{B}{\beta^2 c^2}\right)^{1/4} \left(f + \frac{(2c^2 k - \sigma)v}{4c^2 r^2 - \sigma^2}\right),\]

where
\[
\begin{align*}
A &= \frac{i\beta c^2 k}{\beta c \sqrt{B}} + \frac{k\alpha - i\beta \sigma}{2r} - \frac{c^2 k^2}{4c^2 r^2 - \sigma^2} - \frac{r^2}{4c^2 r^2 - \sigma^2} + \frac{\sigma^2 v^2 (r^2 + c^2 k^2 - k\alpha)}{4c^2 r^2 - \sigma^2}, \\
B &= \frac{4c^2 r^2 - \sigma^2}{c^2 r^2}. 
\end{align*}
\]

The physically meaningful solution to (13) again requires the frequency dispersion to satisfy the relationship
\[\frac{A}{\beta c \sqrt{B}} = n + \frac{1}{2}\]

for nonnegative integer \(n\). Similarly, the meridional distribution of the solution is given by \(D_n\) in (11).

Although the WES feedback is generally suppressed in westerly zones, the slight phase difference between wind and SST anomalies still enables the tropical and subtropical modes to propagate downwind and upwind, respectively (Figs. 2e,f and 3e,f).

In summary, the tropical and subtropical modes, characterized by dipolelike SST anomalies, are robust features of a thermodynamically coupled atmosphere–ocean system under the influence of active WES feedbacks.

4. Effects of mean meridional wind on the coupled modes

The thermodynamical coupled modes identified in section 3 are shown to be symmetric/antisymmetric about the equator for a mean zonal wind. Adding a meridional component into mean winds, however, may break such a south–north symmetry and modify the characteristics of coupled modes. Based on observations (Fig. 1), two special cases are of interest here: 1) a constant meridional wind and 2) a meridional wind that changes direction across the ITCZ.

a. Constant meridional wind

For constant \(\overline{\theta}\) and \(\overline{v}\) (hence \(\overline{v}\)), one can easily show that Eq. (8) is equivalent to the Weber differential equation,
\[\frac{A}{\beta c \sqrt{B}} = n + \frac{1}{2}\]

For a typical southeasterly wind with \(\overline{\theta} = -3 \text{ ms}^{-1}\) and \(\overline{v} = 1 \text{ ms}^{-1}\), Figs. 4a,b show the stability and frequency of the leading coupled modes, and Figs. 5a,b show the large-scale wind and SST patterns. Comparing with Figs. 3a,b, we can see that the meridional component of mean winds brings in asymmetry and causes the coupled modes to intensify in the Southern Hemisphere (for a mean southerly). Taking the subtropical mode, for
example, this can be easily understood as follows. When the mean wind changes direction from easterly to southeasterly, wind anomalies south of the equator, mainly southeasterlies/northwesterlies, will be more effective in triggering the WES feedback than the wind anomalies north of the equator, mainly northeasterlies/southwesterlies. As a result, the southern branch of the coupled mode is enhanced at the expense of its northern branch (Figs. 5a,b), leaving little change in its total stability and frequency (Figs. 4a,b).

b. Linear-varying meridional wind

A more realistic approximation of the trade winds is \( \dot{v} = \delta y \), that is, southeasterly (northeasterly) south (north) of the ITCZ. Taking the \( \beta \)-plane approximation at the ITCZ latitude,

\[ f = f_0 + \beta y, \]

we may rewrite the governing Eq. (8) in a simpler form:

\[
\frac{d^2Z}{dy^2} + \left[ A - \frac{1}{4c^4r^2}(By^2 + 2Cy) \right] Z = 0, \tag{14}
\]

where

\[ A = \frac{i\beta k}{r} - \frac{r^2 + f_0^2}{c^2} - k^2 + \frac{2kr - i(\beta + \delta r)}{2c^2 r^2} \sigma + \frac{f_0^2}{4c^4 r^4 \sigma^2}, \]

\[ B = 4\beta^2 c^2 r^2 + 4\beta \delta c^2 kr \sigma - (\beta + \delta r)^2 \sigma^2, \]

\[ C = f_0 [4\beta c^2 r^2 + 2\delta c^2 kr \sigma - (\beta + \delta r) \sigma^2]. \tag{15} \]

Then, setting a new independent variable,

\[ \xi = \left( \frac{B}{c^4 \sigma^2} \right)^{1/4} \left( y + \frac{C}{B} \right), \]

we obtain the Weber differential equation again:

\[
\frac{d^2Z}{d\xi^2} + \left( \frac{c^2 r A}{\sqrt{B}} + \frac{C^2}{4c^2 r^2 B^{3/2}} - \frac{1}{4} \xi^2 \right) Z = 0. \tag{16}
\]

Consequently, the dispersion relationship is given by

\[
\frac{c^2 r A}{\sqrt{B}} + \frac{C^2}{4c^2 r^2 B^{3/2}} = n + \frac{1}{2},
\]

which is an algebraic equation in \( \sigma \). Similarly, the meridional distribution of the solution is given by \( D_n \) in (11).

For typical trades with the ITCZ at 7°N and \( \delta = \frac{1}{2} \sigma^0 \), Figs. 4c,d and 5c,d show the stability and the wind–SST
anomalies of the leading coupled modes, respectively. In this case, with the ITCZ located north of the equator, the southeasterly dominates over the northeasterly on average. Again, we see the intensification of coupled variability in the Southern Hemisphere (Figs. 5c,d). However, if the ITCZ is located right at the equator, then the equatorial symmetry of the coupled modes would be generally preserved (not shown).

In summary, the meridional component of mean winds introduces asymmetry into the thermodynamical coupled modes such that, for a strong southeasterly mean wind, the subtropical mode could become a solely Southern Hemisphere feature with little signal in the Northern Hemisphere, just like what happens in observations (Fauchereau et al. 2003; Sterl and Hazeleger 2003).

5. Discussion and conclusions

An analytical solution of equatorial trapped modes is derived for a Gill-type simple atmosphere model coupled to a slab ocean model through the WES feedback. Under a mean easterly wind, the mode $n = 0$, antisymmetric about the equator and propagating westward, describes a series of meridional SST dipoles in the tropics, while the mode $n = 1$, symmetric about the equator and propagating eastward, describes a series of zonal SST dipoles in the subtropics. For realistic coupling strength, the latent heat flux associated with the corresponding winds is generally in phase with the SST anomaly; thus, positive WES feedbacks are excited throughout those coupled modes. When the damping effect of SST-driven surface heat exchange is taken into account, however, only the mode $n = 0$ becomes unstable for large zonal-scale perturbations, reflecting the fact that wind responds strongly to SST anomalies in the tropics and weakly in the subtropics; that is, the WES feedback is strong enough only in tropical oceans. In addition, the meridional component of mean winds can selectively enhance/weaken the WES feedback in the upwind/downwind hemisphere, reducing the subtropical mode to an asymmetric pattern with SST dipoles solely in one hemisphere.

FIG. 5. (left) Large-scale tropical meridional mode and (right) subtropical dipole mode under different mean winds: (top) southeasterly with $u = -3 \text{ m s}^{-1}$ and $v = 1 \text{ m s}^{-1}$; (bottom) trades $\nu = \delta y$ with the ITCZ at $7^\circ\text{N}$ and $\delta = 1/40$. The contours are SSTs with a 0.5°C interval and warm anomalies shaded, and the arrows are surface wind response. The $x$ and $y$ axes are drawn in units of $2\sqrt{\beta}/c$, about 17° for parameter values in Table 1. Also, see wavenumber $k = 1$ in Fig. 4 for the growth rate and frequency of these coupled modes.
Using simple atmosphere–ocean coupled models, Xie (1999), Kossin and Vimont (2007), and Lee and Wang (2008) numerically identified the least damped mode that shares similar features with the Atlantic meridional mode, namely a meridional SST dipole associated with cross-equatorial winds blowing from a cold SST anomaly toward a warm SST anomaly. By comparing our Fig. 3a with Fig. 7 of Xie (1996) and Fig. SB2 of Kossin and Vimont (2007), we recognize, after a proper phase shift as discussed in footnote 1, that the mode they found corresponds to the mode $n = 0$ here, suggesting that the predominant meridional dipole mode is an inherent property of a thermodynamical coupled atmosphere–ocean system. Similarly, one can draw a close analogy between the analytical mode $n = 1$ and the subtropical dipoles of the Atlantic and Indian Oceans. To the extent that the Gill model realistically captures the surface wind responses to the SST anomalies, we may conjecture that the tropical Atlantic (Pacific) meridional mode is the manifestation of thermodynamical coupled mode $n = 0$ and the South Atlantic (Indian) subtropical dipole mode is the manifestation of mode $n = 1$.

In the analytical solution, the wind and SST anomalies differ in phase such that the coupled variability propagates westward/eastward freely. In reality, however, the oceans are zonally bounded, and the tropical meridional mode and subtropical dipole mode are stationary in space (by definition). Meanwhile, various studies have shown that the positive WES feedback only exists in limited tropical regions (Frankignoul and Kestenare 2002; Park et al. 2005); thus, the thermodynamical modes are most likely being damped and require external forcing for their excitation (Wang and Chang 2008a,b; Lee and Wang 2008). As a result, how much variability of the observed tropical meridional mode and subtropical dipole mode can be explained in terms of our free oscillatory mode remains unclear. In addition, ocean dynamics can have a large impact on the SST–dipole oscillation (Xie 1999; Lee and Wang 2008), and the atmospheric heating, surface heat flux, and SST usually do not collocate (An 2000). How all those processes interact with the thermodynamical coupled modes identified here will be explored in a future study.

Finally, we point out that in the current study only the basin-scale thermodynamically coupled modes are discussed, which are known to affect interannual to decadal climate variability significantly. As the zonal wave-number increases, however, the action centers for mode $n = 0$ move poleward, whereas the centers for modes $n \geq 1$ move equatorward (not shown). For short zonal scales, an additional pole of strong equatorial SST anomalies appears for mode $n = 1$, and the clear distinction between the tropical mode and subtropical modes no longer applies. Whether these small-scale coupled modes play any roles in the observed tropical climate variability is an open question beyond the scope of this paper.

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