Rain Attenuation of Radar Echoes Considering Finite-Range Resolution and Using Drop Size Distributions

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ABSTRACT

The classical rain attenuation correction scheme of Hitschfeld and Bordan (HIBO) and the newer iterative approach by Hildebrand (HL) are reconsidered. Although the motivation for the HL algorithm was an extension into ranges, where HIBO tends to be unstable, it is shown here that the contrary is the case. The finite-range resolution causes an intrinsic instability of HL already at moderate attenuation, where HIBO would still deliver stable results. Therefore, the authors concentrate the further analysis on HIBO, and confirm that the usual implementation of HIBO does not account correctly for finite-range resolution. They suggest a modified scheme that produces exact retrievals in the ideal case of perfect measurements.

For vertically pointing Doppler radars a new element is explored in the attenuation correction—namely, calculating rain attenuation $\kappa$ and rainfall $R$ from Doppler spectra via the raindrop size distributions (RSDs). Although this spectral scheme (SIBO) avoids the uncertainty of $Z$–$R$ and $Z$–$\kappa$ relations, the superiority of this approach is not a priori obvious because of its sensitivity to vertical wind. Therefore, radar rain rates, based on a $Z$–$R$ relation and on RSDs, respectively, are compared with in situ measurements. The results indicate better agreement for RSD-based retrievals. Because $\kappa$ is closely correlated with $R$, the authors assert the advantage of RSD-based retrievals of $\kappa$.

The application of HIBO and SIBO to real data shows that the uncertainty of standard $Z$–$R$ relations is the main source of deviation between the two versions. In addition, the comparison of profiles suggests that the parameters of $Z$–$R$ relations aloft can deviate considerably from near-surface values. Although artifacts cannot be excluded with certainty, there is some evidence that this observation actually reflects microphysical processes.

1. Introduction

The attenuation $\kappa$ was recognized very early as a major problem for the quantitative radar retrieval of rain rate $R$ and a wealth of techniques and algorithms have been invented in the course of five decades to account for it. Hitschfeld and Bordan (1954) derived an analytical forward attenuation correction of rain rate (HIBO), assuming that only the attenuated radar reflectivity factor $Z_a$ is available from measurements. They pointed to the limited applicability of the correction because of its increasing sensitivity to errors of the measurement of $Z_a$ and of the invoked $Z$–$R$ and $\kappa$–$R$ relations with increasing (two way) path-integrated attenuation (PIA).

\begin{equation}
\text{PIA} = \exp \left[ 2 \int_0^r \kappa(x) \, dx \right].
\end{equation}

Therefore, Hildebrand (1978) suggested replacing the analytical solution by an iterative estimation of the attenuation (HL) and to start the iteration with an intentional underestimate of $\kappa$. Hildebrand argued that the attenuation error of the rain rate could be reduced even in conditions of diverging iteration by aborting the iteration at some earlier stage. Nevertheless, these forward algorithms proved to be of practical use only up to moderate PIAs and we will see that HL is even more limited than the HIBO. The general limitation of these forward correction algorithms was eventually the main driver for the use of lower, less-attenuating frequencies in operational weather radars and for devising advanced attenuation retrievals using polarimetric radars (e.g., Bringi and Chandrasekar 2001; Park et al. 2005a,b), multifrequency radars (e.g., Eccles and Mueller 1971; Amayenc et al. 1996), radar networks with overlapping range (e.g.,
The finite depth of the scattering volume will be taken into account for quantitative applications. In contrast with in situ disdrometers, VPDRs can retrieve RSDs as a function of height, including those heights coincident with the sampling volume of weather radars. In this way, a larger uncertainty of weather radar calibration can be avoided, namely, the poor correlation between rain at the surface and rain aloft. Encouraged by the excellent correlation of collocated reflectivity measurements in rain between radar wind profilers and weather radars, previously Ecklund et al. (1999) and Gage et al. (2004) pointed out the potential of such measurements for weather radar calibration. The range-resolving capability also opens other applications, for example, melting layer height monitoring or the observation of microphysical processes on the fall path Peters et al. (2005). Profilers, tailored for this application, can be built very compact and with low power consumption.

Therefore, we will reconsider here the earlier mentioned forward algorithms and start in section 2 with modified formulations of HIBO and HL. The modification consists basically of two new elements:

- The knowledge of RSDs, as retrieved from Doppler spectra, will be exploited to calculate $\kappa$ and $R$ directly using Mie theory and the drop size versus fall velocity relation, respectively. Thus, the uncertainty of $Z$–$R$ and $Z$–$\kappa$ relations employed in the $Z$-based versions of HIBO and HL is avoided.

- The finite depth of the scattering volume will be taken into account, and the impact of intravolume attenuation on retrieved rain rates will be discussed. Although this modification is only applicable to HIBO, it will be shown that disregarding the finite thickness of the scattering volume has fatal effects on the HL attenuation correction.

The superiority of the spectral retrieval is not obvious a priori because the vertical wind $w$ represents a source of error only for the spectral retrieval. Therefore, we will discuss the impact of $w$ on the retrieval of $\kappa$ and $R$ from Doppler spectra in more detail in section 3 on the basis of simulated spectra, and we will show by comparison of real data with in situ measurements that there is actually an advantage in calculating $R$ from Doppler spectra rather than from fixed $R$–$Z$ relations. Because $R$ and $\kappa$ are very closely related—particularly at K-band frequencies (Atlas and Ulbrich 1977)—we believe that this result can be also transferred to $\kappa$ retrievals.

In section 4 the original version of HIBO and the spectral modification (SIBO) are compared by application to real data.

2. Forward attenuation correction based on Doppler spectra

a. Conversion of Doppler spectra in RSDs

At vertical incidence, the attenuated drop velocity distribution $N_a(v_D)$ in still air is related to the observed Doppler spectrum $\eta_a(v_D)$ by

$$N_a(v_D) dv_D = \frac{\eta_a(v_D) dv_D}{\sigma_b(D(v_D))}$$

(2)

with $v_D$ = terminal fall velocity, $\eta_a(v_D) =$ observed spectral volume backscatter cross section, $\sigma_b(D)$ = single-particle backscatter cross section (calculated by Mie theory) of a raindrop with diameter $D$. The attenuated drop size distribution $N_a(D)$ is derived from $N_a(v_D)$ using the relation between $D$ and the fall velocity $v_D$:

$$N_a(D) dD = N_a(v_D) \frac{\partial v_D(D)}{\partial D} dD.$$  

(3)

The fall velocity versus diameter relation $v_D(D)$ has been established in laboratory measurements by Gunn and Kinzer (1949) and was parameterized in several ways (e.g., by Atlas et al. 1973). The height dependence of this relation due to the air density variation can be included, for example, according to Beard (1985).

Here, $N_a(D)$ can be used for the direct calculation of various attenuated integral rain parameters as, for example,

$$Z_a = \frac{\Lambda^4}{\pi^3} \left[ m^2 + 1 \right] \int_0^\infty \sigma_b N_a(D) dD,$$

(4)

$$R_a = \int_0^\infty D^2 N_a(D) v(D) dD,$$  

and

$$\kappa_a = \int_0^\infty \sigma_e(D) N_a(D) dD.$$  

(5)

(6)
where $Z$ is the equivalent radar reflectivity factor, $\lambda$ is the radar wavelength, $m$ is the complex refractive index, $R$ is the rain rate, $\kappa$ is the attenuation coefficient, and $\sigma_e$ is the single-particle extinction cross section. The subscript $a$ stands for “attenuated.”

b. Analytical correction

Here, $N_a(D)$ is to be multiplied with PIA to obtain the intrinsic drop size distribution $N(D)$. Because PIA is a common factor for all drop sizes, it can be taken out of the integrals in Eqs. (4), (5), and (6). Therefore, all attenuated integral variables $\xi_a$ and the corresponding intrinsic variables $\xi$ are related by

$$\xi = \xi_a \text{PIA}.$$  \hspace{1cm} (7)

In this section, we deduce an analytical relation between PIA$(r)$ at the range $r$ and the attenuated values $\kappa_a(x)$, calculated according to Eq. (6) for each point $x$ on the path between the radar and $r$. This relation can be considered as the spectral counterpart to HIBO referred to as SIBO. For convenience of notation we temporarily introduce the transmission $T = 1/\text{PIA}$. With this substitution and the relation of Eq. (7) we can rewrite Eq. (1):

$$T(r) = \exp \left[ -2 \int_0^r \frac{\kappa_a(x)}{T(x)} dx \right].$$  \hspace{1cm} (8)

By taking the logarithm on both sides and a differentiation with respect to $r$ yields

$$\frac{1}{T(r)} \frac{\partial T(r)}{\partial r} = -2 \frac{\kappa_a(r)}{T(r)},$$

and after the canceling of $T(r)$

$$\frac{\partial [T(r)]}{\partial r} = -2 \kappa_a(r).$$

The integration over $r$ leads to

$$T(r) - T(0) = -2 \int_0^r \kappa_a(x) dx.$$  \hspace{1cm} (9)

Because $T(0) = 1$, we obtain (going back to PIA = $1/T$)

$$\text{PIA}_{\text{SIBO}}(r) = \frac{1}{1 - 2 \int_0^r \kappa_a(x) dx}.$$  \hspace{1cm} (10)

It can be shown that Eq. (10) is in fact identical with Eq. (18) in Hitschfeld and Bordan (1954), if the corresponding empirical $Z$–$R$ and $\kappa$–$R$ relations are inserted.

c. Effect of calibration error

For shortness of notation we omit the indices HIBO and SIBO in this and the following section (2d) because the findings are generally applicable.

We assume a calibration error $\delta$ ($\delta = 1$ corresponds to perfect calibration) leading to the retrieval of a biased PIA denoted by $\text{PIA}_\delta$

$$\text{PIA}_\delta = \frac{1}{1 - 2\delta \int_0^r \kappa_a(x) dx}.$$  \hspace{1cm} (11)

The retrieved biased variable $\xi_\delta$ can be described by modifying Eq. (7),

$$\xi_\delta = \delta \frac{\text{PIA}_\delta}{\text{PIA}}.$$  \hspace{1cm} (12)

Here, Eq. (12) indicates that the total bias of the variable $\xi$ consists of the calibration error $\delta$ itself and of the biased attenuation correction due to the calibration error. The relative total bias is equal to the ratio of $\xi_\delta$ and $\xi$, which according to Eqs. (12) and (7) is

$$\frac{\xi_\delta}{\xi} = \delta \frac{\text{PIA}_\delta}{\text{PIA}}.$$  \hspace{1cm} (13)

We insert $\text{PIA}_\delta$ from Eq. (11) and obtain

$$\frac{\xi_\delta}{\xi} = \frac{\delta}{1 - 2\delta \int_0^r \kappa_a(x) dx},$$

or

$$\frac{\xi_\delta}{\xi} = \frac{\delta \text{PIA}}{1 - 2\delta \int_0^r \kappa_a(x) dx},$$

or

$$\frac{\xi_\delta}{\xi} = \frac{\delta}{1 - \text{PIA}(1 - \delta)}.$$  \hspace{1cm} (14)

Figure 1 shows that for a given total relative error the accuracy of calibration increases for increasing values of PIA. A practical upper limit of PIA seems to be around 10 dB, where the calibration uncertainty must be less than 1 dB to keep the measurement error within ±3 dB. This is in agreement with the analysis of HIBO by Delrieu et al. (1999).

d. Effect of finite-range resolution

The integration in Eq. (10) implies that $\kappa_a$ can be measured with infinitesimal range resolution. In reality, spatial averages over the range resolution $\Delta r$ are observed. Although the integral in Eq. (10) can be replaced...
Fig. 1. Total relative measurement error vs calibration error for various values of PIA. The calibration tolerances are indicated for ±3-dB total relative error.

Fig. 2. Bias of attenuation correction vs τ for different choices of the reference point r in the finite-scattering volume.

\[
\text{PIA}^* = \frac{1}{1 - 2(\kappa^*_a)(\alpha \Delta r)},
\]

where \(0 \leq \alpha \leq 1\) determines the chosen position of the reference point in the scattering volume. The retrieved RSD is calculated (under the false assumption of homogeneity for the attenuated signal) by

\[
N^*(D) = \langle N_a(D) \rangle \text{PIA}^*.
\]

If we insert \(\langle N_a(D) \rangle\) from Eq. (17) and PIA* from Eq. (19), we obtain

\[
N^*(D) = N(D) \frac{1 - \exp(-\tau)}{\tau (1 - \alpha[1 - \exp(-\tau)])}.
\]

The condition for the correct choice of \(\alpha\) is \(N^*(D) = N(D)\), which leads to

\[
\alpha = \frac{1}{1 - \exp(-\tau)} - \frac{1}{\tau}.
\]

Here, Eq. (22) shows that there is no global value \(\alpha\) for an unbiased retrieval, but \(\alpha\) is a function of \(\tau\). The ratio \(N^*(D)/\langle N(D) \rangle\) is shown in Fig. 2 for three choices of \(\alpha\), namely 0, 0.5 and 1.

We recognize that choosing the center of the scattering volume avoids gross errors but that a negative bias remains for larger values of \(\tau\). The exact reconstruction of \(N(D)\) from average observations \(\langle N_a(D) \rangle\) (still under the assumption of homogeneity in the scattering volume) is nevertheless possible by combining Eqs. (17) and (18) so that

\[
N(D) = \langle N_a(D) \rangle \frac{\kappa}{\langle \kappa_a \rangle}.
\]

Because we wish to express \(N(D)\) as function of only observable variables, \(\kappa\) needs to be eliminated from Eq. (23). For this purpose we solve Eq. (18) for \(\kappa\),
and insert Eq. (24) into Eq. (23):

\[ N(D) = -(N_d(D)) \frac{\ln(1 - 2(\kappa_r)\Delta r)}{2(\kappa_r)\Delta r}. \]  

(25)

e. Application of exact attenuation correction to multiple finite-range gates

For the attenuation correction of a profile consisting of multiple range gates with finite depth \( \Delta r \), the PIAs and RSDs for each range gate \( r_i \) can now be calculated in the usual way, proceeding stepwise from the lowest to the highest range gate according to the flowchart shown in Fig. 3. The lower and upper boundaries of range gate \( r_i \) are at \((i - 1)\Delta r\) and \(i\Delta r\), respectively. The subscript \( p \) stands for “partial attenuation corrected” namely, corrected for the attenuation on the path between the radar and the lower boundary of range gate \( r_i \) but without the correction of attenuation within \( r_i \). Here, \( N(D, r_i) \) is the retrieved RSD in range gate \( r_i \) with a full attenuation correction. [Note that \( N(D, r_i) = \langle N(D, r) \rangle \) because we assumed homogeneity in each range gate.]

The flowchart shows both the conventional biased and the new unbiased provision for finite-range resolution as discussed in section 2d. In the first range gate, \( \text{PIA}_{\text{biased}} \) and \( N_{\text{biased}} \) are identical with \( \text{PIA}^* \) and \( N^* \), respectively [see Eqs. (19) and (20)]. Although it is only slightly biased for \( \alpha = 0.5 \) and realistic values of \( \tau \) (see Fig. 2), the bias can become significant at higher range gates by accumulation with each cycle through the flowchart sketched in Fig. 3. An example is shown for real data in section 4.

**Fig. 3.** HIBO with different provisions for the finite-range resolution. (left) Biased, according to Eqs. (19) and (20). (right) Unbiased, according Eq. (25).
**f. Iterative approach**

Figure 1 showed that a calibration error \( \delta < 1 \) would be less detrimental than \( \delta > 1 \). This was the motivation for devising an iterative retrieval procedure (Hildebrand 1978), with the intentional underestimation of the initial guess of \( \kappa \). For reference we reproduce Eqs. (5) [denoted here as (HL 5)] and (6) [denoted here as (HL 6)] of that paper adopting the original notation:

\[
\log Z'(r) = \log Z_a(r) + 2 \sum_{i=1}^{r-1} K_a(x), \quad \text{(HL 5)}
\]

where \( Z'(r) \) is the first iteration of the attenuation-corrected radar reflectivity factor, \( Z_a(r) \) is the measured attenuated radar reflectivity factor, and \( K_a(x) \) is an (under) estimate of attenuation using a \( K-Z \) relation with \( Z_a(x) \) as the independent variable. The second iteration for \( Z(r) \) is obtained by estimating \( K(x) \) using again the \( K-Z \) relation with \( Z'(r) \) as the independent variable:

\[
\log Z'(r) = \log Z_a(r) + 2 \sum_{i=1}^{r-1} K'(x). \quad \text{(HL 6)}
\]

The general iteration would read \( \log Z_{i+1}(r) = \log Z_a(r) + 2 \sum_{i=1}^{r-1} K_{i+1}(x) \), and the iteration is terminated when the difference between subsequent steps falls below a certain threshold.

To show why this approach contains—in contrast to intuition—a further source of instability in addition to the analytical solution, it is necessary to render more precisely the meaning of \( K \) in Eqs. (HL 5) and (HL 6). Here, \( K \) was introduced in the paper as the specific attenuation, which has the dimension length \(^{-1}\). For clarity we prefer to formulate the equations for a general range resolution \( \Delta r \). Then, Eq. (HL 5) takes the form

\[
\log Z'(r) = \log Z_a(r) + 2 \sum_{i=1}^{r-1} \kappa_a(x) \Delta r. \quad \text{(26)}
\]

We focus here on the second range gate \( (r = 2) \) without the loss of generality and omit the range argument for the sake of brevity in the following. (The choice of \( r = 2 \) is equivalent to considering an arbitrary range gate \( r \) and assuming that the attenuation up to range gate \( r - 1 \) is already known.) Then, Eq. (26) assumes the simpler form of

\[
\log Z'(r) = \log Z_a + 2 \kappa_a \Delta r, \quad \text{(27)}
\]

and generally

\[
\log Z_{i+1} = \log Z_i + 2 \kappa_{i-1} \Delta r \quad \text{for} \quad i = 1, \ldots, n, \quad \text{(28)}
\]

with \( \kappa_{(0)} = \kappa_a \) and \( Z_{(0)} = Z_a \). The index \( a \) was replaced by \( (0) \) as a reminder that the attenuated value is the initial value for the iteration.

Empirical \( \kappa-Z \) relations are usually presented as power laws \( \kappa = \alpha Z^\beta \), where the deviation of \( \beta \) from unity accounts for the change of shape of the RSD with \( Z \). Although the \( \kappa-Z \) relation is defined for the true values of \( \kappa \) and \( Z \), it is still at our discretion how \( \kappa_{(0)} \) is inferred from the attenuated value \( Z_{(0)} \). Hildebrand invoked the same relation between the attenuated and the true values, that is,

\[
\kappa_{(i)} = \alpha Z_{(i)}^\beta. \quad \text{(29)}
\]

Typical values of \( \beta \) are between 0.7 and 0.9 depending on temperature and wavelength (e.g., Gorgucci et al. 1998). For the repeated guesses of \( \kappa_{(i)} \) in the course of higher iterations there is nevertheless some arbitrariness in the choice of the employed \( \kappa_{(i)}-Z_{(i)} \) relation. A linear dependence \( (\beta = 1) \) might be reasonable as well because the shape of the DSD is not affected by attenuation:

\[
\kappa_{(i)} = \frac{Z_{(i)}}{Z_{(i-1)}} \kappa_{(i-1)} \quad \text{for} \quad i > 1. \quad \text{(30)}
\]

In case of a vertically looking Doppler radar, there is in fact no need for any empirical \( \kappa-Z \) relation; however, \( \kappa_{(0)} \) is inferred directly from the attenuated RSD, which is identical with the true RSD except for an attenuation factor. Therefore, the linear relation \( \kappa_{(0)}/\kappa = Z_{(0)}/Z \) holds and, consequently, a nonlinear relation between iterations of \( \kappa \) would be very artificial. Fortunately, the choice of \( \beta \) is irrelevant for the mathematical structure of the iteration. Any choice of \( \beta \) can be reduced to the case \( \beta = 1 \) by replacing \( Z \) by the new variable \( \zeta = Z^\beta \) and performing the iteration for \( \zeta \). Therefore, we set \( \beta = 1 \) for convenience in the following analysis.

The relation between \( \kappa_{(0)} \) and \( \kappa \) can be rewritten remembering that the attenuation of one range gate is \( \text{PIA} = \exp(2\kappa \Delta r) \):

\[
\kappa_{(0)} = \kappa \exp(-2\kappa \Delta r). \quad \text{(31)}
\]

We expand Eq. (31) with \( 2\Delta r \) to obtain \( \kappa_{(0)}2\Delta r = \exp(-2\kappa \Delta r)2\Delta r \) and introduce again the two-way transmission \( T = 1/\text{PIA} = \exp(-2\kappa \Delta r) \) leading to

\[
\ln T_{(0)} = T \ln T. \quad \text{(32)}
\]

After exponentiating, we obtain the final form

\[
T_{(0)} = (T)^T. \quad \text{(33)}
\]
The function \( T(0) \) versus \( T \) is plotted in Fig. 4. It shows that Eq. (33) can be only satisfied for \( T(0) \) larger than a minimum value, which can be shown to be \( T(0),\min = \exp(-1/e) \). We attribute this puzzling behavior to a contradiction between Eqs. (27) and (32). Both equations imply that the specific attenuation is uniform inside the scattering volume [otherwise the product \( \kappa \Delta r \) or \( \kappa(0) \Delta r \) would have to be replaced by the integral \( \int_0^R \kappa(x) \, dx \) or \( \int_0^r \kappa(0)(x) \, dx \) over the scattering volume.]. The assumption of homogeneity in the scattering volume is certainly reasonable for the true attenuation because we cannot resolve structures inside the scattering volume anyway. However, having already decided the homogeneity of \( \kappa \) in the scattering volume, this assumption cannot be applied at the same time to the attenuated attenuation \( \kappa(0) \) because the path-integrated attenuation is increasing inside the scattering volume.

To demonstrate the behavior of HL for different start values \( T(0) \), we cast Eq. (28) in a recursive form. For that purpose we introduce the iterated transmission \( T_i = Z_i(0)/Z_i(0), \)

\[
T(i) = \exp(-2\kappa(i-1)\Delta r).
\]  

Furthermore, we replace \( \kappa(i-1) \) by \( \kappa(0)T(i-1) \) and \( \exp(-2\kappa(0)\Delta r) \) by \( T(0) \), which leads to the form of

\[
T(i) = T(0)^{1/T(i-1)}.
\]  

Here, Eq. (35) allows us to calculate the iterations \( T(i) \) as a function of the start value \( T(0) \) as shown in Fig. 5.

For start values of the transmission close to 0 dB, the iterations seem to converge to the upper branch of true transmission. Although we could not find the analytic proof, numerical experiments showed that the iterations diverge if the start value \( T(0) \) falls below 1/1.444 668 . . . , corresponding to the numerical value of \( T(0),\min = \exp(-1/e) \).

Figure 6 shows results of the iteration scheme applied to a homogeneous profile of simulated RSDs according to Marshall and Palmer (1948), that is, \( N_{\text{MP}}(D) = N_0 \exp(-AD) \) with \( N_0 = 8 \times 10^4 \text{ m}^{-3} \text{ mm}^{-1} \) and \( A/\text{mm}^{-1} = 4.1 \) \((R/\text{mm h}^{-1})^{-0.21} \). A range resolution of 100 m was chosen and the performance of PIAHL in 3 ranges is displayed as function of \( R \). [Note that here we disregard that \( R \) in the \( N_{\text{MP}}(D) \) parameterization is not exactly equal to the rain rate, which is obtained by inserting \( N_{\text{MP}}(D) \) into the spectral retrieval according to Eq. (5).] The iterations were terminated when the ratio of successive iteration steps fell below 1.05 (equivalent to 0.2 dB). Because HL is intrinsically unstable, if PIA(0) per range gate exceeds 1/T(0),min, the display is truncated one step before PIAHL exceeded 30 dB. For reference, the corresponding PIA_SIBO is shown, which is identical with the true PIA in this simulation. One recognizes that PIAHL consistently overestimates PIA even in the range where the “success criterion” \( \text{PIA}_{\text{HL}}/\text{PIA}_{\text{HL},-1} < 1.05 \) is reached—except for very low rain rates. Because the overestimation is accumulated with increasing range, the start value PIA(0) increases with range and consequently the maximum possible rain rate with converging PIAHL decreases with increasing range. For the assumed K-band frequency this limit is already reached at \( R \approx 20 \text{ mm h}^{-1} \) at a range of \( r = 1000 \) m. The tendency of overestimation and instability of HL is in agreement with field observations reported by Johnson and Brandes (1987).

### 3. Vertical wind

RSDs derived from Doppler spectra are sensitive to the vertical wind component \( w \), which therefore represents a source of uncertainty in spectral forward attenuation algorithms. To quantify the influence of \( w \), we go back to Eq. (3). In the presence of nonvanishing vertical wind, the Doppler velocity of the raindrops is \( v = u_r + w \). Therefore, Eq. (3) yields an erroneous RSD if we insert \( N_d(u_r + w) \) of Eq. (2):
We analyze the \(w\)-induced error of attenuation as calculated with Eqs. (6) and (4), again using simulated RSDs according to Marshall and Palmer (1948). The simulation consists of the following steps:

(i) Transform \(N_{MP}(D, R)\) into a Doppler spectrum \(\eta(y, R)\) using

\[
\eta_{MP}(v_i, R) dv_i = \sigma_b(D(v_i)]N_{MP}(D, R) \frac{\partial D}{\partial v_i} dv_i;
\]

(ii) shift the velocity axis by \(w\): \(v = v_i + w\); (iii) transform the shifted Doppler spectrum back into a biased \(N_{MP,w}(D, R)\),

(iv) calculate the biased integral variable \(\kappa_{MP,w}\); and (v) calculate the bias \(\Delta \kappa_{MP}(w) = \kappa_{MP,w} - \kappa_{MP,0}\). For a realistic simulation of the wind-induced error, the details of the radar signal processing must be taken into account. The integration according to Eqs. (4)–(6) is to be replaced by a sum over diameter classes, corresponding to discrete Fourier lines of the Doppler spectrum. More importantly, the summation over \(D\) starting from \(D = 0\) would diverge in case of upwind. For avoiding the associated instability of the retrieval, the integration is truncated at \(v_{\text{min}} = 0.585\) m s\(^{-1}\) (here downward velocities are counted positive). Similarly, an upper limit of integration is introduced at \(v_{\text{max}} = 9.2\) m s\(^{-1}\), which corresponds to the maximum fall speed of stable drops according to Gunn and Kinzer (1949). [These limits are slightly modified at higher ranges to account for the density-dependent terminal fall speed according to Beard (1985).]

With these conditions, the sensitivity of \(\kappa\) to the vertical wind was simulated. Figure 7 shows that upwind (downwind) causes overestimation (underestimation) of \(\kappa\). Similar results are obtained for the rain rate (not shown here).

Because empirical \(\kappa-Z\) relations do not depend on the vertical wind, the superiority of RSD-based retrieval...
methods is not a priori evident. Therefore, we show in Fig. 8 a comparison of rain rates $R_S$ and $R_Z$, obtained with a microrain radar (MRR2) as described in Table 1, with $R_I$ obtained with a standard Hellmann tipping bucket rain gauge. Here, $R_S$ (left panel) and $R_Z$ (right panel) are based on RSDs and an $R$–$Z$ relation, respectively. They were retrieved simultaneously from the Doppler spectra and the radar reflectivity factor of a vertically pointing MRR2. The data were collected at a test site of the German weather service in Westermarkelsdorf, Germany, on the isle of Fehmarn during April 2003. The time resolution was 1 min, the range resolution of the MRR2 was 35 m, and the third range gate (105 m) was used for the comparison. The resolution of the rain gauge was 0.3 mm h$^{-1}$. The total number of rain samples was 1852.

For the retrieval of $R_Z$ the relation $Z = 200 \times R^{1.6}$ was used. Both $R_S$ and $R_Z$ show a bias of about $-2$ dB compared to $R_I$, which we attribute to the MRR2 calibration. A standard wind correction (Rubel and Hantel 1999) was applied to the rain gauge data. Therefore, the samples of the rain gauge are not exactly multiples of 0.3 mm h$^{-1}$. The large scatter of the regression at low rain rates is obviously due to the coarse resolution of the rain gauge. At moderate and higher rain rates, the better agreement between $R_S$ and $R_I$ is clearly visible. Because the attenuation is very closely linked to the rain rate—particularly at K-band frequencies—(Atlas and Ulbrich 1977) we also believe that $\kappa$ retrievals are more accurate if they are based on RSDs rather than on $\kappa$–$Z$ relations.

4. Application of HIBO and SIBO to real data

The original HIBO and the PIA retrieval schemes described in sections 2b and 2e were applied to a set of Doppler spectra obtained with a vertically pointing MRR2 (specifications in Table 1). The data were collected in Wielenbach, Germany, during one day with several hours of rain (3 August 2007) during the course of the field campaign Advances in Quantitative Areal Precipitation Estimation by Radar (AQUARADAR; available online at http://www.meteo.uni-bonn.de/projekte/aquaradar-wiki/doku.php/home). The averaging time for one measurement was 10 s, the range resolution $\Delta r = 100$ m, and the number of range gates was 31.

Figure 9 shows a height–time cross section of the Doppler velocity indicating a rain event of more than 15-h duration. The step in the fall velocity profiles marks the height of the melting layer, where the falling hydrometeors accelerate during the melting process. The melting layer starts at 2200 m and sinks to 1600 m in the course of the rain event. Because the retrieval algorithms are only applicable in the liquid phase, we constrain the analysis to the lower 1500 m.

To demonstrate the isolated effect of a finite-range resolution, we compare in Fig. 10 rain rates retrieved at 1000 m by applying the conventional, biased scheme with $\alpha = 0.5$ and the new unbiased scheme described in

<table>
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<th>Table 1. MRR2 radar specifications.</th>
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section 2e. At the highest rain rates, the biased scheme yields an underestimation up to 3 dB.

The overall plausibility of $R$ and $\kappa$ retrievals with SIBO is checked by converting the SIBO-retrieved values of $\kappa$ into rain rates using the power law $R$–$\kappa$ relation of Atlas and Ulbrich (1977) for the K band. The agreement with SIBO, as shown in Fig. 11, is very good except for rain rates of more than 5 mm h$^{-1}$, where the $R$–$\kappa$ relation causes a bias of about $-2$ dB. Nevertheless, the scatter is small here as well, which suggests that the agreement could be even improved by a slight modification of the $R$–$\kappa$ relation.

Finally, we compared SIBO with HIBO using $Z_a$ as the input and applying the $Z$–$R$ and $R$–$\kappa$ relations as

![Fig. 9. Time–height cross section of the Doppler velocity.](image1)

![Fig. 10. Comparison at 1000-m range of different provisions for finite-range resolution, as discussed in section 2e. Biased rain-rate retrieval according to the left branch of Fig. 3 is shown on the y axis (conventional method) and the rain-rate retrieval according to the right branch of Fig. 3 is shown on the x axis (new method). At this range differences become significant at rain rates exceeding 10 mm h$^{-1}$.](image2)
used in the original version of HIBO. The correlation between HIBO and SIBO retrievals of the rain rate is only moderate, as is shown for the ranges 500 m and 1000 m in Fig. 12.

We recall the comparison of rain rates versus rain gauge, shown in section 3, which indicates the superiority of Doppler spectra over a $Z$–$R$ relation for the retrieval of $R$. Because $R$ and $\kappa$ are highly correlated (see, e.g., Atlas and Ulbrich 1977), we expect that the use of Doppler spectra also leads to a better quality of $\kappa$ retrievals than the retrieval of $\kappa$ from $Z$ via $R$ as done in HIBO.

In addition to the scatter, the regression at 1000 m (right panel of Fig. 12) shows a systematic deviation at high rain rates. The most obvious reason for the discrepancy is overestimation by SIBO due to upwind. Its impact on the retrieved rain rate is similar to the attenuation shown in Fig. 7 (Peters et al. 2005). Although it is not possible to quantify unambiguously the effects of vertical wind in this dataset, we discuss three indicators that in this case the wind-induced bias of SIBO may be less important than a bias of HIBO because of deviations from assumed RSD shapes (which are based on surface observations).

a. Shape of Doppler spectra

Figure 13 shows the spectral reflectivity below the melting layer between 100 and 1500 m during the period of maximum rain rate. The baselines of spectra for adjacent range gates are shifted by the range resolution (100 m) and the maxima are normalized, such that each spectrum extends over 2.5 range gates in this display. At higher levels, there is a shift to smaller fall velocities, which could either be caused by a shift of the RSD toward smaller drops or by an updraft with amplitude increasing with height or a mixture of both. Closer inspection of the spectra shows that they are not only shifted, but that they also show different shapes. The near-surface spectra are left skewed, which is the typical shape for moderate and strong rain. It is caused by the small variation of terminal fall velocity (8–9.5 m s$^{-1}$) of raindrops with diameters larger than 3 mm. At higher levels, the skewness vanishes or even changes sign. This change of shape indicates an abundance of smaller drops aloft, with the consequence of underestimated rain rates by HIBO. Another potential reason for the observed shape of Doppler spectra aloft can be the presence of snow flakes in the considered measuring height, causing overestimated rain rates by SIBO. This is nevertheless unlikely at 1000 m because the melting layer is at 1700 m between 1100 and 1200 UTC.

b. Doppler velocity above the melting layer

A hint on the possible role of upwind can be obtained from inspecting the Doppler velocity above the melting layer. Because the fall velocity of snow is rather small...
and uniform, we expect that potential updrafts during intense phases of precipitation should extend above the melting layer and should cause a noticeable modification of the Doppler velocity there.

Figure 14 shows the time series of the Doppler velocity $v$ at 2300 m (short dashed) together with $R_{\text{HIBO}}$ (long dashed) and $R_{\text{SIBO}}$ (solid) at 300 m. The original 10-s samples were condensed to 60-s averages. Most of the time $v$ is close to 1.5 m $s^{-1}$, which is a typical value for snowfall. This is to be expected at 2300 m, which is according to Fig. 9 well above the melting layer. During the most intense rain phase around $t_p = 1130$ UTC—corresponding to the deviating samples of the regression in Fig. 12 (right panel)—the Doppler velocity indeed shows pronounced excursions, but the sign indicates downdrafts rather than updrafts. Also, a change of the habit of ice crystals may cause the excursion of the Doppler velocity. At any rate, there is no hint for an updraft above the melting layer (not shown).

c. Time series of $R_{\text{HIBO}}$ and $R_{\text{SIBO}}$

An indication on systematic deviations from standard RSDs is provided by the time series of $R_{\text{HIBO}}$ and $R_{\text{SIBO}}$ at 300 m in Fig. 14. The rain event can be separated into two phases with a ratio of $R_{\text{HIBO}}:R_{\text{SIBO}} \approx 2$ before and $R_{\text{HIBO}}:R_{\text{SIBO}} \approx 1$ after $t_p$. Vertical wind differing by about 1 m $s^{-1}$ before and after $t_p$ with a persistence of several hours would be necessary to explain such bias of SIBO. Because this is unrealistic, there was obviously a change of the RSDs toward smaller drops that occurred at $t_p$. This switching between “modes” has been described already by Clemens et al. (2006) and seems to be not uncommon. In comparison to 1000 m, where the ratio $R_{\text{HIBO}}:R_{\text{SIBO}}$ fell below unity during the peak phase, the change at 300 m is less pronounced. This height dependence is also visible in Fig. 12. The bias at high rain rates, observed at 1000 m (right panel), is absent at 500 m (left panel). This difference may be due to the transformation of “young” RSDs close below the melting layer into equilibrium RSDs further down in the rain shaft.
5. Conclusions

The analytic (Hitschfeld and Bordan 1954) and iterative (Hildebrand 1978) forward attenuation correction algorithms HIBO and HL were critically reviewed. Although HL was developed to mitigate the instability of HIBO occurring at larger values of PIA, we found a fundamental instability of HL, which is not related to measurement errors and in fact represents an even narrower restriction of applicability in comparison with HIBO. The instability of HL is related to the finite-range resolution—it diverges if the intrarange attenuation exceeds e^1/e. We also found that the usual implementation of HIBO does not account correctly for the effect of finite-range resolution. Therefore, a modified algorithm was suggested, which exactly reproduces the attenuation in case of perfect measurement accuracy, but it can differ from the original version by up to several decibels.

Further, we analyzed the option to use the full spectral information of vertically pointing Doppler radars (SIBO). The better quality of spectral rain-rate retrievals than Z–R-based retrievals, which were observed in comparisons with in situ measurements, suggest that for the vertically pointing direction the attenuation retrieval via SIBO is to be preferred to HIBO at least in stratiform conditions. The analysis of the vertical distribution of rain rates retrieved by HIBO and SIBO during a stratiform rain event with several hours of persistence showed some evidence for height dependencies of RSDs because of microphysical processes leading to the height-dependent bias of HIBO in this example. Although vertical wind seemed to not be the prevailing factor for HIBO–SIBO differences in this example, it must generally be taken into account.

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REFERENCES


