The Resonant Interaction of a Tropical Cyclone and a Tropopause Front in a Barotropic Model. Part II: Frontal Waves

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ABSTRACT

The influence of frontal waves on the interaction of a tropical cyclone and a tropopause front is investigated in an idealized framework. In a nondivergent barotropic model the front is represented by a vorticity step with a superimposed sinusoidal perturbation. This gives rise to a jet that meanders to the north and south and can be viewed as a sequence of upper-level troughs and ridges. The model evolution depends sensitively on the position of the cyclone relative to the troughs and ridges. Here a bifurcation point is identified that is located on the trough axis at a distance where the zonal speed of the background flow equals the phase speed of the wave. Arbitrarily small displacements from this position determine whether a cyclone is advected toward the front or repelled. Only a limited range of wavelengths can lead to track bifurcations. The largest effects are obtained for resonant frontal waves propagating with a phase speed matching the initial zonal translation speed of the cyclone. Weak and large-scale vortices can be disrupted when approaching the bifurcation point, where they are exposed to continuously strong shear deformation.

1. Introduction

The track, structure, and intensity of a tropical cyclone during and after extratropical transition (ET) are highly dependent on the details of the interaction with the midlatitude flow. Before recurvature, the relative locations of a tropical cyclone and a preexisting midlatitude trough determine whether the tropical cyclone accelerates into the midlatitudes ahead of the trough, or becomes embedded in the equatorward flow behind the trough. If the relative locations are incorrectly forecast, 72-h track errors in excess of 1000 km may result (Beven 2000). Following recurvature, the interaction with the midlatitude flow can modify the flow upstream of the tropical cyclone (Bosart and Lackmann 1995), contribute to enhanced ridging downstream (Henderson et al. 1999; Riemer et al. 2008), and excite a Rossby wave train, resulting in downstream development (Harr and Dea 2009). Ensemble prediction systems often exhibit a large variability in the development downstream of an ET event (Harr et al. 2008), indicating a reduced predictability. This variability has been related to the interaction of the tropical cyclone and the upper-level jet stream during the ET event (Anwender et al. 2008).

In Scheck et al. (2011, hereafter Part I) we investigated the interaction of a tropical cyclone with a zonally oriented front in a barotropic nondivergent framework. The tropical cyclone excites a localized frontal wave that is characterized by strong ridging downstream of the cyclone. The circulation associated with the frontal wave has a strong influence on the vortex motion. The interaction is sensitive to the vortex structure, especially to the presence of anticyclonic relative vorticity around the cyclonic vortex core. For stronger jets the interaction is more marked, in the sense that the tropical cyclone approaches the jet more rapidly.

Resonant waves (i.e., waves propagating with a phase speed matching the zonal translation speed of the cyclone) are of particular importance. A continuous acceleration of the cyclone accompanied by a continuous amplification or damping of the frontal wave is possible.
only for wave modes close to resonance. In general, the spectrum of frontal waves excited by the cyclone is dominated by wave modes that are resonant in the initial phase of the interaction process. As a result, these modes have the strongest influence on the cyclone motion.

In contrast to the idealized straight jets studied in Part I, real jets can be strongly perturbed. Frontal waves characterized by a trough upstream and a ridge downstream of the cyclone are associated with a circulation that advects the cyclone northward, bringing it closer to the front. The inverse configuration with an upstream ridge and a downstream trough acts to advect the cyclone southward, away from the front. The largest effects on the cyclone track are to be expected for near-resonant waves that stay in phase with the cyclone for a long time and cause a continuous meridional acceleration of the latter. Near-resonant frontal waves should actually be a common case, as every stationary (orography) or slow-moving source of frontal deformation will excite standing or slow-moving waves.

Therefore we investigate in this second paper a more realistic scenario in which preexisting frontal waves modify the cyclone–front interaction. Furthermore, the impact of these waves on the predictability of cyclone tracks is discussed. The paper is organized as follows. A short description of the model setup is given in section 2. In section 3 we discuss the influence of preexisting frontal waves on the vortex–jet interaction and on the predictability. Finally, section 4 contains a summary of our results.

2. Model description

a. Numerical methods and grid

In this study we use a nondivergent barotropic model on the β plane with second-order diffusion. We employ an explicit finite difference code to solve these equations numerically (see Part I for details).

In our calculations the numerical domain extends \( L_x = 27,000 \) km in the zonal direction and \( L_y = 9000 \) km in the meridional direction. For the model runs of Part I typically \( 3000 \times 1001 \) grid points were used (corresponding to a grid length of 9 km). To allow for the large number of simulations that are required to explore the parameter space, most of the model runs performed for this study are computed with a reduced resolution of \( 1500 \times 501 \) zones. This results in some damping of the waves that stay in phase with the cyclone for a long time and cause a continuous meridional acceleration of the latter. Near-resonant frontal waves should actually be a common case, as every stationary (orography) or slow-moving source of frontal deformation will excite standing or slow-moving waves.

b. Front and cyclone representations

The barotropic model is initialized by adding the relative vorticity field of a vortex to the absolute vorticity background that represents the front. A simple model for a front in a barotropic framework (e.g., Bell 1990; Schwierz et al. 2004) is given by two regions of constant absolute vorticity \( \eta = f \pm \eta_0 \) separated by an interface at \( y = y_f(x) \). The Coriolis parameter is approximated as \( f = f_0 + \beta y \). For convenience, the constant \( f_0 \) which has no influence on the calculations, is set to zero. Thus we adopt an absolute vorticity distribution

\[
\eta(x, y) = \xi(x, y) + \beta y \operatorname{sgn}[y - y_f(x)] \eta_0. \tag{1}
\]

An infinitely sharp vorticity step cannot be modeled with the numerical methods used for this study. As described in more detail in Part I, we therefore replace the vorticity step by a narrow smooth transition zone with a width of about 100 km.

For a zonally aligned jet \( (y_f = 0 \) km), Eq. (1) gives rise to the zonal velocity profile

\[
\pi(y) = -\operatorname{sgn}(y) \eta_0 y + \frac{1}{2} \beta y^2 + u_0, \tag{2}
\]

where \( u_0 = \bar{u}(0 \) km) is the jet speed (see Fig. 1 in Part I).

For the model runs in this study we set the jet parameters to \( u_0 = 40 \) m s\(^{-1}\) and \( \eta_0 = 4 \times 10^{-5} \) s\(^{-1}\) and use the SUD vortex described in Part I, which reaches the maximum wind speed of \( 40 \) m s\(^{-1}\) at a radius of 100 km. The start position of the vortex is 1500 km south of the front. Thus the initial state is set up as in model run BS from Part I, except that frontal waves are already present at \( t = 0 \) days.

Frontal waves are modeled as a sinusoidal frontal deformation:

\[
y_f(x, t = 0) = b_m^i \sin(k_m x + \phi_m^i), \tag{3}
\]

where \( k_m = 2\pi m/L_x \) is the wavenumber, \( \phi_m^i \) the initial wave phase, and \( b_m^i \) the initial wave amplitude of mode \( m \). In some cases several wave modes are excited.

A frontal wave of mode \( m \) propagates with a phase speed

\[
c = \omega/k_m = u_0 - \eta_0 \lambda_m / 2\pi = u_0 - \eta_0 / k_m, \tag{4}
\]

where \( \lambda_m = L_x / m \) is the wavelength (see Part I).

The circulation associated with the propagating frontal wave is given by the zonal and meridional components

\[
\omega = \frac{\eta_0}{L_x} \sinh \left( \frac{\eta_0 y}{u_0} \right) \left[ 1 - \cos \left( \frac{\eta_0 y}{u_0} \right) \right], \tag{5}
\]
\[ u_f(x,t) = -v_m \exp\left(-|k_m y|\right) \sin(k_m x - \omega t + \phi^i_m) \]  
and
\[ v_f(x,t) = v_m \exp\left(-|k_m y|\right) \cos(k_m x - \omega t + \phi^i_m), \]
respectively, with a velocity amplitude \( v_m = b_m^i \eta_0/2 \).

As in Part I, the letters B and S indicate that the runs are performed on the upstream and a ridge downstream of the cyclone, which causes a northward flow at the cyclone center. The values \( \phi_m^i = \pi/2, \pi, \) and \( 3\pi/2 \) correspond to eastward, southward, and westward flows, respectively. Note that in the meridional direction the circulation is damped exponentially with an e-folding scale of \( \lambda_m/2\pi \).

The flow steering the cyclone contains contributions associated with the vorticity step [Eq. (2)], the preexisting frontal wave, the frontal waves excited by the cyclone in the course of the model run, and the anticyclones sheared from the vortex in the initial phase (see Part I).

### 3. Interaction with a perturbed front

The effects of preexisting frontal waves are demonstrated using several sets of model runs. Within a set of calculations the mode number \( m \) and the initial wave amplitude \( b_m^i \) are kept constant and the initial wave phase \( \phi_m^i \) is varied between 0 and \( 2\pi \). Each set consists of about 40–50 model runs. The naming convention for the model runs is BS\(m\)-\(|\phi_m^i|\pi\) and a full set of calculations is denoted by BS\(m\)-X. As in Part I, the letters B and S indicate that the runs are performed on the \( \beta \) plane and that the SUD vortex is used, respectively.

The mode \( m = 5 \) is closest to resonance at the initial time and thus of special interest for the setup considered here. Given its phase speed, \( c_5 = 5.6 \) m s\(^{-1}\), and the initial zonal translation velocity of the cyclone for a straight jet, \( U_c(t = 0 \text{ days}) = 2.5 \) m s\(^{-1}\), the time scale over which this mode can grow \( [\tau_5 \approx 10 \text{ days}; \text{see Part I, Eq. (23)}] \) is rather long. Because of the preexisting frontal wave, \( U_c(t = 0 \text{ days}) \) varies with longitude in the model runs of set BS\(5\)-X, with \( \tau_5 \approx 1.7 \) m s\(^{-1}\).

a. Sensitivity to the wave phase

As a first step we investigate calculation set BS\(5\)-X, in which the wavelength of the preexisting frontal wave, \( \lambda_5 = L_x/5 = 5400 \) km, is close to the initial resonant wavelength. In Fig. 1 the evolution of the cyclone position is shown for the cases \( \phi_5^i = 0, \pi, 1.43\pi, \) and \( 1.44\pi \) and for the run with an unperturbed front (run BS). Not unexpectedly, the cyclone in run BS\(5\)-0.0 is advected toward the front faster than in run BS and crosses the interface 4 days earlier (Fig. 1a). The cyclone in run BS\(5\)-1.0, which is initially advected southward by the wave circulation, reaches the front 7 days later than in run BS. However, BS\(5\)-1.0 is not the most extreme case in terms of the time needed to reach the front. In model run BS\(5\)-1.43 the cyclone does not cross the interface at all in the 20 days included in the calculation.

An unexpected sensitivity to the initial conditions is found when comparing runs BS\(5\)-1.43 and BS\(5\)-1.44, in which the preexisting frontal wave is shifted only \( \lambda_5 \times 0.01 = 54 \) km farther upstream. The start positions of the cyclones in these runs are close to the trough axis \( (\phi_5^i = 1.5\pi) \). After about 5 days the meridional locations of the cyclones in the two runs diverge dramatically. While in run BS\(5\)-1.43 the cyclone moves southward, it is advected rapidly northward in run BS\(5\)-1.44 and crosses the front at about the same time as in the unperturbed case BS. The huge differences in the temporal evolution of the meridional cyclone coordinate lead also to vastly
have shown that at these critical phases shifting the frontal wave by a distance smaller than the grid length (in this case $\Delta x = 18$ km) is sufficient to switch between vastly different solutions, (e.g., BS5-1.43-like solutions in which the cyclone remains south of the front and solutions in which the cyclone reaches the front in less than 10 days, as in run BS5-1.44). This behavior can be interpreted as a bifurcation in the solution space.

To investigate the cause of the bifurcations it is sufficient to consider only the effect of the freely propagating preexisting wave on the cyclone motion and neglect the changes in wave phase and amplitude that are caused by the cyclonic and anticyclonic circulations, as these become important only after some days. According to this simplification, the motion of the cyclone relative to the basic-state flow is determined by the phase of the preexisting wave at the zonal position of the cyclone:

$$
\phi_c(t) = \phi_m + 2\pi[X(t) - c_m/t]/\lambda_m.
$$

For $-\pi/2 < \phi_{c,m} < \pi/2$ (around a trough–ridge transition) cyclones are advected northward by the wave circulation; otherwise they are advected southward. South of troughs ($\pi < \phi_{c,m} < 2\pi$) cyclones are advected eastward relative to the basic-state flow, and south of ridges ($0 < \phi_{c,m} < \pi$) they are advected westward. The phase $\phi_{c,m}$ is indicated by the shading in Fig. 3, which displays the evolution of the zonal cyclone position for all runs of set BS5-X.

The cyclones starting at phases $\phi_5 < 0.5\pi$ or $\phi_5 > 1.5\pi$ are initially advected northward, thereby increasing their zonal translation velocity sufficiently to stay in the northward advecting range (white background in Fig. 3) for several days and reach the front within 9 days. Cyclones starting at phases slightly smaller than $\phi_5 = 1.5\pi$ [i.e., upstream of the trough center, where the preexisting frontal wave causes a southward directed flow (gray background) initially] are still able to overtake the trough and reach the northward advection zone. This is possible for two reasons. At $t = 0$ days the cyclones are already moving with a translation speed close to the phase speed of the wave (5.6 m s$^{-1}$) because the wave circulation is eastward near the trough center and thus $U_c(t = 0) \approx \bar{U}(t + \nu_5) = 4.4$ m s$^{-1}$. The increase in zonal velocity required to overtake the ridge is provided by the anticyclones, which cause initially a northward advection (see Part I) and thus also a zonal acceleration of the cyclone.

However, at a critical phase $\phi_{c,m} \approx 1.44\pi$ the cyclone attains a zonal velocity matching the phase speed of the wave at a time when it is south of the center of the trough (i.e., on the trough axis). In this position the meridional velocity component at the cyclone center caused by the
wave is zero. Thus, the cyclone remains at the same meridional position and its zonal translation speed remains constant and equal to the phase speed of the wave. In the frame of reference of the wave this is a stationary situation—cyclone and wave move with the same zonal velocity and their separation remains constant. In runs with $f = f_{\text{crit}}$ the cyclone can stay close to this position for a rather long time. Very small changes in the initial wave phase determine whether the cyclone eventually moves toward the next trough–ridge transition downstream of the trough, where it is attracted by the front, or toward the ridge–trough transition upstream of the trough, where it is repelled from the front. The fact that very small changes in the initial state determine which of two vastly different solutions will occur gives rise to the bifurcation.

Neglecting the influence of the anticyclones and the frontal waves excited by the cyclone, the point at which the bifurcation occurs fulfills the conditions $V_c = u_f(X_c, Y_c) = 0$ and

$$U_c = \pi(Y_c) + u_f(X_c, Y_c) = c_m. \quad (8)$$

There are actually two points south of the front that satisfy these conditions, as sketched in Fig. 4: a point T south of the trough center ($\phi_r = 3\pi/2$) where the background flow and the circulation of the preexisting frontal wave are parallel, and a point R south of the ridge center ($\phi_r = \pi/2$), where the background and they are antiparallel. For the set of calculations considered here Eq. (8) yields meridional coordinates $Y_{cT} \approx -1400$ km and $Y_{cR} \approx -1100$ km for points T and R, respectively. The bifurcation at $\phi \approx 1.44\pi$ occurs close to the trough center and is thus clearly related to point T. The bifurcation at $\phi \approx \pi$ (see Fig. 2) is also located close to the trough center.\footnote{This is not clearly visible in Fig. 3 because at $t = 15$ days frontal waves excited by the cyclone have already caused a significant phase shift of the $m = 5$ mode.} No unusual spreading of the tracks is visible for cyclones that attain positions south of the ridge center (dotted lines in Fig. 3), so there are no bifurcations that occur close to point R.

The fact that bifurcations occur at point T but not at point R is related to the different flows in the vicinity of the two points. Close to point T the solutions diverge. As sketched in Fig. 4, cyclones that are located slightly downstream of point T are attracted toward the front and accelerated in the zonal direction, whereas cyclones slightly upstream of point T are advected farther away from the front and slow down. Point T can be approached from the southeast because a cyclone located downstream and farther away from the front than point T moves more slowly than the point itself and experiences...
a northward directed flow component from the wave. Both effects will bring the cyclone closer to point T.

In contrast to point T, point R cannot be reached by cyclones from the south. To get closer to the front, the cyclone must be located upstream of point R. However, in this position the cyclone is moving more slowly than point R, so that by the time the cyclone has reached the meridional position of point R, the latter has already propagated downstream. If a cyclone is located downstream and south of point R, it will move southward and slow down until it is overtaken by point R. Subsequently this cyclone will move toward the front and also pass point R on the upstream side (see Fig. 4). Thus point R cannot be reached by a cyclone moving northward, but the flow around point R causes cyclones to revolve around it (in the frame of reference of the frontal wave).

In accordance with these considerations, cyclone tracks diverging near point T and cyclones tracks orbiting around point R are visible in Fig. 5 (top panel), which displays the cyclone tracks of simulation set BS5-X in the frame moving with the wave. These cyclone tracks are influenced not only by the preexisting frontal wave, but also by frontal waves excited by the cyclone and by the anticyclones. The locations of points R and T indicated in Fig. 5, however, are computed under the simplifying assumption that the influence of the anticyclones and of the additional frontal waves excited by the cyclone can be neglected. Thus it is assumed that the cyclone is advected like a tracer particle in a velocity field \((u_p, v_p)\) that is a superposition of the stationary jet and the flow caused by the propagating preexisting frontal wave. The limits of this approximation are evident from Fig. 5 (bottom panel), which displays cyclone tracks together with contour lines of the streamfunction \(\Psi_p\), that gives rise to \((u_p, v_p)\) in the frame of the wave. The points R and T are located at maxima and saddle points of \(\Psi_p\), respectively. Overall the streamlines are similar to the cyclone tracks—they diverge near point T and orbit around point R, and the angle between the direction of the cyclone motion and the closest streamline is relatively small for most of the time and most of the tracks.

However, clear differences between the tracks and the streamlines are visible. These differences are related to the anticyclones and the frontal waves excited by the cyclone. During the formation of the anticyclones in the first few days the cyclone is advected northward (see Part I). Because of this effect the tracks diverge at a point slightly upstream of the position marked with T, where the southward flow component caused by the wave compensates for the northward flow due to the anticyclones. At later times, and in particular when the cyclone comes close to the interface, the waves excited by the cyclone can no longer be neglected compared to the preexisting wave, and the tracks no longer follow the streamlines. Therefore, considering the cyclone as a tracer particle can explain some basic features of the tracks, but this is not useful to determine the cyclone tracks quantitatively. When the amplitude of the preexisting frontal wave is reduced, the cyclone-induced waves become important earlier and eventually the bifurcation should vanish. However, the bifurcation of tracks near point T is still clearly visible even for a preexisting wave amplitude of only 100 km.

Figure 5 illustrates also why a bifurcation occurs at \(\phi_5 \approx \pi\) (see Fig. 2). The cyclone starting at \(\phi_5 = \pi\) circles around point R, approaches point T from the northwest, and is deflected toward the front. The cyclone starting at \(\phi_5 = 1.02\pi\) takes the same route initially but is then deflected to the south. For slightly higher values of \(\phi_5\) the cyclone is advected northward before it can come closer to point T. Thus, a rather large
However, for all phases the solution changes smoothly to approach the front varies between 8 and 14 days (Fig. 6).

With the highest wind speeds [see Eqs. (5) and (6)]. The modes with the largest wavelengths are associated with the preexisting wave to advect the cyclone southward. Consequently, the range of phases for which the cyclone does not cross the front during the simulated 20 days becomes narrower. While for $m = 5$ a considerable fraction $\phi_5^b \in [0.85\pi, 1.43\pi]$ causes the cyclone to reach the front only after 10 days or later (Fig. 2), this is the case for $m = 7$ only when $\phi_7^b \in [0.6\pi, 0.7\pi]$. For higher mode numbers there is more time for the cyclone to excite frontal waves.

### Table 1. Parameters of the sets of runs with preexisting frontal waves.

<table>
<thead>
<tr>
<th>Name</th>
<th>$r_{max}$ (km)</th>
<th>$v_{max}$ ($\text{s}^{-1}$)</th>
<th>Perturbation</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS4-X</td>
<td>100</td>
<td>40</td>
<td>$m = 4, b_4 = 250$ km</td>
</tr>
<tr>
<td>BS5-X</td>
<td>100</td>
<td>40</td>
<td>$m = 5, b_5 = 250$ km</td>
</tr>
<tr>
<td>BS7-X</td>
<td>100</td>
<td>40</td>
<td>$m = 7, b_7 = 250$ km</td>
</tr>
<tr>
<td>BS10-X</td>
<td>100</td>
<td>40</td>
<td>$m = 10, b_{10} = 250$ km</td>
</tr>
<tr>
<td>BS15-X</td>
<td>100</td>
<td>40</td>
<td>$m = 15, b_{15} = 250$ km</td>
</tr>
<tr>
<td>BS5a-X</td>
<td>100</td>
<td>40</td>
<td>$m = 4, 5, b_4 = 3\pi/2, b_5 = 250$ km</td>
</tr>
<tr>
<td>BS5b-X</td>
<td>100</td>
<td>40</td>
<td>$m = 4, 5, b_4 = 3\pi/2, b_5 = 250$ km</td>
</tr>
<tr>
<td>BS5c-X</td>
<td>100</td>
<td>40</td>
<td>$m = 4, 5, 6, 8, b_4 = 3\pi/2, b_5 = 250$ km</td>
</tr>
<tr>
<td>BNS-X</td>
<td>500</td>
<td>-8</td>
<td>$m = 5, b_5 = 250$ km</td>
</tr>
</tbody>
</table>

area downstream of point R cannot be reached by any cyclone.

### b. Influence of perturbation wavelength

The wavelength of the $m = 5$ frontal wave mode used above is slightly smaller than the initial resonant wavelength and the phase speed is slightly faster than the initial zonal translation velocity of the cyclone. In the following, the influence of waves with larger or smaller wavelengths is discussed and cases are investigated in which several wave modes are excited. For this purpose, several additional sets of calculations are considered (see Table 1 for a list of all sets discussed in this section). In particular, we consider several calculation sets similar to BS5-X, but with frontal waves of the modes $m = 4, 7, 10,$ and 15.

The wave amplitude $b_m^f = 250$ km is the same in all runs, which means that at a fixed distance from the front the modes with the largest wavelengths are associated with the highest wind speeds [see Eqs. (5) and (6)]. Therefore the $m = 4$ wave has a strong impact on the cyclone track—the time required by the cyclone to approach the front varies between 8 and 14 days (Fig. 6). However, for all phases the solution changes smoothly with $\phi_m^f$ and there are no bifurcations. This fundamental difference is related to the fact that $m = 5$ waves propagate in the same direction as the fluid itself, whereas $m = 4$ waves propagate in the upstream direction. The bifurcations for $m = 5$ can arise because two kinds of solutions can develop from very similar initial conditions: one in which the cyclone overtakes the wave, and one in which the wave overtakes the cyclone. This situation can be encountered only if the cyclone can actually obtain a zonal translation speed matching the phase speed of the wave. As the zonal translation speed of the cyclone is close to the zonal speed of the background flow (unless the cyclone is close to the front), this requires that the phase speed is contained in the zonal speed profile of the jet. This is not the case for $m = 4(c_4 < 0$, but $\pi > 0$), so the cyclone will always overtake the wave. These results indicate that preexisting frontal waves with wavelengths larger than the initial resonant wavelength can have a strong influence on the cyclone track, but they are not important for the sensitivity of the track with respect to small changes in the initial conditions.

For smaller wavelengths [i.e., higher values of $m$ and faster (positive) phase speeds], $U_c = c_m$ is again possible. With decreasing wavelength the frontal wave circulation decays faster in the meridional direction [Eqs. (5) and (6)] and the meridional positions of points R and T move closer to the front. Thus the strongest effects of preexisting frontal waves with smaller wavelengths, including bifurcations, are expected to occur only after the cyclone has moved sufficiently close to the front. Indeed, for the $m \geq 7$ cases the cyclone position in the first 7 days hardly varies for different phases $\phi_m^f$. During this period the cyclone motion is not yet influenced significantly by the preexisting frontal wave, but the cyclone circulation excites waves with the current resonant wavelength. This wave component causes a northward acceleration of the cyclone, making it considerably more difficult for the preexisting wave to advect the cyclone southward. Consequently, the range of phases for which the cyclone does not cross the front during the simulated 20 days becomes narrower.
before the bifurcation process takes place. Thus the influence of the preexisting wave becomes weaker compared to the other waves, and the impact of the bifurcation process on the cyclone track becomes less severe. At $m = 10$ the bifurcation process is able to change the time required by the cyclone to reach the front by about 3 days (Fig. 7) but does not lead to such strongly diverging evolutions as in the case of runs BS5-1.43 and BS5-1.44 (Fig. 1).

For preexisting frontal waves with very small wavelengths and high phase speeds, bifurcations become impossible again because the wave is always faster than the cyclone. In our calculations with a jet speed $u_0 = 40$ m s$^{-1}$ the maximum zonal translation velocity obtained by the cyclone is about 30 m s$^{-1}$, which corresponds to $m \approx 15$ for the $\beta$-plane cases discussed here (see Fig. 13 in Part I). For $m = 10$ bifurcations are still present whereas for $m = 15$ no bifurcations are encountered (Fig. 6). Thus, a rather narrow band of wavelengths smaller than the initial resonant wavelength has a strong influence on the sensitivity of the solution on the initial conditions. Assuming that the zonal translation velocity of the cyclone is in the range $[0, u_0/2]$, bifurcations can only be caused by waves with a phase speed in the same range. The corresponding range of wavelengths can be computed using the relation $u_0 = \frac{\eta}{2\beta}$ (see Part I, section 3a) and the dispersion relation [Part I, Eq. (6)]. The maximum wavelength (i.e., the wavelength of the wave with phase speed zero) is thus $\lambda_b = \pi \sqrt{2u_0/\beta}$, which amounts to about 6000 and 8000 km for jet speeds of 40 and 70 m s$^{-1}$, respectively. The minimum wavelength is about $\lambda_0/2$.

To examine whether bifurcations can still occur when preexisting waves containing several wave modes are present, several further sets of calculations are considered. In a first experiment, set BS5a-X, the preexisting frontal wave consists of an $m = 4$ and an $m = 5$ component with equal amplitudes (250 km). The phase of the $m = 5$ component is varied continuously, whereas for the $m = 4$ mode only some phases are investigated. We show only the case $\phi_4 = 1.5\pi$, which should be the least favorable for the occurrence of bifurcations, as the $m = 4$ wave causes the maximum northward advection for this phase. Yet bifurcations are present even in this case (Fig. 8). The bifurcation is suppressed only when the amplitude of the $m = 5$ mode is reduced to 100 km and $\phi_4 = 1.5\pi$ (set BS5b-X, Fig. 8). If either the amplitude is increased or a more favorable phase is chosen, bifurcations are present.

Finally, in set BS5c-X four wave modes ($m = 4, 5, 6, 8$) are excited in the preexisting frontal wave, each with an amplitude of 250 km. In this case several bifurcations are present (Fig. 8). These results demonstrate that bifurcations due to waves that are in resonance with the cyclone at some time are not easily suppressed by the presence of other resonant or nonresonant waves.

c. Influence of frontal wave–induced shear

In the previous sections we showed that the main features of the interaction between the cyclone and the frontal wave can be explained by treating the cyclone as a point vortex. A secondary effect was the modification of the cyclone structure by the background shear associated with the jet, leading to the formation of anticyclones. The tropopause evolution, however, is sensitive to the interaction with the upper-level anticyclone of a tropical cyclone that has a much larger horizontal scale than the cyclonic core. Thus it is important to consider the structural evolution of an anticyclone as a function of its position relative to the frontal wave. Because of its large horizontal scale, the anticyclone is more sensitive to the horizontal shear of the jet and the frontal wave than the cyclonic vortex considered previously.
We investigate this interaction with a set of calculations, BN5-X, for which the anticyclonic part of the ANTI vortex (see section 3b in Part I) is used. In this anticyclonic vortex \( \eta_{\text{max}} = 52.8 \text{ m s}^{-1}, r_{\text{max}} = 500 \text{ km} \) the velocity gradient is of a size comparable to the background shear. Furthermore, for the \( m = 5 \) frontal wave with amplitude 250 km in BN5-X the shear of the frontal wave circulation is also of the same order as the background shear. Consequently, the vortex structure can be changed considerably because of the shear and the changes depend on the initial position of the vortex.

Two extreme cases are shown in Fig. 9. In run BN5-0.6 the anticyclone is initially located south of a ridge, in the vicinity of point R. In this region the shear of the frontal wave circulation partially compensates for the background shear. The shape of the anticyclone is basically preserved as it circles around point R (Fig. 9, top panel). In run BN5-1.5 the anticyclone starts south of a trough, close to point T. Because of the strong shear deformation caused by the jet profile and the wave circulation in this region, the vortex is deformed to an elongated shape and then disrupted into two filaments within several days. The two filaments are advected to the northeast and the southwest, respectively, and end up south of the two neighboring ridges (Fig. 9, bottom panel). In this model run the anticyclone cannot influence the frontal wave substantially, as its vorticity distribution soon extends over a zonal range comparable to the wavelength of the frontal wave. In contrast, the anticyclone in run BN5-0.6, which remains compact and stays in phase with the wave, can influence the wave more strongly and causes a phase shift of about 500 km in 10 days (Fig. 9, top panel).

4. Summary

We investigated the interaction of a tropical cyclone and a perturbed tropopause front using an idealized model. In a barotropic framework a vorticity step is used to model the front and gives rise to a jetlike velocity profile. To study the influence of preexisting troughs and ridges on the cyclone–front interaction, we performed model runs in which a sinusoidal perturbation was applied to the jet in the initial state. In these cases a high sensitivity of the cyclone track to the initial cyclone position is observed. A bifurcation of the cyclone tracks occurs close to a point on the trough axis where the zonal translation speed of the cyclone matches the phase speed of the wave and the meridional translation speed is zero. Arbitrarily small displacements from this location determine whether a cyclone is advected northward and accelerated in the zonal direction or is advected southward and slowed down. The difference in the cyclone position for these two cases increases by up to 2000 km day\(^{-1}\). A second point on the ridge axis fulfilling the same conditions does not give rise to bifurcations, but the flow in its vicinity causes cyclones to revolve around it (in the frame moving with the wave). To a first approximation the cyclone behaves like a tracer particle advected with the flow due to the jet and the preexisting frontal wave. Two factors cause the cyclone motion to deviate from that of a tracer particle. First, the anticyclones sheared away from the outer region of the vortex modify the track in the first few days. Second, frontal waves excited by the cyclone influence the motion, particularly in later phases.

The strongest sensitivity to the initial cyclone position is found for perturbation wavelengths slightly smaller than the initial resonant wavelength. Waves with larger wavelengths still have a strong influence on the cyclone motion, but they cannot cause bifurcations. For smaller wavelengths the bifurcation occurs closer to the front and leads to less severe differences in the cyclone tracks. For very small wavelengths bifurcations become impossible again. Thus, bifurcations can occur only for a limited range of wavelengths of the preexisting wave (in our standard case between about 3000 and 6000 km). If several wave modes are excited, this leads in general to additional bifurcation points.

For weak, broad vortices with parameters typical for anticyclonic outflows of real tropical cyclones, a preexisting frontal wave can also have a strong influence on the vortex structure. The shear deformation of the background flow is stronger south of troughs and weaker south of ridges. Weak vortices that come close to the bifurcation point are disrupted under the influence of the strong shear deformation and their remnants are advected into the regions south of ridges.
Despite the strong idealizations, the model runs performed for this paper show a variety of processes that are important for the interaction of real cyclones with jets. The strongly idealized model has the advantage that the causes and effects of these processes are easier to analyze than in three-dimensional full-physics models. For instance, through the idealized approach we were able to identify bifurcations in the cyclone tracks and explain why and where they occur. These scenarios are interesting as test problems for sensitivity and predictability studies. In a future study we will investigate the behavior of singular vectors for these cases, in particular for cyclone trajectories that come close to a bifurcation point. Furthermore, the scenarios discussed in this paper will be used as test problems for adaptive grid refinement techniques, which are evaluated for the use in tropical cyclone–related problems.

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