The Relative Roles of Different Physical Processes in Unstable Midlatitude Ocean–Atmosphere Interactions

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ABSTRACT

Following Goodman and Marshall (hereinafter GM), an improved intermediate midlatitude coupled ocean–atmosphere model linearized around a basic state is developed. The model assumes a two-layer quasigeostrophic atmosphere overlying a quasigeostrophic upper ocean that consists of a constant-depth mixed layer, a thin entrainment layer, and a thermocline layer. The SST evolution is determined within the mixed layer by advection, entrainment, and air–sea flux. The atmospheric heating is specified at midlevel, which is parameterized in terms of both the SST and atmospheric temperature anomalies. With this coupled model, the dynamical features of unstable ocean–atmosphere interactions in the midlatitudes are investigated. The coupled model exhibits two types of coupled modes: the coupled oceanic Rossby wave mode and the SST-only mode. The SST-only mode decays over the entire range of wavelengths, whereas the coupled oceanic Rossby wave mode destabilizes over a certain range of wavelengths (10 000 km) when the atmospheric response to the heating is equivalent barotropic. The relative roles of different physical processes in destabilizing the coupled oceanic Rossby wave are examined. The main processes emphasized are the influence of entrainment and advection for determining SST evolution, and the atmospheric heating profile. Although either entrainment or advection can lead to unstable growth of the coupled oceanic Rossby wave with similar wavelength dependence for each case, the advection process is found to play the more important role, which differs from GM’s results in which the entrainment process is dominant. The structure of the unstable coupled mode is sensitive to the atmospheric heating profile. The inclusion of surface heating largely reduces the growth rate and stabilizes the coupled oceanic Rossby wave. In comparison with observations, it is demonstrated that the structure of the growing coupled mode derived from the authors’ model is closer to reality than that from the previous model.

1. Introduction

Over the past two decades, a variety of observational studies have suggested the existence of midlatitude climate variability on decadal-to-interdecadal time scales in both the atmosphere and the ocean (e.g., Van Loon and Rogers 1978; Kushnir 1994; Dickson et al. 1996; Nitta and Yamada 1989; Trenberth 1990; Miller et al. 1994; Mantua et al. 1997). However, the maintenance or causative mechanism responsible for climate variability in the midlatitudes has been debated. Some early studies suggested that the midlatitude atmosphere generates the climate variations on its own, while the ocean just reacts passively to that stimulus (e.g., Bjerknes 1962; Davis 1978; Trenberth 1975; Harworth 1978). Meanwhile, the role of ocean–atmosphere interaction in midlatitude variability was also noticed by several other early investigations (e.g., Namias 1959, 1963, 1969; Bjerknes 1964). Recent GCM simulations have shown that the midlatitude SST anomalies can have a significant influence on the atmospheric circulation (e.g., Palmer and Sun 1985; Kushnir and Lau 1992; Lau and Nath 1994; Graham et al. 1994; Ferranti et al. 1994; Latif and Barnett 1994; Peng et al. 1995; Peng and Whitaker 1999; Kushnir et al. 2002; Liu and Wu 2004; Zhong and Liu 2008). These studies provided strong support for the potential role of ocean–atmosphere interaction in midlatitude climate variability. As a candidate mechanism, ocean–atmosphere interaction has been used to explain decadal-to-interdecadal variability in GCMs in the North Pacific (Latif and Barnett 1994, 1996; Latif 1999; Miller and Schneider 2000) and in the North Atlantic (Robertson 1996; Zorita and Frankignoul 1997). Therefore, it was often assumed that the atmosphere can respond quickly to the midlatitude
SST anomaly in a number of theoretical studies on midlatitude decadal variability (e.g., Jin 1997; Münich et al. 1998; Neelin and Weng 1999; Tillery 1999; Watanabe and Kimoto 2000; Cessi 2000; Jin et al. 2001; Primeau and Cessi 2001; Solomon et al. 2003).

Since unstable ocean–atmosphere interaction may play an important role in decadal-to-interdecadal climate variability in the midlatitudes, it is worthwhile to investigate the dynamical behavior of the midlatitude coupled ocean–atmosphere system. Although the GCMs can give us tremendous knowledge in this regard, the fundamental mechanisms contributing to decadal variability are always difficult to understand, because of complicated physical processes involved in the GCMs. Therefore, simplified or intermediate-coupled ocean–atmosphere models become useful tools, in which the dynamics and physics behind air–sea interaction are modeled in an ideal but clear way. With such models, the extratropical unstable coupled modes have been exhibited in several theoretical analyses. Liu (1993) performed a theoretical analysis with a highly simplified coupled model and suggested that the phase relationship between the atmospheric response and the SST anomaly is of crucial importance for the extratropical unstable ocean–atmosphere interaction. Goodman and Marshall (1999, hereinafter GM) formulated an intermediate midlatitude coupled ocean–atmosphere model and found that the unstable growth only occurs if the atmospheric response to thermal forcing is equivalent barotropic. Following their work, Ferreira et al. (2001) considered the effect of the finite width of the ocean on the unstable modes and investigated the oscillatory behavior of the coupled midlatitude model with a stochastic forcing. Similarly, Van der Avoird et al. (2002) analyzed the linear stability of an intermediate-complexity coupled model and revealed that a large-scale mode can become unstable once the coupling strength is large enough. Arzel and Huck (2003) illustrated that the interactions between zonal winds and ocean gyres would become unstable in a specific range of parameters and then they can induce decadal variability.

The occurrence of the midlatitude unstable modes derived from the above simplified coupled air–sea model largely depends on whether the key physical processes central to the unstable growth are represented properly. Unfortunately, these physical processes were often treated crudely and sometimes even arbitrarily in previous studies, thus probably leading to model-dependent results. One issue is the role of the processes that determine SST change in unstable growth. Such processes include entrainment, advection, and air–sea flux. Whether all of these processes are considered properly in a coupled model can directly affect the behavior of the coupled system. For example, the unstable coupled mode derived from GM’s model is found to depend considerably on the entrainment process, while the Ekman flow in the mixed layer is ignored and the entrainment is assumed to occur in the entire thermocline layer in their model. This assumption would overestimate the effect of entrainment on SST. The other issue is the role of the atmospheric heating profile in unstable growth. The atmospheric heating profile is a crucial factor that determines how the atmosphere responds to the thermal forcing (eventually to the SST anomaly). In previous models, the heating profile is set in such a way that the heating is assumed to occur only in the midlevel of the atmosphere. It is of interest to understand whether the feature of unstable coupled modes is sensitive to the specification of the vertical heating structure.

This study aims at investigating the dynamical features of unstable midlatitude air–sea interaction with an improved intermediate midlatitude coupled model. The focus will be on the relative roles of different physical processes in unstable coupled modes. The main processes emphasized here include the entrainment and advection processes that determine SST evolution and the atmospheric heating profile. The paper is organized as follows. An improved intermediate midlatitude coupled air–sea model is described in section 2 and formulated in the appendix. In section 3, the coupled modes of the model linearized about a basic state are derived and their instabilities are analyzed. The relative roles of different physical processes in contributing to the unstable coupled modes are discussed in section 4. In section 5, the results deduced from present dynamical analysis are compared with GM’s work and with the midlatitude coupled air–sea pattern in the North Pacific statistically identified from observations. The last section is devoted to the conclusions and discussion.

2. Description of the coupled model

The model used in this study is an intermediate type of midlatitude coupled ocean–atmosphere model linearized around a basic state. It assumes a two-layer quasigeostrophic atmosphere overlying a quasigeostrophic upper ocean that consists of a constant-depth mixed layer, a thin entrainment layer, and a thermocline layer, as illustrated in Fig. 1. Both the atmosphere and the upper ocean are governed by their respective quasigeostrophic potential vorticity (QGPV) equations on a midlatitude beta plane. A linearized, relatively full mixed layer temperature equation is considered to describe the SST evolution. The atmospheric heating with a fixed profile is parameterized in terms of both the SST anomaly and the atmospheric temperature anomaly. The atmosphere and the ocean are coupled as follows:
an initial SST anomaly results in an anomalous atmospheric heating that changes the atmospheric circulation, leading to an anomalous wind field. The anomalous wind stress then drives the upper-ocean motions, which in turn alter the initial SST anomaly. A detailed description of the coupled model equations can be seen in the appendix. The values of all the parameters and constants taken for the model are listed in Table 1.

The coupled model is similar to GM’s model. However, it differs from GM’s model in the following aspects. First, a constant-depth mixed layer is considered in our model. The horizontal velocity that influences the SST evolution is determined by the motion of the mixed layer, which is the sum of an Ekman plus a geostrophic component. In contrast, because of the lack of the mixed layer, the horizontal velocity that influences the SST evolution in GM’s model is determined by the quasigeostrophic motion of the whole upper ocean. Second, a relatively full form of SST equation is used for linearization in this study. This SST equation [see (A12)] includes all the important processes responsible for the SST evolution, while in GM’s model the role of anomalous entrainment velocity is omitted from the linearized SST evolution. Last, the entrainment process in our model is assumed to happen within a thin entrainment layer instead of the entire thermocline layer as in GM’s model. Our model can be reduced to GM’s model if the Ekman flow is neglected [i.e., $V_S = 0$ in (A9)], the entrainment layer depth $\Delta h_c$ is specified equal to the mean thermocline depth $H_2$, and $c_3$ is set to be zero.

### 3. Unstable midlatitude coupled modes

#### a. Dispersion relation

Here we seek the plane wave solutions of the coupled model described above. The dispersion relations of (A23)–(A28) are derived to find unstable midlatitude coupled modes. We assume that the plane wave solutions have the form

$$\hat{\psi}, \hat{\psi}_f, \psi_o, T_1 = (\hat{\psi}, \hat{\psi}_f, \psi_o, T_1) e^{i(kx - \sigma t)} \sin \psi,$$

where $\hat{\psi}, \hat{\psi}_f, \psi_o$ and $T_1$ are the amplitudes of these variables, $\sigma$ is the frequency, and $k, l$ are the horizontal wavenumbers for the directions $x$ and $y$, respectively. Inserting (1) into (A23)–(A28) and dropping the primes for notational convenience, we can obtain the following relations of the amplitudes of these variables:

$$\hat{\psi} = -\mu \hat{\psi}_f,$$

$$\left(1 + i \frac{\nu}{\Gamma} \right) \hat{\psi}_f = \frac{1}{r_o} T_1,$$

$$\left(\alpha - \omega \right) \psi_o = i \alpha \kappa^2 L_\sigma \left( \frac{\hat{\psi}}{2} - \hat{\psi}_f \right),$$

$$-i \alpha T_1 = c_1 \psi_o - c_2 T_1 + c_3 \frac{f k^2}{r_o^2} \left( \frac{\hat{\psi}}{2} - \hat{\psi}_f \right) + c_4 \frac{r k + f l}{f^2 + r^2} i \alpha \left( \frac{\hat{\psi}}{2} - \hat{\psi}_f \right) + c_4 i k \psi_o - \gamma_o \left( T_1 - r_o \hat{\psi}_f \right).$$

### Table 1. Numeric parameter values used in the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>$1 \times 10^{-4}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$1.8 \times 10^{-11}$ (ms)$^{-1}$</td>
</tr>
<tr>
<td>$H_a$</td>
<td>10 km</td>
</tr>
<tr>
<td>$H_1$</td>
<td>100 m</td>
</tr>
<tr>
<td>$H$</td>
<td>500 m</td>
</tr>
<tr>
<td>$\Delta h_c$</td>
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</tr>
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<td>660 km</td>
</tr>
<tr>
<td>$L_o$</td>
<td>45 km</td>
</tr>
<tr>
<td>$\theta_{0}$</td>
<td>290 K</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>10$^3$ kg m$^{-3}$</td>
</tr>
<tr>
<td>$T_r$</td>
<td>283 K</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>2.5 \times 10^{-4}$ K$^{-1}$</td>
</tr>
<tr>
<td>$r_s$</td>
<td>1.3 \times 10^{-7}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>3 \times 10^{-8}$ s$^{-1}$</td>
</tr>
<tr>
<td>$C_p$</td>
<td>4 \times 10^7$ J kg$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$\gamma_o$</td>
<td>8 \times 10^{-7}$ s$^{-1}$</td>
</tr>
<tr>
<td>$l$</td>
<td>$\eta/3200$ km</td>
</tr>
<tr>
<td>$c_4$</td>
<td>3 \times 10^{-6}$ K m$^{-1}$</td>
</tr>
<tr>
<td>$U_1$</td>
<td>17 m s$^{-1}$</td>
</tr>
<tr>
<td>$U_2$</td>
<td>8 m s$^{-1}$</td>
</tr>
</tbody>
</table>
where \( \omega_r = -\beta k L_o^2 \) is the oceanic baroclinic Rossby wave frequency; \( c'_1 = c_1/b, c'_2 = c_2/(H - H_1) \), and \( c'_3 = c_3(H - H_1)/H_1 \) are positive coefficients; \( \kappa^2 = k^2 + \ell^2 \) and \( \kappa_n^2 = \kappa^2 + 2/L_o^2 \) are defined for simple expression; \( \mu \) determines the relative strength of barotropic and baroclinic modes; and \( \Gamma \) and \( \nu \) represent the thermal damping time scale and advective-propagation time scale, respectively. They are defined as

\[
\mu = \frac{\bar{U}}{U - \beta/k^2}, \quad \Gamma = \frac{4\gamma_a}{\kappa^2 L_o^2}, \quad \text{and} \quad \nu = \frac{\bar{U}}{U - \beta/k^2}\mu + \frac{\bar{U}}{U - \beta/k^2}k^2 = \frac{\beta k}{\kappa^2}.
\]

Equation (3) describes the atmospheric response to the SST anomaly forcing. Both the phase and the amplitude of this response are determined by the factor \( \nu/T \), the ratio of the thermal equilibrium time scale to the advection time scale. Equation (3) can also be rewritten as

\[
(T_1 - r_a \tilde{a}) = m T_1,
\]

where \( m = (\nu/T)(i + \nu/T)[1 + (\nu/T)^2] \) is a complex. Thus the air–sea flux term in (5) is expressed only in terms of \( T_1 \).

Inserting (2)–(4), and (6) into (5), we obtain the quadratic equation of the frequency \( \sigma \), the solutions of which are the dispersion relations of the coupled modes, given by

\[
\sigma = \frac{1}{2} \left[ \omega_r - c'_2 i - im \gamma_o - \frac{c'_3 (r_2 k + f_2) l i + c'_4 f k^2}{(r_2^2 + f_2^2) r_a} \frac{\nu}{\Gamma} + i \frac{\mu}{2} + 1 \right]
\]

\[
\pm i \left[ \frac{1}{4} \left[ \omega_r + c'_2 i + im \gamma_o + \frac{c'_3 (r_2 k + f_2) l i + c'_4 f k^2}{(r_2^2 + f_2^2) r_a} \frac{\nu}{\Gamma} + i \frac{\mu}{2} + 1 \right] \right]^2 + \frac{\alpha k^2 L_o^2 \left( \frac{\nu}{\Gamma} + i \right)}{r_a^2 \left[ 1 + \left( \frac{\nu}{\Gamma} \right)^2 \right]^{1/2}} \right).
\]

The two solutions of the coupled dispersion relation (7) represent two coupled modes caused by air–sea interactions. The coupled mode is unstable if the imaginary part \( \text{Im}(\sigma) \) of the solution is nonzero and positive.

b. Coupled oceanic Rossby wave mode

The real and imaginary parts of the first solution \( (\sigma_+) \) of (7), which denote frequency and growth rate, respectively, are shown as a function of the zonal wavelength in Figs. 2a and 2c. Note that a meridional wavelength \( 2\pi/l = 6400 \text{ km} \) is specified hereinafter, which is comparable to the meridional scale of the SST and sea level pressure (SLP) anomalies observed in the North Pacific. It can be seen from Fig. 2a that the frequency curve of the coupled mode is close to that of the free oceanic Rossby wave [the dashed line; \( \alpha = 0 \) in (7)], except within a certain range of wavelengths where the coupled mode becomes unstable. The associated growth rate shown in Fig. 2c indicates that this coupled mode grows within a narrow range of wavelengths. The unstable growth rate peaks at a wavelength around 10 500 km, corresponding to the fastest-growing mode. The coupled mode is basically a coupled oceanic Rossby wave modified by air–sea interaction. The air–sea interaction destabilizes the oceanic Rossby wave over a certain range of wavelengths.

The left panel of Fig. 3 shows the phase relationships between the SST anomaly and the upper- and lower-atmospheric streamfunction and upper-oceanic streamfunction anomalies for the fastest-growing coupled mode. One striking feature seen in Figs. 3a and 3b is that for the growing mode, the upper- and lower-atmospheric streamfunction anomalies are both in phase with the SST anomaly. This indicates that the atmospheric response to the SST anomaly is equivalent barotropic with highs (lows) over warm (cold) water. This vertical structure supports the view of many previous studies that the equivalent barotropic response of atmosphere to the thermal forcing and its in-phase relationship with the SST anomaly are necessary conditions to yield a growing mode (GM; Hoskins and Karoly 1981; Palmer and Sun 1985; Peng et al. 1997; Peng and Robinson 2001). The other striking feature shown in Fig. 3c is that the oceanic current anomaly is found to lead the SST anomaly, indicating the role of horizontal advection process in the unstable coupled mode, which will be discussed in the next section.

c. Coupled SST-only mode

The second solution \( (\sigma_-) \) of (7) reflects the other coupled mode induced by the air–sea interaction. It
is the SST-only mode, which is fundamentally determined by the SST evolution (A28). Figures 2b and 2d show the frequency and growth (decay) rate for the mode. It is evident that the SST-only mode is a damping mode with the propagation direction depending on the wavelength. The mode appears stationary at a critical wavelength when damping is the weakest and propagates westward (eastward) when the wavelength is shorter (longer) than this critical wavelength. Since our focus in this study is mainly on the unstable air–sea interaction, we will conclude discussing the damping SST-only mode.

4. Relative roles of different physical processes

We examine below the relative roles of different physical processes in destabilizing coupled oceanic Rossby waves. The main processes we emphasize here are entrainment and advection determining SST evolution and the atmospheric heating profile.

a. Entrainment and advection

Setting \( c_4 = 0 \) and \( \gamma_o = 0 \) to neglect the advection and air–sea flux terms, we only include the effect of entrainment in (7). In this case, an approximate form of the first solution of (7) is given as

\[
\sigma_+ = \omega_r - \frac{\alpha \kappa^2 L_o^2 (\mu^2 + 1)}{\left(1 + \frac{\nu^2}{\Gamma^2}\right)} \left(\omega^2 + c_2^2\right)^2 \frac{(\nu^2 + c_2^2) i + \left(c_2^2 \frac{\nu}{\Gamma} - \omega_r\right)}{\left(1 + \frac{\nu^2}{\Gamma^2}\right)^2 \left(\omega^2 + c_2^2\right)}.
\]

Figure 4b shows the growth rate as a function of wavelength for the coupled oceanic Rossby wave mode for the case of entrainment alone. It can be seen from Fig. 4b that entrainment alone can destabilize the oceanic Rossby wave with a wider range of wavelengths. However, the unstable growth rate in this case is much smaller than that
in the full case, which is illustrated in Fig. 4a (the same as Fig. 2c). The unstable mode in this case also features an equivalent barotropic structure of atmospheric response (the left panel of Fig. 5). This can be explained by the imaginary part of (8):

$$\text{Im}(\sigma) = -\frac{\alpha \kappa^2 L^2 \epsilon_{2} (\frac{\mu}{2} + 1)(\omega_r \frac{\nu}{\Gamma} + c_2)}{r_a \left[ 1 + \left( \frac{\nu}{\Gamma} \right)^2 \right] (\omega_r^2 + c_2^2)}.$$  \hspace{1cm} (9)

Since growth occurs only when the imaginary part of frequency is positive, to satisfy this for (9), we need

$$\left( \frac{\mu}{2} + 1 \right)(\omega_r \frac{\nu}{\Gamma} + c_2) < 0,$$  \hspace{1cm} (10)

where $\omega_r$ is negative. Therefore, the possibility of growth is determined by the quantities $\mu$ and $\nu/\Gamma$. For the fast-growing mode, the atmosphere and the ocean interact adequately and efficiently. The thermal equilibration time scale is thus much shorter than the advection–propagation time scale, that is, $\nu \ll \Gamma$. Thus, the second bracket in (10) has been positive and the necessary condition for growth is reduced to $\mu/2 + 1 < 0$, which means that the atmospheric response is equivalent barotropic.

The effect of advection is isolated in (7) by setting $c_2 = 0$, $c_1 = 0$, $c_5 = 0$, and $\gamma_r = 0$, and the corresponding approximate solution has the form of

$$\sigma = \omega_r + \frac{\alpha \kappa^2 L^2 \epsilon_{2} (\frac{\mu}{2} + 1)(\nu/\Gamma + i)}{r_a \left[ 1 + (\nu/\Gamma)^2 \right] \omega_r},$$  \hspace{1cm} (11)
From its imaginary part, growth can easily be seen occurring also only when the atmospheric response structure is equivalent barotropic ($\mu/2 + 1 < 0$) (also seen in the right panel of Fig. 5). Figure 4c shows the growth rate for the case of advection alone. A striking feature is that this case yields an unstable growing mode with the largest growth rate almost the same as that for the full case, including entrainment shown in Fig. 2c.

Comparing Figs. 4b and 4c with Fig. 4a, we can find that although either entrainment or advection alone can lead to unstable growth of coupled oceanic Rossby wave with similar wavelength dependence, the advection process for determining SST evolution plays a more important role than the entrainment process in our model. This feature can also be confirmed by comparing Fig. 5 with Fig. 3. The phase relationships between the SST anomaly and the upper- and lower-atmospheric streamfunction and upper-oceanic streamfunction anomalies for the fastest-growing coupled mode for the advection-only case shown in the right panel of Fig. 5 are different from that for the entrainment-only case shown in the left panel, but they are identical to that for the full case shown in Fig. 3. All of these results indicate that the advection process in the SST equation dominates the growing mode. Moreover, no matter which process works, the appearance of growing coupled modes is characterized by the equivalent barotropic response of the atmosphere to the thermal forcing with highs (lows) over warm (cold) SST.

b. Atmospheric heating profile

The atmospheric heating profile is a crucial factor that determines how the atmosphere responds to the thermal forcing. For the two-layer quasigeostrophic atmospheric model used in this study, the atmospheric heating that is relevant to the SST anomaly is generally assumed to be located at the middle level. With this configuration of the midlevel-only heating profile, the dynamical features of the unstable coupled mode have been investigated in the last section. Figure 6a shows the phase relationships among SST, heating, and atmospheric response for the growing coupled mode with the midlevel-only heating profile. The atmospheric streamfunction perturbations for the two layers are in phase. The SST anomaly leads the heating by $\pi/2$ phase, while the heating lags the atmospheric perturbation by $\pi/2$ phase. Thus, the unstable coupled mode exhibits equivalent barotropic atmospheric structure with highs over warm SST.

Besides the radiation and latent heat sources that occur in large portion in the midtroposphere, the sensible heating near the air–sea interface in midlatitudes is also an important heating source for the atmosphere. It is equally important to investigate what happens if the heating is assumed to be at the surface. Following (A5), suppose that the surface heating has a Newtonian relaxation form as well,

$$S_s = -\gamma_a \left( \frac{\theta_{as}}{\theta_{a0}} - \frac{\theta^*}{\theta_{a0}} \right),$$

(12)

where $\theta_{as}$ is the atmospheric temperature just above the sea surface. Other quantities are defined to be the same as in (A5). To parameterize $\theta_{as}$, we directly assume a linear dependence of $\theta_{as}$ on both the SST ($T_1$) and the atmospheric temperature in the middle level ($\theta_a$), given by $\theta_{as} = \gamma_1 T_1 + \gamma_2 \theta_a$. Here $\gamma_1$ and $\gamma_2$ are the thermal coupling coefficients with $\gamma_1 = 0.8$ and $\gamma_2 = 0.2$ as standard values (Van der Avoird et al. 2002). Thus, the surface heating can be written as

$$S_s = -\gamma_a \gamma_2 \left( 2\frac{f}{\beta H_a} \psi - \frac{T_1}{\theta_{a0}} \right) = \gamma_2 S.$$

(13)
The above relation implies that the surface heating is proportional to the midlevel heating.

With the configuration of surface-only heating profile, the barotropic and baroclinic atmospheric equations (A1) and (A2) are modified to

\[
\frac{\partial}{\partial x} \nabla^2 \psi + \beta \frac{\partial}{\partial x} \psi + \hat{U} \frac{\partial}{\partial x} \nabla^2 \hat{\psi} = -\frac{gH_a S}{fL_a^2} \quad \text{and} \quad (14)
\]

\[
\hat{U} \frac{\partial}{\partial x} \nabla^2 \hat{\psi} + \beta \frac{\partial}{\partial x} \hat{\psi} + \hat{U} \frac{\partial}{\partial x} \left( \nabla^2 \hat{\psi} - \frac{2}{L_a^2} \hat{\psi} \right) + \beta \frac{\partial}{\partial x} \hat{\psi} = \frac{gH_a S}{fL_a^2} . \quad (15)
\]

The dispersion relations for the coupled model with this surface-only heating assumption still keep the same form as (7), while the definitions for the variables \(\mu, \nu, \text{ and } \Gamma\) are modified accordingly as

\[
\mu = \frac{\hat{U}k^2 + \hat{U}k_a^2 - \beta}{(\hat{U} + \hat{U})k_a^2 - (\beta + \beta)}; \quad \Gamma = \frac{4\gamma_a \gamma_2}{k^2 L_a^2}; \quad \text{and} \quad \nu = (\hat{U} - \hat{U})k\mu + \frac{(\beta - \beta)k}{k^2} + \hat{U}k - \hat{U}k \frac{k_a^2}{k^2} + \hat{\beta} k .
\]

Figure 7 shows the growth rate of coupled oceanic Rossby wave mode as a function of wavelength for the surface-only heating case. Some features of coupled
Rossby wave mode for this heating profile are similar to those for the midlevel-only heating profile. For instance, the coupled oceanic Rossby wave mode becomes unstable over a certain wavelength range, and the advection in SST evolution is more important in generating the growing coupled mode than entrainment. However, with the surface-only heating the phase relationships among SST, surface heating and atmospheric response for the growing coupled mode display distinctive features, as seen in Fig. 6b. The surface heating in this case leads both SST and atmospheric streamfunction perturbations by $\pi/2$ phase, although the atmospheric response is still equivalent barotropic and in phase with the SST anomaly.

Another heating profile considered is that the heating occurs at both atmospheric midlevel and air–sea interface. With this configuration of heating profile, the barotropic and baroclinic atmospheric equations (A1) and (A2) are modified to

$$\begin{align*}
\hat{U} \frac{\partial}{\partial x} \nabla^2 \psi + \hat{\beta} \frac{\partial}{\partial x} \hat{\psi} + \hat{U} \frac{\partial}{\partial x} \nabla^2 \hat{\psi} &= -\frac{g H S}{fL_a^2}, \quad \text{and} \quad (16)
\end{align*}$$

and

$$\begin{align*}
\hat{U} \frac{\partial}{\partial x} \nabla^2 \hat{\psi} + \hat{\beta} \frac{\partial}{\partial x} \hat{\psi} + \hat{U} \frac{\partial}{\partial x} \hat{\psi} &= -\frac{2g H S}{fL_a^2} + \frac{g H S}{fL_a^2}. \quad \text{and} \quad (17)
\end{align*}$$

Accordingly, the variables $\mu$, $\nu$, and $\Gamma$ are redefined as

$$\begin{align*}
\mu &= \frac{\hat{U} \kappa^2 (2 - \gamma_2) + \gamma_2 \hat{\beta} - \hat{U} \kappa^2 \gamma_2}{[\hat{U} (2 - \gamma_2) - \hat{U} \gamma_2] \kappa^2 - [(2 - \gamma_2) \hat{\beta} - \gamma_2 \hat{\beta}]}; \\
\nu &= -\hat{U} k \mu + \frac{\hat{\beta} k}{\kappa^2} \mu + \hat{U} k; \quad \text{and} \quad \Gamma = \frac{2 \gamma_2 \gamma_2}{\kappa^2 L_a^2}.
\end{align*}$$

Figure 8 shows the growth rate of coupled oceanic Rossby modes as a function of wavelength for the midlevel-plus-surface-heating case. In this case, the coupled oceanic Rossby wave can become unstable; however, the wavelength range for growing tends to be much narrower and, more importantly, the unstable growth rate is considerably reduced when compared with the midlevel-only heating case as well as the surface-only
heating case. Moreover, the contribution of advection process to the growth rate is comparable to that of the entrainment process. All of these results suggest that, given the midlevel heating, the inclusion of the surface heating largely reduces the growth rate and stabilizes the coupled oceanic Rossby wave. Furthermore, in this case the atmospheric perturbation is no longer exactly in phase with the SST anomaly while either the midlevel heating or the surface heating lags the atmospheric response by $\pi/2$ phase (Fig. 6c).

5. Comparison and verification

As mentioned in section 2, our model can be reduced to GM’s model. Figure 9 displays the growth rate as a function of wavelength for GM’s model. It can be clearly seen that there is a discrepancy between the result of our model and that of GM’s model. In our model the advection process is found to play a dominant role in the growing coupled mode (Figs. 4 and 5) and the oceanic current anomaly is found to lead the SST anomaly (Fig. 3c), whereas the entrainment process is dominant in GM’s model (Fig. 9) and the oceanic current anomaly is found to be in phase with the SST anomaly (Fig. 3f). This discrepancy is mainly due to the different considerations of the processes that determine SST evolution in the two models. As described in the appendix, major improvements of our model lie mainly in the ocean part. A thin entrainment layer, the influence of anomalous entrainment on the SST evolution and the dynamical difference between the mixed layer and thermocline layer (i.e., Ekman flow), is included into our model. In particular, the assumption of a thin entrainment layer in this study makes it more reasonable to consider the contributions of both entrainment and advection to the SST evolution. However, the assumption of GM’s model that entrainment occurs in the whole upper layer seems to overestimate the contribution of entrainment to the SST evolution.

To verify the analytical results of our model as well as GM’s model, we identify the ocean–atmosphere pattern of decadal variability in the North Pacific with singular value decomposition (SVD) analysis. The data used here are taken from the Global Sea Ice and Sea Surface Temperature dataset (GISST), the oceanic analysis dataset Simple Ocean Data Assimilation version 6 (SODA 6), and National Centers for Environmental Prediction–National
Center for Atmospheric Research (NCEP–NCAR) atmospheric reanalysis dataset for 1950–98. The SVD is performed between the SST anomaly and the atmospheric geopotential height anomaly at different levels over the North Pacific region (\( \sim 20^\circ-60^\circ\)N, \( \sim 110^\circ\)E–110\(^\circ\)W) as well as between the SST anomaly and the upper-ocean current anomaly. Figure 10 displays the heterogeneous correlation pattern of the leading SVD mode for the observed SST anomaly (shaded), the atmospheric geopotential height anomalies at different levels (contours in Figs. 10a–c), and the surface oceanic current anomaly (streamline in Fig. 10d) in the North Pacific. Note that the heterogeneous correlation means the correlation between the time series of the SVD mode for one variable (say, the SST) and the anomaly field for the other variable (say, the geopotential height). From Figs. 10a–c, we do see equivalent barotropic atmospheric structure and in-phase relation between the atmospheric anomaly and the SST anomaly with highs over warm water. More importantly, we see clearly in Fig. 10d that the surface oceanic current anomaly leads the SST anomaly by nearly \( \pi/2 \) phase, suggesting the direct observational evidence that meridional advection rather than entrainment dominates the SST anomaly in the coupled air–sea pattern.

As a verification, similar coupled air–sea patterns for the fastest-growing mode are illustrated for our model in the left panel of Fig. 3 and for GM’s model in the right panel of Fig. 3. Overall, we find that the two models give similar atmospheric response patterns with equivalent barotropic structure and similar phases relative to the SST anomaly, which qualitatively agrees with observations as shown in Figs. 10a–c. However, the phase relationship between the oceanic current anomaly and the SST anomaly for our model significantly differs from that for GM’s model. The oceanic current anomaly is found to lead the SST anomaly in our model (left panel of Fig. 3), which is similar to the observation (Fig. 10d), whereas a near-in-phase relation is found in GM’s model (right panel of Fig. 3). The former suggests the dominant role of meridional advection for determining the SST anomaly, while the latter indicates that of entrainment.

6. Conclusions and discussion

Following GM, an improved midlatitude coupled ocean–atmosphere model linearized around a basic state is developed in this study. The atmospheric component of the coupled model is a two-layer quasigeostrophic atmosphere linearized around a basic zonal flow with vertical shear. The oceanic component of the coupled model is a quasigeostrophic upper ocean that consists of a constant-depth mixed layer, a thin entrainment layer, and a thermocline layer. A linearized, relatively full mixed layer temperature equation is considered to describe the SST evolution. The atmospheric heating with a fixed heating profile (heating at either midlevel or surface or both levels) is parameterized in terms of both SST anomaly and atmospheric temperature anomaly.

With the coupled model of intermediate complexity, dynamical features of unstable midlatitude ocean–atmosphere interaction are investigated. The coupled model developed in this study exhibits two types of coupled modes: the coupled oceanic Rossby wave mode and the SST-only mode. The SST-only mode decays over the entire range of wavelengths, whereas the coupled oceanic Rossby wave mode, which is the oceanic Rossby wave modulated by the midlatitude air–sea interaction, destabilizes over a certain range of wavelengths (\( \sim 10 \) 500 km) when the atmospheric response to the heating is equivalent barotropic. For the fastest-growing coupled oceanic Rossby wave mode, the SST anomaly leads the heating by \( \pi/2 \) phase while the heating lags the atmospheric perturbation by \( \pi/2 \) phase. The unstable coupled oceanic Rossby mode thus exhibits...
equivalent barotropic atmospheric structure with highs (lows) over warm (cold) SST.

The relative roles of different physical processes in destabilizing the coupled oceanic Rossby wave are examined. The main processes emphasized in this study include entrainment and advection, that determine SST evolution, and the atmospheric heating profile, respectively. Although either entrainment or advection can lead to the unstable growth of the coupled oceanic Rossby wave with similar wavelength dependence, the advection process for determining SST evolution plays a more important role than the entrainment process. The advection process in the SST equation dominates the growing mode. As a verification of the analytical results of our coupled model, the typical ocean–atmosphere pattern of decadal variability in the North Pacific is statistically identified with singular value decomposition (SVD) analysis. It is demonstrated that the structure of growing coupled mode derived from our model coincides more closely with reality, although the coupled model seems to be simple.

The atmospheric heating profile is a crucial factor that determines how the atmosphere responds to the thermal forcing. The above conclusions are deduced from the midlevel heating. The role of surface heating in unstable coupled mode is also examined in this study. With the surface-only heating the phase relationships among SST, surface heating, and atmospheric response for the growing coupled mode display distinctive features. In this case, the surface heating leads both SST and atmospheric streamfunction perturbations by \( \pi/2 \) phase, although the atmospheric response is still equivalent barotropic and in phase with the SST anomaly. Furthermore, given the midlevel heating, the inclusion of the surface heating largely reduces the growth rate and stabilizes the coupled oceanic Rossby wave. In this case the atmospheric perturbation is no longer exactly in phase with the SST anomaly while either the midlevel heating or the surface heating lags the atmospheric response by \( \pi/2 \) phase.

It should be mentioned that the conclusion that the advection process plays a dominant role in the growing coupled mode is significantly different from that derived from GM in which the entrainment process was found to be a dominant contributor to the growing coupled mode. This discrepancy is mainly due to the different consideration of the processes that determine SST evolution. In the present study, as described in the appendix, we modify GM’s model mainly in the ocean part by introducing a thin entrainment layer, the influence of anomalous entrainment and the Ekman flow on the SST evolution. In particular, the assumption of a thin entrainment layer in this study makes it more reasonable to consider the contributions of both entrainment and advection to the SST evolution. However, the GM’s assumption that entrainment...
acts over the whole upper layer seems to overestimate the contribution of entrainment to the SST evolution.

The results of the improved dynamical analysis in this study could provide reasonable theoretical basis for the understanding of positive feedback mechanisms of ocean–atmosphere interaction in the midlatitudes. However, the drawbacks of this study lies in that the model used is still highly simplified, from which the results are still considerably qualitative despite easily being physically understood. In particular, some key physical processes involved in the midlatitude air–sea interaction are empirically parameterized. For example, the atmospheric heating together with its profile is parameterized to be simply related to the SST anomaly. In fact, the relationship between the atmospheric heating source and the SST anomaly is complicated in the midlatitudes. How to precisely specify the relationship in a simplified model remains unsolved. Moreover, the parameters used in present coupled model are actually difficult to ascertain. Change in some parameters may cause a change in results. Further analysis of the sensitivity of the dynamical behavior of the coupled model to those key parameters is needed. Last, the linearization of the present coupled model about a fixed basic state eliminates the nonlinearity of either the ocean or the atmosphere in the midlatitudes, where interaction of waves and the mean flow is prominent. Further improvement with the inclusion of feedback of wave activity on the mean flow in the midlatitude model is valuable. There have been a few studies to explore the dynamical behavior of midlatitude air–sea interaction with the nonlinear configuration of a coupled model (Hogg et al. 2006; Kravtsov et al. 2007).

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APPENDIX

Coupled Model Equations

a. Atmospheric equations

The atmospheric component of the coupled model is described by a simple two-layer quasigeostrophic model, with a lid upper boundary and a rigid ocean surface (see Fig. 1). The heating is imagined to occur only in the middle troposphere. Atmospheric fluctuations are assumed to be a steady-state response to the heating. Following GM, the two-layer steady-state QGPV equations for the atmosphere, linearized around a basic zonal wind with vertical shear, can be written in terms of the following barotropic and baroclinic parts:

\[ \dot{U} \frac{\partial}{\partial x} \nabla^2 \psi + \beta \frac{\partial}{\partial x} \dot{\psi} + \dot{U} \frac{\partial}{\partial x} \nabla^2 \psi = 0 \text{ and } \]  
\[ \frac{\partial}{\partial x} \nabla^2 \psi + \beta \frac{\partial}{\partial x} \dot{\psi} + \frac{\partial^2}{\partial x^2} \left( \nabla^2 \psi - \frac{2}{L_a^2} \psi \right) + \beta \frac{\partial}{\partial x} \dot{\psi} \]
\[ = -\frac{2gH_a S}{fL_a^2}, \]  

where \( \psi \) is the streamfunction, \( U \) is the basic zonal wind, and \( S \) represents the heating at the interface between two atmospheric layers. Other relevant parameters are defined by \( \beta = 2\beta, \tilde{\beta} = 2\tilde{U}/L_a^2, \) and \( L_a = (N_a H_a)/(2f) \), where \( L_a \) is the atmospheric baroclinic Rossby radius of deformation with \( N_a \) the atmospheric Brunt–Väisälä buoyancy frequency; \( H_a \) is the depth of the troposphere. The barotropic and baroclinic components for the streamfunction and the basic zonal wind are defined by

\[ \dot{\psi} = \psi_1 + \psi_2; \dot{\psi} = \psi_1 - \psi_2 \text{ and } \]
\[ \dot{U} = U_1 + U_2; \dot{U} = U_1 - U_2. \]

The heating term \( S \) is prescribed to have the form of Newtonian relaxation of the atmospheric potential temperature perturbation at middle level \( \theta_a \) to a radiative–convective equilibrium temperature anomaly \( \theta_a^* \), given by

\[ S = -\gamma_a \left( \frac{\theta_a - \theta_a^*}{\theta_a^*} \right) = -\gamma_a \left( \frac{2f \tilde{U} \dot{\psi}}{gH_a} - \tilde{\theta}_a^* \right), \]  

where \( \theta_a \) has been converted to \( \tilde{\theta} \) using the thermal wind relation

\[ \theta_a = \frac{2f \theta_a^*}{gH_a} \tilde{\theta}, \]  

where \( \theta_a^* \) is a typical value of \( \theta_a \), \( \gamma_a \) is the atmospheric air–sea exchange parameter, and \( \tilde{\theta}_a^* \) is set to be a function of the SST anomaly. With (A5), (A2) can be rewritten as

\[ \dot{U} \frac{\partial}{\partial x} \nabla^2 \psi + \tilde{\beta} \frac{\partial}{\partial x} \dot{\psi} + \dot{U} \frac{\partial}{\partial x} \nabla^2 \psi \]
\[ = \frac{4\gamma_a}{L_a^2} \left( \tilde{\theta} - \frac{1}{r_a} \tilde{\theta}_a^* \right), \]  

where \( r_a = (2f \theta_a^*)/(gH_a) \).
b. Oceanic equations

The oceanic component of the coupled model contains a constant-depth mixed layer \((H_1)\), a thin entrainment layer \((\Delta h)\), a thermocline layer (with a mean depth \(H_2)\), and a resting deep layer (see Fig. 1). The first three layers constitute a moving upper layer (with a mean depth \(H = H_1 + H_2)\) that satisfies the quasigeostrophic dynamics. The motion for the upper layer represented by the first baroclinic Rossby mode is controlled by a linear QGPV equation with a rest basic state driven by the curl of the surface wind stress. With longwave approximation, it evolves according to

\[
-\frac{1}{L_o^2} \frac{\partial}{\partial t} \psi_o + \rho_o \frac{\partial}{\partial x} \psi_o = \frac{1}{\rho_o} \mathbf{k} \cdot \mathbf{\nabla} \otimes \frac{\tau}{H},
\]

where \(\psi_o\) is the oceanic geostrophic streamfunction of the upper layer, \(L_o\) is the oceanic baroclinic Rossby radius of deformation, \(\rho_o\) is a constant reference value of upper-ocean density, and \(\tau\) is the surface wind stress.

The motion in the mixed layer that is ignored in GM’s model is considered in this study. Defining an anomalous vertical shear current \(\mathbf{V}_e\) between the mixed layer and the quasigeostrophic upper ocean (i.e., Ekman flow), we can express the horizontal current perturbation in the mixed layer simply by

\[
\mathbf{V}_1 = \mathbf{k} \otimes \mathbf{\nabla} \psi_o + \left(1 - \frac{H_1}{H}\right) \mathbf{V}_e,
\]

where \(\mathbf{V}_e\) can be crudely represented in terms of a steady-state response to the surface wind stress (see Zebiak and Cane 1987; Wang et al. 1995; Yang et al. 1996, 1997) by

\[
f \mathbf{k} \otimes \mathbf{V}_s = \frac{\tau}{H_1 \rho_o} - r_3 \mathbf{V}_s,
\]

where \(r_3\) is a Rayleigh damping coefficient. With inclusion of such a motion in the mixed layer, the perturbation of the entrainment velocity at the base of the mixed layer that links the mixed layer and the thermocline can be diagnostically determined by

\[
W_e = H_1 \mathbf{V} \cdot \mathbf{V}_1 = \frac{H_1 (H - H_1)}{H} \mathbf{V} \cdot \mathbf{V}_s.
\]

Since the mixed layer directly links air–sea interaction, the proper simulation of the mixed layer temperature (identical to SST) evolution is crucial to the behavior of a coupled model. Following Frankignoul (1985), here we start with a full evolution equation of the mixed layer total temperature:

\[
\frac{\partial}{\partial t} T_1 + \mathbf{V}_1 \cdot \nabla T_1 = -\frac{W_1^2}{H_1} (T_1 - T_e) + \frac{\Delta Q}{\rho_o C_p H_1} + \gamma \nabla^2 T_1,
\]

where \(T_1\) and \(T_e\) denote the temperature in the mixed layer and at the base of the entrainment layer, respectively; \(\Delta Q\) is the net downward heat flux in the mixed layer, and \(C_p\) is specific heat; \(\gamma\) is the horizontal mixing coefficient. Assuming that the vertical temperature gradient in the entrainment layer is equal to that in the thermocline layer (Wang et al. 1995), the entrained temperature is then expressed as

\[
T_e = T_1 - \Delta h \frac{T_1 - T_r}{\eta},
\]

where \(\eta\) is an instantaneous depth of the thermocline layer and \(T_r\) is the reference temperature for the resting deep layer. Decomposing each variable into a mean and an anomaly,

\[
T_1 = T_{1} + \bar{T}_1; \quad \mathbf{V}_1 = \mathbf{V}_{1} + \mathbf{V}_{1}'; \quad W_e = W_{e} + W_{e}'; \quad \eta = H_2 + \eta'; \quad \text{and} \quad \Delta Q = \Delta Q' + \Delta Q''.
\]

With (A13), (A12) is linearized as

\[
\frac{\partial}{\partial t} T_1 + \mathbf{V}_1 \cdot \nabla T_1 + c_1 \eta' - c_2 T_1' - c_3 W_e' + \frac{\Delta Q'}{\rho_o C_p H_1} + \gamma \nabla^2 T_1',
\]

where \(c_1, c_2,\) and \(c_3\) are all positive coefficients, defined by

\[
c_1 = \frac{W_e \Delta h}{H_1 H_2} (T_1 - T_r); \quad c_2 = \frac{W_e \Delta h}{H_1 H_2}; \quad \text{and} \quad c_3 = \frac{\Delta h}{H_1 H_2} (T_1 - \bar{T}_1).
\]

Equation (A15) is a relatively full expression of the SST anomaly evolution equation, and its right-hand terms describe the effect of the following processes: (1) the horizontal advection of mean temperature gradient by anomalous current, (2) the vertical advection of the subsurface temperature anomaly (due to the thermocline depth anomaly) by mean entrainment velocity, (3) the vertical advection of the mixed layer temperature anomaly by mean entrainment velocity, (4) the vertical advection of mean temperature gradient by anomalous entrainment velocity, (5) the net heat flux anomaly, and (6) the horizontal diffusion. Employing the quasigeostrophic linear equilibrium equation \(f \nabla \psi_o = b \nabla^2 \eta'\) for
the upper-ocean layer, the anomalous depth of the thermocline can be expressed in terms of the streamfunction for the upper layer as
\[ \eta' = \frac{f}{b} \psi'_o, \quad (A17) \]
where \( b = \partial p/\partial z = \lambda g(T_1 - T_0) \) is the buoyancy, with \( \lambda \) as the thermal expansion coefficient of seawater. Neglecting the mean zonal SST gradient and the horizontal mixing, and dropping the primes of the notations, with (A9) and (A17), (A15) can be rewritten as
\[ \frac{\partial T_1}{\partial t} = c_1 \frac{f}{b} \psi_o - c_2 T_1 - c_3 W_x + c_4 \frac{\partial \psi_o}{\partial x} + \frac{H - H_1}{H} v_x + \frac{\Delta Q}{\rho_r C_p H_1}, \quad (A18) \]
where \( c_4 \) is the mean meridional SST gradient, given by
\[ c_4 = -\frac{\partial T_1}{\partial y}. \quad (A19) \]

### c. Air–sea coupling

The atmosphere and the ocean are coupled as follows: an initial SST anomaly results in anomalous heating that would change the atmospheric circulation, and thus leads to an anomalous wind field. The anomalous wind stress then drives the upper-ocean motions, which in turn alter the initial SST anomaly. As we assumed, the heating perturbation is induced by the SST anomaly, thus the radiative–convective equilibrium temperature anomaly \( \theta^e_o \) in (A7) is a function of the SST anomaly. Here, for simplicity, we set them equal, by
\[ \theta^e_o = T_1. \quad (A20) \]

The term \( \Delta Q \) is the counteraction of \( S \) because of the SST anomaly, which can be expressed as
\[ \frac{\Delta Q}{\rho_r C_p H_1} = -\gamma_o (T_1 - r_o \psi^e), \quad (A21) \]
where \( \gamma_o \) is the oceanic air–sea heat exchange parameter.

Defining an anomalous surface streamfunction \( \psi'_s \), which is expressed in terms of \( \psi \) and \( \psi^e \), using linear interpolation, as \( \psi'_s = \psi/2 - \psi^e \), the surface wind stress anomaly is then written as
\[ \frac{\tau}{\rho_r H} = \alpha k \wedge \nabla \psi'_s = \alpha k \wedge \nabla (\psi/2 - \psi^e), \quad (A22) \]
where \( \alpha \) is a dynamical coupling coefficient. Thus, by inserting the coupling relations (A20)–(A22) into (A7), (A8), (A10), and (A18), together with (A1) and (A11), we get the following final closed equations for the coupled model:
\[ \begin{align*}
U \frac{\partial}{\partial x} \psi^s + \bar{\beta} \frac{\partial}{\partial x} \psi^s + U \frac{\partial}{\partial x} \psi^e = 0, \quad (A23) \\
\bar{U} \frac{\partial}{\partial x} \psi^s + \bar{\beta} \frac{\partial}{\partial x} \psi^s + \bar{U} \frac{\partial}{\partial x} \left( \psi^e - \frac{2}{L_a} \psi^e \right) + \bar{\beta} \frac{\partial}{\partial x} \psi^e = \frac{4\gamma^s}{L_a^2} \left( \psi - r_o T_1 \right), \\
fv^s = \frac{\alpha}{H_1} \frac{\partial}{\partial x} \left( \psi - \psi^e \right) + r_s u^s, \\
fu^s = \frac{\alpha}{H_1} \frac{\partial}{\partial x} \left( \psi - \psi^e \right) - r_s v^s, \\
W_e = \frac{H (H - H_1)}{H} \left( \frac{\partial u^s}{\partial x} + \frac{\partial v^s}{\partial y} \right), \\
-\frac{1}{L_o^2} \frac{\partial}{\partial t} \psi'_o + \beta \frac{\partial}{\partial x} \psi'_o = \alpha \nabla^2 (\psi^s + \psi^e), \quad (A26) \\
\frac{\partial}{\partial t} T_1 = c_1 \frac{f}{b} \psi_o - c_2 T_1 - c_3 W_x + c_4 \frac{\partial \psi_o}{\partial x} + \frac{H - H_1}{H} v_x + \gamma_o (T_1 - r_o \psi^e). \quad (A28)
\end{align*} \]

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