The Impact of Evaporation on Polarimetric Characteristics of Rain: Theoretical Model and Practical Implications

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ABSTRACT

Soon, the National Weather Service’s Weather Surveillance Radar-1988 Doppler (WSR-88D) network will be upgraded to allow dual-polarization capabilities. Therefore, it is imperative to understand and identify microphysical processes using the polarimetric variables. Though melting and size sorting of hydrometeors have been investigated, there has been relatively little focus devoted to the impacts of evaporation on the polarimetric characteristics of rainfall. In this study, a simple explicit bin microphysics one-dimensional rainshaft model is constructed to quantify the impacts of evaporation (neglecting the collisional processes) on vertical profiles of polarimetric radar variables in rain. The results of this model are applicable for light to moderate rain ($< 10$ mm h$^{-1}$). The modeling results indicate that the amount of evaporation that occurs in the subcloud layer is strongly dependent on the initial shape of the drop size distribution aloft, which can be assessed with polarimetric measurements. Understanding how radar-estimated rainfall rates may change in height due to evaporation is important for quantitative precipitation estimates, especially in regions far from the radar or in regions of complex terrain where low levels may not be adequately sampled. In addition to quantifying the effects of evaporation, a simple method of estimating the amount of evaporation that occurs in a given environment based on polarimetric radar measurements of the reflectivity factor $Z_H$ and differential reflectivity $Z_{DR}$ aloft is offered. Such a technique may be useful to operational meteorologists and hydrologists in estimating the amount of precipitation reaching the surface, especially in regions of poor low-level radar coverage such as mountainous regions or locations at large distances from the radar.

1. Introduction

With the upgrade of the National Weather Service’s Weather Surveillance Radar-1988 Doppler (WSR-88D) network to dual-polarization capabilities comes additional information that operational meteorologists can use to better diagnose and predict the local weather (e.g., Zrnic and Ryzhkov 1999; Ryzhkov et al. 2005b). Because polarimetric radars are quite sensitive to certain phenomena, including hydrometeor phase transitions and size sorting, it is important to understand how such microphysical processes are detectable with measured polarimetric variables. For example, the important polarimetric “bright band” signature associated with melting graupel and snow is well documented (e.g., Brandes and Ikeda 2004; Giangrande et al. 2005, 2008). Additionally, polarimetric observations of size sorting can reveal important information about storm kinematics, including the low-level storm-relative helicity in supercell environments (Kumjian and Ryzhkov 2009).

In contrast, the impacts of warm rain precipitation processes on the polarimetric variables have received comparatively little attention. As rain falls from a precipitating cloud, the distribution of mass among different-sized drops is governed by several microphysical processes. The evolution of this drop size distribution (DSD) is a complex problem that has received considerable attention in the literature over the past few decades. Subcloud microphysical processes affecting the shape of the DSD at the ground include differential sedimentation, spontaneous breakup, collisional breakup, coalescence, and vapor diffusion (i.e., evaporation). Early works by Young (1975), Srivastava (1978), and Johnson (1982) determined that spontaneous breakup is relatively unimportant in comparison with collisional breakup, especially for higher rainwater contents. Laboratory studies by Low and List (1982a) and Beard and Ochs (1995) have facilitated our
understanding of collisional processes such as coalescence and breakup. Based on their experimental data, Low and List (1982a,b) developed coalescence efficiencies and a parameterization for the fragment size distribution of particles resulting from filament, sheet, and disk breakup following the collision of two raindrops. These and similar parameterizations have been widely analyzed, improved, and utilized in zero-dimensional box models and one-dimensional rainshaft models (e.g., Gillespie and List 1976; Low and List 1982b; Brown 1986, 1987; List and McFarquhar 1990). Evaporation beneath the cloud base was included in the models by List et al. (1987), Tzivion et al. (1989), Brown (1993, 1994), Hu and Srivastava (1995, hereinafter HS), and Seifert (2008).

These studies and others have found that the collisional processes of coalescence and breakup tend to dominate the evolution of the DSD shape. These processes tend to drive an arbitrary initial DSD toward a family of equilibrium shapes that are related through simple multiplicative factors (List et al. 1987; Brown 1987, 1993; HS). The latter authors found that DSD evolution can be categorized into two phases: 1) collisional processes dominating the evolution of the spectrum as it approaches equilibrium, followed by 2) evaporation dominating the spectral evolution, smoothing maxima, and decreasing the overall mass (while only slowly changing the shape of the distribution). If the initial drop size distribution is already close to its equilibrium shape, HS found that the first stage does not occur. Note that some of the equilibrium DSDs in the aforementioned works are the result of artifacts in the original Low and List (1982a,b) parameterization; the parameterization by McFarquhar (2004) based on the Low and List data alleviates some of these shortcomings and produces equilibrium distributions that are quite different than those found in the previous studies.

The collisional processes and differential sedimentation by themselves do not contribute to or deplete the liquid water content in the subcloud layer: they simply redistribute mass to different sizes. However, net evaporation (the diffusion of water vapor away from drops) depletes the total rainwater content, which has significant implications for quantitative precipitation estimation (QPE). In addition, the generation of negative buoyancy via evaporational cooling plays an important role in storm evolution, including the production of severe downdrafts (e.g., Srivastava 1985, 1987) and even possibly affecting a supercell storm’s likelihood of producing a tornado (e.g., Markowski et al. 2002, 2003; Grzych et al. 2007). Despite the many studies that quantify the impacts of environmental conditions on the rate of evaporation and how evaporation affects the rainfall rate, drop size distribution, downdrafts, and radar reflectivity factor (e.g., Srivastava 1985, 1987; Rosenfeld and Mintz 1988; HS), there is a paucity of studies investigating how varying evaporation rates are manifest in the polarimetric variables. A notable exception is Li and Srivastava (2001), who quantified the impacts of evaporation on differential reflectivity, Z_{DR}.

In contrast, the purpose of this paper is to quantify the sensitivity of all polarimetric variables to various environmental thermodynamic conditions, and variations in DSDs for light rainfall rates, as well as to formulate recommendations to aid in hydrometeorological rainfall estimation.

Most frequently, such rainfall estimates are made by using remote sensing techniques, especially radar (and, soon, dual-polarization radar). Hence, this paper will focus on evaporation. As such, this work should not be viewed as an attempt to investigate DSD evolution; rather, our goal is to quantify the impacts of evaporation on the polarimetric variables as a preliminary step in understanding how warm rain microphysics are manifested in the radar measurements. The inherent limitations of such an approach reduce the general applicability of such a study. Nonetheless, the results of this work should help with the aforementioned questions and may contribute to improving QPE.

The following section will outline the polarimetric variables and their physical interpretation as well as the physics of the evaporation of raindrops. The qualitative impacts of evaporation on the polarimetric variables will be conceptualized to provide a framework for the quantitative analysis conducted in the remainder of the paper. This quantification will be done using an explicit microphysics model that is described in section 3. Section 4 details the sensitivity tests, outlining the model sensitivity and exploring the parameter space. The model is then used to perform simulations in different evaporation scenarios in section 5. The impacts of evaporation on rainfall estimation are explored in the discussion in section 6, followed by a brief summary of the important conclusions in section 7.

2. Background
   a. Polarimetric variables

Following the nationwide upgrade of the WSR-88D radar network, meteorologists will have for their use a full slate of polarimetric variables in addition to the conventional radar reflectivity factor at horizontal polarization (Z_H), Doppler velocity (υ_D), and spectral width (σ_v). These additional variables are the differential reflectivity (Z_{DR}), differential phase (Φ_{DP}) and specific differential phase (K_{DP}), and the copolar cross-correlation coefficient at zero lag (ρ_{HV}). Here, we will briefly describe

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the meaning of these measurands, but the reader is directed to more thorough descriptions in the literature (e.g., Herzegh and Jameson 1992; Doviak and Zrnić 1993; Zrnić and Ryzhkov 1999; Straka et al. 2000; Ryzhkov et al. 2005b).

Seliga and Bringi (1976) first proposed the use of the ratio of backscattered power at orthogonal polarizations for observations of rainfall. The so-called differential reflectivity $Z_{DR}$ is the difference between reflectivity factors at horizontal and vertical polarizations. Differential reflectivity $Z_{DR}$ is dependent on particle size, shape, orientation, and dielectric constant but is independent of concentration. Since raindrops become increasingly oblate with increasing diameter (e.g., Pruppacher and Beard 1970; Pruppacher and Pitter 1971), $Z_{DR}$ is considered a good measure of the median drop size of a distribution. For spherical or randomly oriented hydrometeors, $Z_{DR}$ is 0 dB.

The use of a differential propagation phase $\Phi_{DP}$ for rainfall measurements was introduced by Sachidananda and Zrnić (1986, 1987). Due to the oblateness of raindrops, the horizontally polarized wave lags progressively behind the vertically polarized wave in rain, resulting in a positive differential phase shift. The radar measures the cumulative phase shift along each radial. Because $\Phi_{DP}$ is immune to radar calibration, attenuation, and beam blockage (Zrnić and Ryzhkov 1999), and is not biased by noise, it is an attractive variable for quantitative precipitation estimation and attenuation correction.

A more convenient way of interpreting the path-integrated differential phase shift is its range derivative, the specific differential phase $K_{DP}$, which provides the phase shift per unit distance in the radial direction. Here, $K_{DP}$ increases as the oblateness and dielectric constant of hydrometeors increase and is not affected by spherical or isotropic hydrometeors such as tumbling dry hail (Balakrishnan and Zrnić 1990). Sachidananda and Zrnić (1987) found that $K_{DP}$ is almost linearly related to rainfall rate. In addition, $K_{DP}$ is less sensitive to the particle diameter than radar reflectivity factor ($Z_H \sim D^6$ whereas $K_{DP} \sim D^{1/2}$); thus, $K_{DP}$ is more sensitive to the smaller drop sizes than $Z_H$ and $Z_{DR}$, which are dominated by larger particles within the radar sampling volume.

The correlation coefficient between the horizontally and vertically polarized signals at zero lag $\rho_{HV}$ is useful for discriminating between meteorological and nonmeteorological echoes such as biological scatterers (Zrnić and Ryzhkov 1999), chaff (Zrnić and Ryzhkov 2004), tornadic debris (Ryzhkov et al. 2005c), and smoke plumes (Melnikov et al. 2008). In pure rain at S band, $\rho_{HV}$ generally does not fall below 0.98. Further decreases in the measured $\rho_{HV}$ are due to the diversity of the particle sizes, orientations, shapes, and irregularities, and to the phase compositions within the radar sampling volume. For example, observations of rain mixed with melting hail will produce lower $\rho_{HV}$ than pure rain or pure dry hail. Hydrometeors of the size at which resonance scattering effects occur will also contribute to substantially lower values of $\rho_{HV}$. Resonance scattering, or Mie scattering, occurs when the combined microwave radiation reflected from the near side of the particle and the radiation that penetrates the particle is reflected from its back side interfere in such a way that the resulting backscattered wave is augmented or diminished.

### b. Evaporation

Liquid drops in clouds can grow by the diffusion of water vapor from the ambient environment if the ambient vapor density is greater than the vapor density at the droplet’s surface. On the other hand, if the vapor density at the surface of the drop exceeds the vapor density in the ambient environment, vapor is diffused away from the drop (evaporation). The rate of mass diffusion from a falling drop can be written as

$$\frac{dm}{dt} = 4\pi r_f \rho_v (\rho_v - \rho_{yr}).$$

where $r$ is the radius of the drop, $f_v$ is the ventilation coefficient for the vapor diffusivity, $D_v$ is the molecular diffusion coefficient, and $\rho_v$ and $\rho_{yr}$ are the vapor densities of the ambient environment and at the surface of the drop, respectively. Following Pruppacher and Klett (1978) and Rogers and Yau (1989), one can derive an approximate expression describing the rate of change of the drop radius:

$$\frac{dr}{dt} = \frac{S - 1}{F_K + F_D},$$

where $S$ is the saturation ratio, and $F_K$ and $F_D$ are terms related to heat conduction and vapor diffusion, respectively, that include ventilation effects and are detailed in the appendix. For now, we simply focus on a qualitative interpretation of (2). For subsaturated environments (where $S < 1$), the drop radius will decrease, indicating decay through evaporation. Also note that $dr/dt$ is inversely proportional to the drop radius, meaning that the radii of smaller drops will decrease more rapidly under evaporation than larger drops. This well-known fact is important when considering the impacts of evaporation on polarimetric variables.

Theoretically, the preferential depletion of smaller drops will result in a decrease in the observed $Z_H$ and $K_{DP}$ with an increase in the observed $Z_{DR}$. The observed
decrease in $ZH$ has been well documented and is intuitive. Decreasing drop diameters across the spectrum and a decrease in the concentration of smaller drops (those that are totally evaporated) will result in a decreased magnitude of the backscattered signal. Recall that $K_{DP}$ is less sensitive than $ZH$ to large drops, and as a corollary, more sensitive to changes in the lower end of the drop size spectrum. Therefore, one can expect evaporation to affect $K_{DP}$ more substantially than $ZH$.

The expected increase in $Z_{DR}$ is less intuitive because all drops are losing mass (size). However, since $Z_{DR}$ is a measure of the median drop size in a distribution, a preferential depletion of smaller drops (which generally have a large concentration) causes an increase in the median drop size of a given DSD. This effect is shown schematically in Fig. 1. At S band, the change in $\rho_{HV}$ due to evaporation in pure rain is not expected to be significant for reasons discussed in a later section. The magnitude of the changes in all polarimetric variables should be dependent on the relative contributions of small drops and large drops and thus is strongly dependent on the DSD.

### 3. Evaporation model

#### a. General description

In an effort to quantify the impacts of evaporation on the polarimetric variables under different conditions and assumptions, a simple numerical model is constructed. The idealized one-dimensional model explicitly computes the changes in size of raindrops falling through subsaturated air. The model domain is the subcloud layer, with 100-m vertical resolution. Eighty initial drop sizes are considered, ranging from 0.05 to 7.95 mm in 0.1-mm increments. Each drop size “bin” is tracked independently in order to isolate the effects of evaporation. Hence, no drop interactions such as collisions, coalescence, or breakup are taken into account. Note that coalescence and breakup significantly contribute to the evolution of the drop size spectrum, as found in the numerous theoretical and modeling studies mentioned in the introduction. These collisional processes become increasingly important in heavier rainfall. On the other hand, evaporation tends to only change the slope of the DSD slowly, instead mainly affecting the total water content (e.g., Srivastava 1978; HS). For rainfall estimation, evaporation is important since it is the only subcloud process that directly affects the total mass of rainwater reaching the ground. The relative contributions to the depletion of total water content from combinations of coalescence, breakup, and evaporation have been investigated previously (e.g., HS; Seifert 2008). HS found that the effects of coalescence and breakup tend to approximately balance. In their model, the total rainwater mass depletion in simulations that employed full microphysics (coalescence, breakup, and evaporation) was similar to that in which only evaporation was considered. Thus, only including evaporation in this model, although inherently limiting its applicability toward simulating the evolution of the DSD, is justifiable to improve the computational efficiency given our focus on the radar measurements and associated rainfall estimation.

At the top of the domain, or in the “cloud,” any DSD model can be prescribed. In the subcloud domain, any vertical profile of temperature and relative humidity can be administered. In our model, the feedback on the environmental thermodynamic profiles due to evaporation may be turned on or off. The feedback adjustment scheme is described below. Since the model is one-dimensional, no size sorting due to vertical shear (e.g., Kumjian and Ryzhkov 2009) is considered. At the initial time, drops begin to fall into the top of the domain, as in many rainshaft models. After numerous tests, the time step was selected to be 0.25 s, maximizing the computational efficiency while maintaining stable solutions.

#### b. Equations

We are interested in the vertical profiles of the polarimetric variables, so we convert (2) into an expression for the change in drop radius with height by dividing (2) by the fall speed of raindrops as a function of size. To simplify the ensuing integration, the empirical power law fall speed relation suggested by Atlas and Ulbrich (1977) is used for the terminal velocity of the raindrops:

![Conceptual schematic illustrating how evaporation can cause an increase in the median drop size $D_m$ of a distribution. The solid black line indicates the DSD before evaporation occurs; the dashed gray line represents the modified DSD due to evaporation.](http://journals.ametsoc.org/doi/pdf/10.1175/2010JAMC2243.1)
\[ v_r(D) = \alpha D^\beta \left( \frac{\rho_a}{\rho_0} \right)^{0.4}, \]  

where \( \alpha = 3.78 \text{ m s}^{-1} \text{ mm}^{-0.67} \), \( \beta = 0.67 \), and the equivalent spherical diameter \( D \) is given in millimeters. The multiplicative factor \( (\rho_0/\rho_a)^{0.4} \) is a density correction, where \( \rho_0 \) is the surface reference density following Foote and duToit (1969). Though (3) is less accurate for large drops than more recent empirical models (e.g., Brandes et al. 2002), it well represents the fall speeds of smaller drops (<5 mm) for which the evaporation effects are most significant. Thus, performing the change of variables in (2) and using the velocity relation (3) yields an analytic expression for the change in diameter of a raindrop with initial size \( D_0 \) as a function of height:

\[ D(h) = \left[ \frac{4(\beta + 2)}{\alpha} \xi(h') \left( \frac{\rho_a}{\rho_0} \right)^{0.4} \Delta h \right]^{1/(\beta + 2)}, \]

where we have defined \( \xi \) as the right-hand side of (2). To numerically integrate this expression, \( \xi \) and \( \rho_a \) are assumed to be constant over each height step, \( \Delta h = h_i - h_{i+1} \) (where \( h_{i+1} < h_i \)), which should be relatively small (100 m is used in this study), resulting in

\[ D(h_{i+1}) = \left[ \frac{4(\beta + 2)}{\alpha} \xi(h_i) \left( \frac{\rho_a}{\rho_0} \right)^{0.4} \Delta h \right]^{1/(\beta + 2)}. \]  

(5)

In this way, the simple expression (5) is used to calculate what a given initial drop size should be at any height beneath the cloud base for given thermodynamic profiles, assuming the drops are falling at terminal velocity.

The mass concentration of raindrops \( n(m) \) in many of the aforementioned modeling studies is governed by the following equation:

\[ \frac{\partial}{\partial t}[n(m)] + \frac{\partial}{\partial z}[v_r(n(m))] + \frac{\partial}{\partial m}[\rho_v n(m)] = B + C, \]  

where the second term on the left-hand side is the change in drop concentration due to differential sedimentation. The third term describes the change in concentration of drops of mass \( m \) due to growth or decay by vapor diffusion, \( \dot{m} \) (i.e., \( \frac{dm}{dt} \), which is the time rate of change of the drop mass due to condensation or evaporation). On the right-hand side, the terms \( B \) and \( C \) account for the collisional processes of breakup and coalescence, respectively; these will be ignored in this study. For the calculation of radar variables, it is more convenient to work in terms of diameter \( D \). Thus, we seek an expression of (6) in terms of \( N(D) \), where \( N(D)dD \) is the number of raindrops of size \( D \) to \( D + dD \). The concentrations \( n(m) \) and \( N(D) \) are related by

\[ n(m) = N(D) \frac{dD}{dm}, \]  

Assuming spherical symmetry of the raindrops, \( m = (\pi/6)\rho_lD^3 \), and thus

\[ \frac{dD}{dm} = \frac{2}{\pi \rho_l D^2}, \]  

where \( \rho_l \) is the density of liquid water. When computing the polarimetric radar variables, drop shape is important and must be dealt with appropriately. However, assuming a spherical shape of drops does not significantly impact the computation of evaporation (Straka and Gilmore 2006). Thus, the drop shape inconsistency will not affect the results of this study. The mass diffusion term must be written in terms of \( D \):

\[ \dot{m} = \frac{d}{dt}\left( \frac{\pi}{6} \rho_l D^3 \right) = \frac{\pi}{2} \rho_l D^2 \dot{D}. \]

Thus, substituting (7)–(9) into (6), and making use of the change of variables,

\[ \frac{\partial}{\partial m} \frac{\partial}{\partial m} \]  

yields the governing equation in terms of drop diameter \( D \):

\[ \frac{\partial}{\partial t} [N(D)D^{-2}] = -\frac{\partial}{\partial z} [v_r(N(D)D^{-2})] - \frac{1}{D^2} \frac{\partial}{\partial D} \left[ \frac{\partial N(D)}{\partial D} \right]. \]  

(10)

For the most general case in a 1D rainshaft model, the drop concentration is a function of diameter, height, and time, that is, \( N = N(D, z, t) \). For brevity, the functional dependence of \( N \) will herein be omitted. Thus, expanding the derivatives in (10) results in

\[ \frac{\partial N}{\partial t} = \frac{2N}{D} \frac{\partial D}{\partial z} - \frac{\partial}{\partial z} [v_r(N)D^{-2}] + \frac{2N v_r \partial D}{D} - \frac{\partial}{\partial D} [\dot{N}(D)]. \]

(11)

For drops falling at terminal velocity \( \dot{v}_r \),

\[ \frac{\partial D}{\partial z} = \frac{d}{dt} \frac{\partial}{\partial z} = \dot{D}. \]

So, (11) becomes

\[ \frac{\partial N}{\partial t} = \frac{\partial}{\partial z} [v_r(N)D^{-2}] - \frac{\partial}{\partial D} [\dot{N}(D)] + \frac{4N}{D} \dot{D}, \]  

(12)
which governs the time rate of change of the raindrop concentration. Using (3) for terminal velocity and noting that \( \dot{D} = 4\xi D^{-1} \), we can write (12) in discrete form,

\[
N_{j,k}^{i+1} = N_{j,k}^i - \frac{\Delta t}{2\Delta z} \left[ \alpha_j D_{j+1,k}^i N_{j+1,k}^i \right] - \frac{\Delta t}{D_{j,k}^i \Delta D} \left( \xi_{j,k} N_{j,k+1}^i \right) - \xi_j N_{j,1}^i + \frac{2\Delta t}{(D_{j,k}^i)^2} \xi_{j,k} N_{j,k}^i,
\]

for each time step \( i \), height level \( j \), and drop size bin \( k \).

As raindrops begin to evaporate in subsaturated air, the thermodynamic properties of the air in the rainshaft are affected. Specifically, evaporation causes the shaft to gradually cool and moisten in time, thereby decreasing the subsaturations and limiting the amount of evaporation that occurs. The decrease in temperature and precipitation loading contributes to negative buoyancy relative to the surrounding precipitation-free air, which forces a localized downdraft. The downdraft also serves to limit the amount of mass evaporated at each level, and as-into two groups: sensitivity studies, which will focus on

The evaporation model calculations will be separated immediately cools the surrounding air), the incremental change in the temperature of the air is calculated at each time step. Similarly, the entire mass of liquid water lost due to evaporation goes directly into vapor, increasing the vapor density in the air. Using the new temperature and vapor density, and the ideal gas law for water vapor, the new relative humidity can be calculated.

This environmental feedback scheme may be turned on or off in the model. Since the model is one-dimensional and is not tied to any dynamics, we have omitted the generation of a downdraft. The velocity relation (3) overestimates the fall speeds of the moderate and large drops already, which may partially account for the errors introduced when neglecting a downdraft. Also, to be realistic and dynamically consistent, the generation of a downdraft would require some parameterization of entrainment, a process that still has some uncertainty. Without this entrainment, the 1D column gradually moistens and cools until it becomes saturated, halting the evaporation.

The changes in diameter of the drop size bins at each height level and the calculated concentration of drops at each level are used to compute the vertical profiles of polarimetric variables according to the T-matrix method (e.g., Mishchenko 2000). For raindrop shapes, we assume the corrected relation suggested by Brandes et al. (2002):

\[
r = 0.9951 + 0.0250D - 0.0364D^2 + 0.0053D^3 - 0.0002952D^4,
\]

where \( D \) is in millimeters. See Brandes et al. (2002) for a discussion of the differences between this relation and others that are used frequently in the literature. Note that this relation produces slightly more spherical drops, especially in the range 1–4 mm. The polarimetric variables (especially \( Z_{DR} \) and \( \rho_{HV} \)) are more strongly affected by the larger drops, the shapes of which are similar to other relations. Thus, we expect only slight differences if other drop shape relations are used instead of (15). The raindrops are assumed to have a distribution of canting angles with mean \( 0^\circ \) and standard deviation \( 20^\circ \) with respect to the vertical. In general, \( 20^\circ \) is too high for typical rainfall; a standard deviation of \( 10^\circ \) is more appropriate (e.g., Ryzhkov 2001; Ryzhkov et al. 2002). However, the higher value was chosen to increase the impact of this wider distribution of canting angles is to dampen the effects of evaporation on the other variables very slightly. The wavelength of the radar will be discussed using conventional, according to which “S band” refers to the 10.9-cm wavelength (as in the WSR-88D network) and “C band” refers to a wavelength of 5.3 cm.

The evaporation model calculations will be separated into two groups: sensitivity studies, which will focus on
the exploration of parameter space, and simulations, which will model the vertical profiles of polarimetric variables using observed thermodynamic soundings from different climate regions. The sensitivity studies are designed to test the model sensitivity as well as the sensitivity of the polarimetric variables to different thermodynamic conditions, drop size distribution models, and rainfall rates. The simulations are intended to illustrate potential variations in vertical profiles of polarimetric variables in different regions of the United States that may be observed following the nationwide weather radar upgrade to polarization diversity. For the experiments, the gradual moistening and cooling feedback mechanism has been turned off in order to create “snapshots” of the profiles of polarimetric variables. For applied simulations, the mechanism should be turned on to avoid overaggressive evaporation.

4. Sensitivity studies

a. Design

Because of the aforementioned differences in the drop size dependency for each of the polarimetric variables, sensitivity to selection of the DSD is expected. To address this, each model sensitivity experiment is repeated using different initial DSD models. The standard exponential and gamma models are used:

\[ N(D) = N_0 \exp(-\Lambda D) \]  
(16)

and

\[ N(D) = N_0 D^\mu \exp(-\Lambda D). \]  
(17)

To conduct a fair comparison between the different DSD models, the rainfall rate for each has been fixed at the top of the domain. A Marshall–Palmer DSD (herein MP) is used for the exponential model. Note that the intercept parameter most often used with the MP distribution, \(N_{00} = 8000 \text{ m}^{-3} \text{ mm}^{-1}\), is calculated for use at sea level; since we are using fixed rainfall rates aloft, we must use \(N_0 = N_{00}\rho(h_0)/\rho_0\)^0.4. For the MP distribution, with the velocity relation (3), the slope parameter \(\Lambda\) is calculated using

\[ \Lambda = \left[ \frac{\pi}{6} \alpha N_0 \Gamma(4 + \beta) \right]^{1/4.67} R^{-1/4.67}. \]  
(18)

The gamma model has three parameters, though these are usually constrained and thus not entirely independent (e.g., Zhang et al. 2001, 2006; Brandes et al. 2004; Cao et al. 2008). For Oklahoma precipitation, Cao et al. (2008) found an empirical relation between the shape parameter \(\mu\) and the slope parameter \(\Lambda\):

\[ \mu = -0.0201\Lambda^2 + 0.902\Lambda - 1.718, \]  
(19)

where the equation is applicable for \(0 \leq \Lambda \leq 20\). Constrained gamma models with \(\mu\) varying between \(-1\) and \(5\) are used in the calculations. These values encompass the bulk of the observations in Oklahoma rain (e.g., Cao et al. 2008). A slightly broader range of values of \(\mu\) was found in rain in Florida (Zhang et al. 2001), so there is some dependence on climate region. Since \(\Lambda\) is determined from the preselected \(\mu\) values using (19) and we have a fixed rainfall rate \(R\),

\[ R = \frac{\pi}{6} \int_0^\infty D^3 N(D) \nu_c(D) dD, \]  
(20)

the intercept parameter can be determined by solving (20) for \(N_0\) using (17) for \(N(D)\):

\[ N_0 = \frac{\pi}{6} \alpha \left[ \frac{\rho_0}{\rho_c(z)} \right]^{0.4} \frac{\Gamma(\mu + 4 + \beta)}{\Lambda^{\mu+4+\beta}}. \]  
(21)

where the height-density correction is included. Figure 2 displays each modeled DSD. It is evident that a variety of DSD shapes have been selected, covering a broad spectrum of precipitation regimes. Note that in the Oklahoma precipitation, the negative shape parameters are most often associated with convective precipitation events, so the relatively high concentration of larger drops makes sense. Also note the similarities between the MP DSD (corresponding to a shape parameter of 0) and the \(\Gamma, \mu = 1\) distribution, which nearly overlap for all drop sizes except at the small-drop end of the spectrum.

For each set of model runs the polarimetric variables are calculated for both S and C bands. In the first set of experiments, idealized isothermal layers 2 km deep are considered. The temperature is fixed at 20°C, and the relative humidity (RH) is set constant in height but varies from 10% to 95% in each experiment. Though not necessarily realistic, these experiments simply explore the parameter space of the model. In the second set of sensitivity runs, a dry-adiabatic lapse rate is prescribed with a surface temperature of 30°C. The RH profile is assumed to increase linearly from the given surface value to 100% at the cloud base (top of the domain, taken to be 3 km). The surface values are varied from 55% to 95% in 5% increments. These idealized well-mixed boundary layer cases apply to warm season precipitation events. Simulations using observed thermodynamic profiles, including both warm and cool season events, are explored in section 5. In these two sets of sensitivity tests, each of the DSD models is used. The final set of sensitivity tests
varies $R$ from 0.1 to 20 mm h$^{-1}$ using the MP DSD for a dry-adiabatic environment with a surface RH of 75\% and a surface temperature of 30$^\circ$C.

\textit{b. Transient differential sedimentation}

At the onset of precipitation, a transient effect known as differential sedimentation occurs, where the largest raindrops fall to the ground before the smaller raindrops. The reason for this is simply the difference in the terminal velocity of the large drops versus the smaller drops; the large drops fall faster and thus reach the ground before the smaller drops. Once enough time has elapsed, all drops that do not evaporate reach the ground, and the visible effects of differential sedimentation disappear.

Recall that size sorting is an extremely important effect as observed by polarimetric radars (e.g., see a review by Kumjian and Ryzhkov 2008). With the largest, most oblate drops initially falling farther from the cloud than smaller drops, the polarimetric variables (especially $Z_{DR}$) are significantly enhanced beneath the cloud, with $Z_{DR}$ increasing toward the ground. At S band, $\rho_{HV}$ tends to change very little, only slightly decreasing toward the ground. This is because the resonance effects are still minimal even for the largest drops at S band. At C band, however, the large drops are within the band of sizes where resonance effects are quite strong. Thus, in addition to increased $Z_{DR}$ toward the ground, C-band measurements will show a significant decrease in $\rho_{HV}$. Figure 3 shows an example of the impacts of differential sedimentation on the polarimetric variables. Note that this effect dominates any signal from evaporation, which produces more subtle changes in the vertical profiles of the polarimetric variables. Thus, for the remainder of the sensitivity studies and experiments where we quantify the effects of evaporation, the calculations are carried out for long times (equivalently, waiting for the profiles to attain steady-state solutions where the environmental feedback mechanism has been turned off).

\textit{c. Results of sensitivity tests}

It is convenient to define the “evaporative change” in the polarimetric variables over the depth of the model
domain, which is simply the value of the polarimetric variable at the ground minus its initial value aloft. The evaporative change will be denoted as a $\Delta$ before the variable. As discussed above, evaporation will produce negative $\Delta Z_H$ and $\Delta K_{DP}$ (indicative of a decrease) and a positive $\Delta Z_{DR}$ (indicative of an increase). To make a more meaningful comparison between $\Delta Z_H$ and $\Delta K_{DP}$, the evaporative change in $K_{DP}$ will be converted into logarithmic units:

In all cases, the changes in the polarimetric variables due to evaporation are not constant in height and depend on the environmental thermodynamic profiles as expressed by the variable $\xi(z)$.

FIG. 3. Example simulation in a well-mixed boundary layer of 3-km depth, with surface temperature of 30°C and surface relative humidity of 70%. The simulation uses the MP DSD aloft with a rainfall rate of 5 mm h$^{-1}$. The left column is after 60 s, showing the transient effect of differential sedimentation in vertical profiles of each of the polarimetric variables. The right column shows the steady-state profiles for the same environment. Solid lines are for S band; dashed lines indicate C-band values.
Fig. 4. Sensitivity of the evaporative change in the S-band dual-polarization variables to various constant RH profiles for the five DSD models. The domain depth is 2 km and is isothermal at 20°C. The plotting convention for the DSD models is the same as in Fig. 2. Shown are evaporative changes in (a) $Z_H$, (b) $Z_{DR}$, (c) $K_{DP}$, and (d) $\rho_{HV}$.

\[ \Delta K_{DP} (\text{dB}) = 10 \log_{10} \left( \frac{(K_{DP})_{\text{ground}}}{(K_{DP})_{\text{aloft}}} \right). \]  

(22)

Recall that the logarithmic differences for $Z_H$, $Z_{DR}$, and $K_{DP}$ can be converted into relative changes (e.g., a 3-dB decrease corresponds to about a 50% reduction from its original value).

The results from the first set of experiments (considering isothermal layers) are summarized in Figs. 4 and 5 for S and C bands, respectively. It is clear that all of the variables are sensitive to the relative humidity in the layer, but perhaps more importantly the results are sensitive to the initial DSD model selected. The $\Gamma, \mu = 5$ model exhibits much greater evaporative changes in $Z_H$ and $K_{DP}$ than do the other models (Figs. 4a,c and 5a,c). This is explained by two factors. First, the distribution contains a large concentration of small drops, which are preferentially evaporated, resulting in a substantial decrease in mass. Second, the $\Gamma, \mu = 5$ has fewer large drops than any other distribution. These large drops, which do not evaporate as efficiently as smaller drops, tend to overwhelm the contribution to the observed $Z_H$ (and, to a lesser extent, $K_{DP}$) for the other DSD models, which all exhibit lower magnitudes of $\Delta Z_H$ and $\Delta K_{DP}$ (dB).

Since $Z_{DR}$ is more sensitive to drop size than $Z_H$, it follows that the large $\Delta Z_{DR}$ values occur for the DSDs with relatively large concentrations of big drops. However, also playing an important role in producing the significant $\Delta Z_{DR}$ values is a large concentration of small drops. The preferential evaporation of a significant portion of the spectrum will substantially increase the median drop size of the spectrum. This is why the MP model has the highest $\Delta Z_{DR}$ (Figs. 4b and 5b): it has the highest concentration of small drops and a comparatively large concentration of big drops. Also note that the $\Delta Z_{DR}$ at C band (Fig. 5b) is somewhat higher for the DSD models due to resonance scattering effects.

At S band, the $\Delta \rho_{HV}$ magnitudes are quite small (<0.002 even for extreme evaporation) for all DSD shapes, at least for the electromagnetic scattering model employed in this study (Fig. 4d). Such small changes are insignificant and likely within the uncertainty of the WSR-88D measurements, or about ±0.005–0.01. At C band, the evaporative changes in $\rho_{HV}$ are slightly larger in magnitude and negative for all models (Fig. 5d). For modest evaporation rates, changes in $\rho_{HV}$ at both radar wavelengths are insignificant and probably difficult to detect operationally.
Next, idealized 3-km-deep mixed layers are considered for the calculations. The results for S and C bands are summarized in Fig. 6. The resulting $\Delta Z_H$ and $\Delta K_{DP}$ are dependent on the surface RH and the DSD prescribed aloft, as expected. However, a feature of note is that $\Delta Z_H$ and $\Delta K_{DP}$ reverse sign for three of the DSD models (MP, $\Gamma, \mu = -1$, and $\Gamma, \mu = 1$) for high values of surface RH. This is because the relatively cool and moist conditions beneath the cloud base do cause enough evaporation to counteract the effects of “raindrop convergence,” a result of the drops encountering increasing air density with fall distance. In the absence of any evaporation, the concentration of drops increases slightly with decreasing height, which affects $Z_H$ and $K_{DP}$, but not $Z_{DR}$ or $\rho_{HV}$. Also of note is that the difference between the S- and C-band values of $\Delta Z_{DR}$ and $\Delta \rho_{HV}$ increases with higher concentrations of large drops, a result of the resonance scattering effects prevalent in big drops at C band.

The results of varying the rainfall rate $R$ are provided in Fig. 7. For the higher rainfall rates considered, one should expect a greater change in $K_{DP}$ for a given amount of evaporation than is observed in $Z_H$. At both S and C bands, $\Delta \rho_{HV}$ is insignificant for all rainfall rates with the MP model. For the S-band calculations, $\Delta Z_{DR}$ decreases with increasing rainfall rate whereas the $\Delta Z_{DR}$ at C band increases slightly. The difference in behavior may be attributable to the enhanced resonance scattering effects at C band.

The sensitivity experiments have shown, for given thermodynamic conditions, that the largest $\Delta Z_H$ and $\Delta K_{DP}$ occur for the $\Gamma, \mu = 5$ DSD model, with the $\Gamma, \mu = -1$ model having the smallest evaporative changes. Again, these results are directly related to the shape of the DSD: the smaller the concentration of large (>4 mm) drops, the greater the impacts of evaporation will be on $Z_H$ and $K_{DP}$. The logarithmic changes in $K_{DP}$ are slightly greater in magnitude than those of $Z_H$ since $K_{DP}$ is less dependent on drop diameter than $Z_H$; the preferential depletion of smaller drops has a greater relative impact on $K_{DP}$. Note that since $Z_H$ typically is displayed in logarithmic units (dBZ), whereas $K_{DP}$ generally is shown in degrees per kilometer, $\Delta K_{DP}$ may be more obvious to operational meteorologists. For example, an equivalent evaporative change of 3 dB for $Z_H$ (e.g., 33–30 dBZ) “appears” to be less significant than a 3-dB relative change in $K_{DP}$ (e.g., 1.0°–0.5° km⁻¹). However, it should be noted that for the relatively light rainfall rates considered in this study and at S band, $K_{DP}$ can be noisy and difficult to estimate. At shorter radar wavelengths, the $K_{DP}$ estimates will be less noisy as the $K_{DP}$ values will be larger (since $K_{DP}$ is inversely proportional to radar wavelength), possibly making such evaporative changes
easier to detect operationally with C- and X-band polarimetric radars.

For given conditions, the largest values of $\Delta Z_{DR}$ occur for the MP model, with $\Gamma, \mu = -1$ having the smallest values. The changes depend on the relative contributions from both the small- and large-drop ends of the spectrum. While having the largest concentration of big drops of any of the models used in this study, the $\Gamma, \mu = -1$ model resulted in the smallest $\Delta Z_{DR}$. This is because of the comparatively low concentration of small drops ($<2$ mm). With fewer small drops being depleted, there is a lesser shift in the median drop size of the spectrum. Coupled with this is the fact that the substantial concentrations of big drops, which are not as affected by evaporation, dominate the contribution to the backscattered signal.

5. Simulations with observed soundings

a. Design

Once the model sensitivity is explored, it is informative to look at real cases in an attempt to see the types of variations in vertical profiles of the polarimetric variables that may be observed in different evaporation scenarios. To accomplish this, we have selected three soundings from different regions that display different thermodynamic characteristics. These observed temperature and relative humidity profiles are used to initialize the model.

The three soundings are shown in Fig. 8. The first sounding comes from Albuquerque, New Mexico, at 0000 UTC 16 August 2007 (Fig. 8a) and represents the summer season in the western United States with a deep, dry well-mixed layer. Light precipitation echoes were observed by the Albuquerque WSR-88D about the time of the sounding. Next is a spring sounding (Fig. 8b), taken from Norman, Oklahoma, at 1800 UTC 24 May 2008 and may be said to represent an atmosphere primed for severe weather: a cyclic tornadic supercell developed north of Oklahoma City, Oklahoma, in the afternoon, producing at least 10 tornadoes. The third sounding (Fig. 8c) comes from Wallops Island, Virginia, at 0000 UTC 7 October 2009 and represents a cool,
moist boundary layer in which light rain was observed throughout the day. Note that the environmental feedback mechanism has been turned off for these simulations.

b. Results of simulations

The MP distribution with a 5 mm h$^{-1}$ rainfall rate aloft is used in the simulations. The MP model is employed because it is perhaps the most widely known DSD model and provides intermediate magnitudes of evaporative changes. As previously demonstrated, the choice of DSD model can have a significant impact on the evaporative changes of the polarimetric variables. This should be considered when interpreting the outcome of the simulations and the computed vertical profiles of the polarimetric variables presented herein. Nonetheless, the purpose of these simulations is to illustrate the potential variations in vertical profiles of polarimetric observables that may occur in different climate regions and seasons throughout the United States.

Figure 9 displays the vertical profiles of polarimetric variables for the Albuquerque sounding (Fig. 8a). For both radar wavelengths, $\Delta Z_H$ is less than $-6$ dB. The $\Delta Z_{DR}$ is nearly 0.3 dB at S band, which may be detectable by the WSR-88D, and greater than 0.3 dB at C band. The magnitude of the change in $\rho_{HV}$ is negligible at S band. At C band, the $\Delta \rho_{HV}$ is about $-0.003$, which may be observable with very high quality radar systems. The evaporative changes in all polarimetric variables for each of the simulated soundings are summarized in Table 1.

Because of the uncertainty involved with the choice of the initial DSD, the same simulations are performed with the $\Gamma, \mu = -1$ distribution as well as the $\Gamma, \mu = 5$ model. These models were selected for comparison since they encompass the extremes for evaporative changes in the different polarimetric variables (Table 2). The $\Gamma, \mu = -1$ model exhibits much smaller magnitudes of $\Delta Z_H$ and $\Delta K_{DP}$ than do the MP simulations. Also note that $\Delta \rho_{HV}$ changes sign with the $\Gamma, \mu = -1$ model depending on the radar wavelength. In contrast, the $\Gamma, \mu = 5$ model produces the most significant $\Delta Z_H$ and $\Delta K_{DP}$ values and very small changes in $Z_{DR}$ and $\rho_{HV}$. These simulations can be viewed as the approximate quantitative bounds on the possible changes in polarimetric characteristics of light rain due to evaporation in differing environments.

6. Rainfall-rate estimation

In addition to calculating vertical profiles of the polarimetric variables, we calculate the vertical profiles of the rainfall rate using the true value based on the DSD itself [Eq. (20)] as well as radar relations at S band based on DSD measurements from the Joint Polarization Experiment (JPOLE; Ryzhkov et al. 2005b; also see Giangrande and Ryzhkov 2008),

$$R(Z_H) = 0.017(Z_H^{0.714})$$

(23)
and DSD simulations by Bringi and Chandrasekar (2001),

$$R(K_{DP}) = 44.0|K_{DP}|^{0.822} \text{sgn}(K_{DP}),$$  \hspace{1cm} (24)

where $R$ is given in millimeters per hour and the lowercase subscripts for $Z_h$ and $Z_{dr}$ indicate values in linear units. Understanding how radar-estimated rainfall rates may change in height due to evaporation is important for QPE, especially in regions far from the radar or in regions of complex terrain where low levels may not be adequately sampled (e.g., Maddox et al. 2002). Computing rainfall rates based on accepted algorithms [(23)–(25)] for observed soundings provides an illustration of the types of variations possible across the country. Additionally, comparing these estimated rainfall rates with the actual rainfall rate demonstrates the performance of each of the algorithms in situations where the DSD is changing with height through evaporation.

Vertical profiles of the rainfall rate are plotted in Fig. 10. An MP distribution is assumed at the top of the domain with a rainfall rate of 5 mm h$^{-1}$. In all cases, the true rainfall rate changes more rapidly than do any of the estimates. Note that the initial large decrease in $R$ at Wallops Island is due to the dry layer near 700 mb (see Fig. 8c). At the top of the domain, the $R(Z_h, Z_{DR})$ estimate is closest to the actual $R$, but as the raindrops evaporate, the $R(K_{DP})$ better matches the true $R$. It should be noted, however, that actual measurements of $K_{DP}$ can be noisy in light rain, so $R(K_{DP})$ is not necessarily the best estimate for operational use in the case of evaporation and light rain. The evaporative changes at the ground using different rainfall rate estimates are summarized in Table 3. Overall, the $R(Z_h, Z_{DR})$ relation best captures the change in rainfall rate with height (i.e., it is least sensitive to the changes in the DSD at low rainfall rates, which was frequently claimed as an advantage of the polarimetric radar estimates), though all relations underestimate the change as calculated from
the actual DSD. It is clear that substantial decreases in rainfall rate are possible in dry environments, and even in the moist Wallops Island environment the true rainfall rate decreased by almost 3 mm h\(^{-1}\). If the rainfall rate at low levels is not adequately estimated based on radar measurements, this error can accumulate with time.

There are numerous \(R(Z_H, Z_{DR})\) relations suggested in the literature (see Ryzhkov et al. 2005a for a review). For comparison, the calculations are reproduced comparing the \(R(Z_H, Z_{DR})\) relation from (25) to the one recommended by Giangrande and Ryzhkov (2008),

\[
R(Z_H, Z_{DR}) = 0.0142(Z_h^{0.770})(Z_{dr}^{-1.67}),
\]

and the relation of Brandes et al. (2002),

\[
R(Z_H, Z_{DR}) = 0.00746(Z_h^{0.945})(Z_{dr}^{-4.76}).
\]

The relations in (25) and (27) produced similar results and outperform the relation in (26) in gauging the decrease in rainfall rate due to evaporation. This is likely due to the stronger dependence of \(R\) on \(Z_{DR}\) in (25) and (27): the subtle changes in shape of the DSD are not captured by \(Z_h\), whereas \(Z_{DR}\) does provide some information about the shape of the DSD.

Reproducing the calculations with the \(\Gamma, \mu = -1\) model, the relative errors (between the estimates of evaporative change in the rainfall rate and the true evaporative change in the rainfall rate) depend on the sounding but are all within 2 mm h\(^{-1}\) of the true \(\Delta R\). In contrast, these errors are minimized with the \(R(Z_H, Z_{DR})\) relation for the \(\Gamma, \mu = 5\) and MP models. The difficulty for operational forecasters is in determining the initial DSD aloft. There are rigorous methods for DSD retrieval based on polarimetric radar measurements, such as the procedures described in Zhang et al. (2001, 2006)

![Fig. 9. Simulated vertical profiles of polarimetric variables for the Albuquerque sounding (Fig. 8a). The solid line indicates S band and the dashed line indicates C band. Variables shown are (top) \(Z_H\), (top middle) \(Z_{DR}\), (bottom middle) \(K_{DP}\), and (bottom) \(\rho_{HV}\). The MP DSD model is used at the cloud base, with a rainfall rate of 5 mm h\(^{-1}\) aloft.](image)

TABLE 1. Results of simulated evaporative changes in the polarimetric variables at S and C bands from the three soundings in Fig. 8. Simulations used the MP distribution with a rainfall rate aloft of 5 mm h\(^{-1}\).

<table>
<thead>
<tr>
<th></th>
<th>Albuquerque</th>
<th>Norman</th>
<th>Wallops Island</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta Z_H)</td>
<td>-6.23</td>
<td>-2.29</td>
<td>-1.25</td>
</tr>
<tr>
<td>(\Delta Z_H)</td>
<td>-6.32</td>
<td>-2.33</td>
<td>-1.28</td>
</tr>
<tr>
<td>(\Delta Z_{DR})</td>
<td>0.27</td>
<td>0.13</td>
<td>0.09</td>
</tr>
<tr>
<td>(\Delta Z_{DR})</td>
<td>0.32</td>
<td>0.14</td>
<td>0.09</td>
</tr>
<tr>
<td>(\Delta K_{DP})</td>
<td>-6.88</td>
<td>-2.59</td>
<td>-1.45</td>
</tr>
<tr>
<td>(\Delta K_{DP})</td>
<td>-6.80</td>
<td>-2.56</td>
<td>-1.43</td>
</tr>
<tr>
<td>(\Delta \rho_{HV})</td>
<td>-0.0006</td>
<td>-0.0003</td>
<td>-0.0002</td>
</tr>
<tr>
<td>(\Delta \rho_{HV})</td>
<td>-0.0031</td>
<td>-0.0011</td>
<td>-0.0007</td>
</tr>
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</table>
and Brandes et al. (2002, 2004). These methods mainly rely on $Z_H$ and $Z_{DR}$ measurements.

Using the idealized well-mixed layer environments, and initializing the model with a variety of constrained gamma DSD models with varying rainfall rates (0.1–20 mm h$^{-1}$) and shape parameters ($-1 \leq \mu \leq 5$), the relation between the initial $Z_H$ and $Z_{DR}$ aloft and the evaporative change in rainfall rate over the domain (in this case, 3 km) is explored. With the constrained gamma DSD models, the relative change in rainfall rate due to evaporation ($\Delta R/R$) is independent of the initial $Z_H$ aloft. Thus, $\Delta R/R$ is a function of the environment (including temperature, RH, and depth) and initial $Z_{DR}$ aloft. Observations of $Z_H$ and $Z_{DR}$ at cloud base can be used to

Table 2. As in Table 1 but using the $\Gamma, \mu = -1$ and $\Gamma, \mu = 5$ distributions, respectively, with the values resulting from each DSD model separated by a comma in the table. Magnitudes of $\Delta \rho_{HV}$ less than $10^{-4}$ are indicated as $-0$.

<table>
<thead>
<tr>
<th></th>
<th>Albuquerque</th>
<th>Norman</th>
<th>Wallops Island</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta Z_H$—S band (dB)</td>
<td>$-1.22, -21.0$</td>
<td>$-0.12, -8.06$</td>
<td>$+0.16, -4.86$</td>
</tr>
<tr>
<td>$\Delta Z_H$—C band (dB)</td>
<td>$-1.32, -21.0$</td>
<td>$-0.13, -8.07$</td>
<td>$+0.15, -4.86$</td>
</tr>
<tr>
<td>$\Delta Z_{DR}$—S band (dB)</td>
<td>$0.00, 0.08$</td>
<td>$0.02, 0.03$</td>
<td>$0.01, 0.02$</td>
</tr>
<tr>
<td>$\Delta Z_{DR}$—C band (dB)</td>
<td>$0.21, 0.08$</td>
<td>$0.07, 0.03$</td>
<td>$0.05, 0.02$</td>
</tr>
<tr>
<td>$\Delta K_{DP}$—S band (dB)</td>
<td>$-1.32, -21.2$</td>
<td>$-0.18, -8.14$</td>
<td>$+0.12, -4.91$</td>
</tr>
<tr>
<td>$\Delta K_{DP}$—C band (dB)</td>
<td>$-1.44, -21.2$</td>
<td>$-0.24, -8.14$</td>
<td>$+0.08, -4.91$</td>
</tr>
<tr>
<td>$\Delta \rho_{HV}$—S band</td>
<td>$+0.0016, -0.0001$</td>
<td>$+0.0006, -0.0001$</td>
<td>$+0.0004, -0.0001$</td>
</tr>
<tr>
<td>$\Delta \rho_{HV}$—C band</td>
<td>$-0.0030, -0.0001$</td>
<td>$-0.0006, -0.0001$</td>
<td>$-0.0004, -0.0001$</td>
</tr>
</tbody>
</table>

Fig. 10. Vertical profiles of rainfall rates for the three soundings. The thick solid line is the actual $R$ determined from the DSD at each level. The dash–dot line is $R(Z_H)$ [(23)], the dashed line is $R(K_{DP})$ [(24)], and the dotted line is $R(Z_H, Z_{DR})$ [(25)]. At cloud base, the MP model is used for these simulations, with a rainfall rate of 5 mm h$^{-1}$ aloft.
identify the initial rainfall rate aloft. Obviously, these parameters may be a function of time in evolving storms, but for QPE one must assume a steady state for at least the duration of a radar volume scan. Next, the RH (or another moisture variable) and temperature profiles in the layer beneath the radar horizon can be determined from observations or numerical weather prediction model output [e.g., the Rapid Update Cycle (RUC); e.g., Benjamin et al. (1991), (1994), (2004)], assuming it is sufficiently accurate. In this manner, environmental conditions and the observed $Z_{DR}$ (which does not change much with height) can be utilized efficiently to estimate $\Delta R/R$ (Fig. 11). Such a process may aid in QPE, especially in regions far from the radar or in poor low-level radar coverage.

7. Summary of conclusions

In this paper we have investigated and quantified the impacts of evaporation on the polarimetric variables that will be available following the national upgrade of the WSR-88D radar network, which include $Z_H$, $Z_{DR}$, $K_{DP}$, and $\rho_{HV}$. Evaporation results in significant decreases in $Z_H$ and $K_{DP}$ and subtle increases in $Z_{DR}$, with no significant change in $\rho_{HV}$ at S band. At C band, changes in $K_{DP}$, $Z_{DR}$, and $\rho_{HV}$ are amplified for a given amount of evaporation. The change in the true rainfall rate due to evaporation is dependent on the initial shape of the DSD aloft, which illuminates the importance of polarimetric radar observations in reducing this uncertainty. To test the sensitivity of the polarimetric variables to the thermodynamic conditions in the subcloud environment, the initial DSD aloft, and the rainfall rate, a simple explicit bin microphysics model was constructed and numerous experiments were conducted. Because it neglects raindrop collisional processes, the model is limited to relatively light rainfall rates and should not be used to simulate DSD evolution. We summarize the conclusions reached from the model runs as follows:

1) As expected, warmer and/or drier environments produce more evaporation and thus more substantial changes in the polarimetric variables, as well as the rainfall rates inferred from these variables.

<table>
<thead>
<tr>
<th></th>
<th>Albuquerque</th>
<th>Norman</th>
<th>Wallops Island</th>
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<tbody>
<tr>
<td>$\Delta R(Z_H)$ (mm h$^{-1}$)</td>
<td>-3.05</td>
<td>-1.48</td>
<td>-0.87</td>
</tr>
<tr>
<td>$\Delta R(K_{DP})$ (mm h$^{-1}$)</td>
<td>-2.70</td>
<td>-1.42</td>
<td>-0.87</td>
</tr>
<tr>
<td>$\Delta R(Z_H, Z_{DR})$ (mm h$^{-1}$)</td>
<td>-3.82</td>
<td>-2.13</td>
<td>-1.34</td>
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<tr>
<td>$\Delta R$ (true) (mm h$^{-1}$)</td>
<td>-4.55</td>
<td>-3.46</td>
<td>-2.86</td>
</tr>
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</table>

**TABLE 3.** Evaporative change in the rainfall rate for each of the three soundings for the three rainfall algorithms $R(Z_H)$, $R(Z_H, Z_{DR})$, and $R(K_{DP})$ [(23)–(25), respectively] based on the MP DSD along with the “true” rainfall rate calculated from the actual DSD.

**FIG. 11.** The dependence of the relative change in rainfall rate $\Delta R/R$ on $Z_{DR}$ aloft and surface RH. Results are from calculations using a 3-km-deep idealized well-mixed layer with a surface temperature of 30°C. Each curve corresponds to a different value of the surface RH (solid black line for 55%, solid gray line for 75%, and dashed gray line for 95%). Only constrained gamma models (with 5 mm h$^{-1}$ rainfall rate) are used in the calculations, where $\Delta R/R$ is independent of $Z_H$ aloft. The gray numbers along the abscissa correspond to the shape parameter ($\mu$) values associated with the given $Z_{DR}$. (left) The S-band values, and (right) the C-band values.
The resulting vertical profiles of polarimetric variables from the simplistic model are sensitive to the initial DSD aloft, which is of considerable uncertainty in various precipitation regimes. Thus, polarimetric radar measurements \((Z_H, Z_{DR})\) aloft are needed to diminish the uncertainty due to DSD variability. Drop size distributions with large concentrations of smaller drops (<2 mm) and relatively low number concentrations of big drops (>4 mm) exhibit the largest evaporative changes in \(Z_H\) and \(K_{DP}\). This is because the small drops are preferentially evaporated, while the backscattered signals are not overwhelmed by contributions from a large number of big drops. Distributions with a comparatively large concentration of big drops and a sufficiently high concentration of smaller drops (such as the MP model) will exhibit the largest increase in \(Z_{DR}\). Evaporative changes in \(\rho_{HV}\) are generally quite small in magnitude and likely unobservable at both S and C bands.

2) When the evaporative changes are converted into logarithmic units, the difference in \(K_{DP}\) is larger in magnitude than that of \(Z_H\). This is because \(K_{DP}\) is less sensitive to large drops than \(Z_H\). Since \(K_{DP}\) is generally displayed in linear units \((^\circ \text{ km}^{-1})\), evaporative changes may be more evident in \(K_{DP}\) than in \(Z_H\), especially at shorter radar wavelengths. It is important to note that estimates of \(K_{DP}\) at S band can be noisy, especially in light rain as considered in this study. The relative changes of \(K_{DP}\) are about the same at S and C bands.

3) When the evaporative changes are converted into logarithmic units, the difference in \(K_{DP}\) is larger in magnitude than that of \(Z_H\). This is because \(K_{DP}\) is less sensitive to large drops than \(Z_H\). Since \(K_{DP}\) is generally displayed in linear units \((^\circ \text{ km}^{-1})\), evaporative changes may be more evident in \(K_{DP}\) than in \(Z_H\), especially at shorter radar wavelengths. It is important to note that estimates of \(K_{DP}\) at S band can be noisy, especially in light rain as considered in this study. The relative changes of \(K_{DP}\) are about the same at S and C bands.

4) Except for evaporative changes in \(Z_{DR}\) at C band, the changes in the polarimetric variables become less significant with increasing rainfall rate. For rainfall rates larger than about 10 mm h\(^{-1}\), the effects of evaporation on the evolution of the drop size spectrum as well as the vertical profiles of polarimetric variables are less significant than for other microphysical processes such as coalescence and breakup, which should be considered the dominant processes and should be included in more general applications.

Additionally, the use of observed soundings to initialize the model produces realistic vertical profiles of the polarimetric variables. Different environments produce different degrees of evaporation that are in principle measurable by operational S- and C-band polarimetric radars. Quantitative bounds to the evaporative changes in the polarimetric variables are suggested using the extremes in the variations in DSD shape. However, we should again emphasize the limitations of the model. Since drop coalescence and breakup are ignored, the model is applicable only to relatively light rain. To more accurately model subcloud microphysics for more general applicability, these effects must be considered. Also, the moistening and cooling of the shaft must be turned “on” for simulation studies, especially when coupled with dynamical models.

In situations where low levels are unobserved by radar, modeling based on thermodynamic information from soundings or numerical weather prediction models such as the RUC may provide guidance for precipitation estimates, allowing for adjustments to be made to observed rainfall rates aloft. Assuming the DSD can be described by a constrained gamma model, the relative change in rainfall rate due to evaporation \(\Delta R/R\) is independent of \(Z_H\) aloft. Thus, \(Z_{DR}\) (together with the environmental thermodynamic profiles) can be used in a simple evaporation model to estimate \(\Delta R/R\). This method differs from conventional statistical or climatological techniques because the simple modeling approach in this study is physically based. The use of polarimetric measurements provides crucial information that is not available with single-polarization radar measurements of \(Z_H\). Such adjustments to conventional techniques may improve the estimates of rainfall rates at the surface, benefiting hydrology models and forecasts.

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APPENDIX

Equations for the Thermodynamic Terms

In (2), the heat conduction term and diffusion term are written as

\[
F_K = \left( \frac{L_e}{R_e T} - 1 \right) \frac{L_e \rho_i}{f_i K T} \tag{A1}
\]

and

\[
F_D = \frac{\rho_i R_e T}{f_e D_s c_s(T)} \tag{A2}
\]
following Rogers and Yau (1989), where \( L_v \) is the latent heat of vaporization; \( R_v \) is the gas constant for water vapor; \( T \) is the air temperature; \( \rho_l \) is the density of liquid water; \( K \) and \( D_v \) are the thermal conductivity of air and diffusivity of water vapor, respectively; \( \epsilon_s(T) \) is the saturation vapor pressure as a function of \( T \); and \( f_v \) and \( f_h \) are the ventilation coefficients for vapor and heat,

\[
f_v = 0.78 + 0.308N_{Sc}^{1/3}N_{Re}^{1/2} \quad \text{and} \quad f_h = 0.78 + 0.308N_{Pr}^{1/3}N_{Re}^{1/2},
\]

as in Pruppacher and Klett (1978) and Rasmussen and Heymsfield (1987). The ventilation coefficients depend on the Schmidt \( (N_{Sc}) \), Reynolds \( (N_{Re}) \), and Prandtl \( (N_{Pr}) \) numbers. The functional dependence of \( L_v \), \( K \), \( D_v \), and \( \epsilon_s(T) \) on temperature follows Rasmussen and Heymsfield (1987) and is provided in SI units here for convenience. The latent enthalpy of vaporization \((J \text{ kg}^{-1})\) as a function of temperature \((K)\) is given as

\[
L_v = 2.499 \times 10^6 \left( \frac{273.15}{T} \right)^{\gamma},
\]

where the exponent \( \gamma \) is

\[
\gamma = 0.167 + (3.67 \times 10^{-4}) T.
\]

The thermal conductivity of the air \((J \text{ m}^{-1} \text{s}^{-1} \text{K}^{-1})\) is expressed as

\[
K = (0.441 \times 10^3 + 0.0071 T) \times 10^{-2}.
\]

Similarly, the diffusivity of the water vapor in air \((\text{m}^2 \text{s}^{-1})\) is expressed as

\[
D_v = 2.11 \times 10^{-5} \left( \frac{T}{273.15} \right)^{1.94} \left( \frac{p_0}{p} \right),
\]

where \( p_0 \) is the reference level pressure, taken as 1000 hPa in this study. The saturation vapor pressure as a function of temperature is approximated following Rogers and Yau (1989):

\[
\epsilon_s(T) = A \exp \left( \frac{5420}{T} \right),
\]

where \( A = 2.53 \times 10^9 \) hPa. To calculate the Schmidt, Prandtl, and Reynolds numbers, the kinematic viscosity of the air \((\nu_a)\) is required. The kinematic viscosity is dependent on the dynamic viscosity of the air \((\eta_a)\), as well as the air density, \( \rho_a \):

\[
\nu_a = \eta_a / \rho_a.
\]

The dynamic viscosity of the air \((\text{kg m}^{-1} \text{s}^{-1})\) for \( T > 273 \) K is assumed to be

\[
\eta_a = (0.379565 + 0.00497 T) \times 10^{-4},
\]

and the air density is calculated from the thermodynamic profiles via the ideal gas law.

REFERENCES


