Bias in Differential Reflectivity due to Cross Coupling through the Radiation Patterns of Polarimetric Weather Radars

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ABSTRACT
Examined is bias in differential reflectivity and its effect on estimates of rain rate due to coupling of the vertically and horizontally polarized fields through the radiation patterns. To that end, a brief review of the effects of the bias on quantitative rainfall measurements is given. Suggestions for tolerable values of this bias are made. Of utmost interest is the bias produced by radars simultaneously transmitting horizontally and vertically polarized fields, as this configuration has been chosen for pending upgrades to the U.S. national network of radars (Weather Surveillance Radar-1988 Doppler; WSR-88D). The bias strongly depends on the cross-polar radiation pattern. Two patterns, documented in the literature, are considered.

1. Introduction
The proposed embodiment of dual linear polarization technology on the Weather Surveillance Radar-1988 Doppler (WSR-88D) weather radars is the mode whereby horizontal (H) and vertical (V) polarizations are transmitted and received simultaneously (Doviak et al. 2000). This mode is sometimes referred to as “hybrid” (Wang et al. 2006); to shorten notation, we label it SHV (for simultaneous horizontal and vertical). The U.S. National Weather Service is slated to begin retrofitting its WSR-88Ds with this mode in about 2010. By far, the overriding reason for choosing the SHV mode is its total transparency to all the current automated algorithms used in the radar network. That is, algorithms used to detect mesocyclones and tornadoes, track storms, etc. will continue to accept data from the horizontal channel, process these, and produce products as is presently done (Doviak and Zrnić 1998). The only minor difference is a 3-dB loss in the signal-to-noise (SNR) ratio, but the effects of this loss on algorithm performance will be mitigated by using thresholds that depend on signal coherency (Ivić et al. 2009).

Advantages of the SHV mode are 1) direct measurement of the cross correlation between the copolar signals, 2) 360° unambiguous span for differential phase measurement, 3) decoupling of the differential phase and Doppler velocity measurements, 4) smaller error of estimates, 5) no degradation of the performance of the ground clutter filters, and 6) avoidance of a costly high-power microwave ferrite switch and its associated problems. Nonetheless, there are also disadvantages. For example, if hydrometeors along a propagation path have a mean canting angle, Sachidananda and Zrnić (1985) show bias errors in differential reflectivity $Z_{DR}$ can be an order of magnitude larger than errors ensuing from the mode whereby H and V fields are alternately transmitted (i.e., the AHV mode). Furthermore, as demonstrated herein, the bias associated with the SHV mode depends on the cross-polar radiation to the first order whereas second-order terms are important for the AHV mode. Finally, the SHV mode is not fully polarimetric.
because it precludes cross-polar measurements. On the other hand, cross-polar SNR (measured using the AHV mode) is typically weak, limiting this mode’s utility to high-reflectivity regions.

The effects on polarimetric variables of cross-polar radiation due to coupling within the radar, feed horn misalignment, and hydrometeor canting along the propagation path and in the resolution volume have been quantified by Doviak and Zrnić (1998) and Doviak et al. (2000). These authors also presented the copolar and cross-polar radiation patterns made on the National Severe Storms Laboratory’s (NSSL) research and development WSR-88D (i.e., KOUN, sited near Norman, Oklahoma), but did not quantify polarimetric parameter biases due to the coupling of copolar and cross-polar radiation patterns.

The effects of radiation pattern coupling on the measurement of polarimetric variables were first examined by Chandrasekar and Keeler (1993), specifically for the AHV mode. A detailed investigation and explanation of the effects of pattern coupling on precipitation measurements is presented in the paper by Moisseev et al. (2002), who argue in favor of incorporating the patterns into a calibration procedure using vertically pointed measurements in light rain. Although cross-polar radiation affects the accuracy of polarimetric measurements, neither paper addresses these accuracies for the SHV mode.

Even if there is no coupling of the H and V channels within the radar and its antenna (i.e., no coupling in the absence of backscatter), measurements of polarimetric parameters using the SHV mode are affected by depolarization due to propagation through oriented scatterers (Ryzhkov and Zrnić 2007). The effects for various types of precipitation were thoroughly investigated by Wang et al. (2006), who indicate that the biases in all polarimetric variables increase due to increases in the cross-polarized echoes. Because $Z_{DR}$ is a principal variable for estimating rain rate, we examine $Z_{DR}$ biases caused by coupling of copolar and cross-polar patterns.

Hubbert et al. (2010a) have evaluated the $Z_{DR}$ bias caused by depolarizing media. However, even if media are not depolarizing (e.g., propagation paths filled with oblate drops having a vertical axis of symmetry), a differential phase shift caused by precipitation along the propagation path will influence the level of $Z_{DR}$ bias. They have also computed the bias caused by cross-polar to copolar pattern coupling assumed to be uniform over the significant part of the copolar pattern. Under this assumption of uniform coupling, Hubbert et al. (2010a) were also able to relate pattern coupling to the lower limit of the linear depolarization ratio measurements. These results are backed by experiment indicating that the $Z_{DR}$ bias (up to 0.27 dB) observed in the SHV mode is much larger compared to that observed in the AHV mode (Hubbert et al. 2010b).

Wang and Chandrasekar (2006) investigated biases in the polarimetric variables caused by the cross-polar pattern. They have developed pertinent equations building on the formalism in Bringi and Chandrasekar (2001) and quantified biases for a wide range of general conditions. Moreover, they present curves for the upper bounds of the errors as a function of the precipitation type. In its essence, their methodology and ours are similar. But we examine causes of cross-polar radiation, consider theoretical cross-polar patterns, account for differences in the angular dependence of cross-polar and copolar radiation, and reduce the theoretical expressions of the $Z_{DR}$ bias to simple compact forms. To quantify the effects, we approximate radiation patterns with Gaussian shapes for two types of cross-polar radiation. Applying the methodology to these patterns, we obtain the dependence of error bounds on the relative levels of copolar and cross-polar radiation, on the differences in the functional dependence of these patterns on spherical angles, as well as on differential reflectivity.

Biases in differential reflectivity caused by cross-polar and copolar radiation coupling are examined with the aim of setting reasonable limits on cross-polar radiation pattern. In section 2 we set and justify a bound to the $Z_{DR}$ bias based on the accuracy of rain-rate measurements, and use that bound to derive limits on the cross-polar radiation pattern. Section 3 quantifies the relation between the cross-polar coupling levels and bias, and includes examples of measured patterns. Section 4 compares the $Z_{DR}$ biases in the SHV and AHV modes.

2. Effects of $Z_{DR}$ bias on rain-rate measurements

Accurate polarimetric measurement has two purposes. One is to allow the correct classification of precipitation, and the other is to improve quantitative precipitation estimation. In fuzzy logic classification (Zrnić et al. 2001), performance depends on $Z_{DR}$ through the membership (weighting) functions $W_i(Z, Z_{DR}, \text{etc.})$. The effects of the $Z_{DR}$ bias on classification can be reduced by appropriately broadening the membership functions. Therefore, accurate rainfall measurement imposes a more stringent requirement on the bias of $Z_{DR}$.

To compute light rain rates (i.e., <6 mm h$^{-1}$), the following relation has been proposed for the network of WSR-88Ds (Ryzhkov et al. 2005a):

$$ R = \frac{1.70 \times 10^{-2} Z_h^{0.714}}{0.4 + 5.0|Z_{dr}|^{-1/3}} \text{ (mm h}^{-1}) \tag{1} $$

where $Z_h$ is in units of mm$^6$ m$^{-3}$, $Z_{H}(\text{dB}) < 36$ dBZ, $Z_{dr} = 10^{0.4|Z_{dr}|}$, and $Z_{DR}$ is in decibels. We focus our attention...
on this rain-rate regime because it is affected more by the $Z_{DR}$ bias. Assuming no error in $Z_b$, the fractional bias, $\Delta R/R$, in the rain rate is

$$\Delta R/R = f(Z_{DR})/f(Z_{DRb}) - 1,$$  \hspace{1cm} (2a)

where

$$f(Z_{DR}) = 0.4 + 5.0|10^{0.1Z_{DR}} - 1|^{1.3}$$  \hspace{1cm} (2b)

and $Z_{DRb} = Z_{DR} + \delta Z_{DR}$ is the biased differential reflectivity. It follows from (2) that the fractional error is slightly larger if the decibel bias in the differential reflectivity is negative. Hence, the fractional biases in $R$ are plotted (Fig. 1) for three negative values of $Z_{DR}$ bias.

Implications of bias can be assessed by comparing the polarimetric estimates of $R$ with those obtained by using commonly accepted $R(Z)$ relations. For example, a $\pm 1\text{ dB}$ bias (e.g., due to an error in calibration) in estimating $Z$ results in about a $\pm 15\%$ fractional rain-rate bias if the Marshall–Palmer $R(Z)$ relation is used. For such a stand-alone relation (i.e., no adjustment with gauge data), the RMS errors are about $35\%$ (Brandes et al. 2002; Balakrishnan et al. 1989; Ryzhkov and Zrnić 1995). But, with judicious use of polarimetric data, $R$ errors could be reduced to between $15\%$ and $22\%$ (Zhang et al. 2001; Ryzhkov et al. 2005b; Matrosov et al. 2002). Thus, it is reasonable to strive to keep $\Delta R/R$ less than about $20\%$, implying that the absolute bias in the differential reflectivity should be less than $0.15 \text{ dB}$. As $Z_{DR}$ increases, it takes larger $\delta Z_{DR}$ to produce the same fractional bias in rain rate. This suggests that at larger $Z_{DR}$ a larger $\delta Z_{DR}$ could be tolerated. Two independent mechanisms produce the $Z_{DR}$ bias. One is a small but constant offset due to calibration error [this can be kept within $\pm 0.1 \text{ dB}$; Zrnić et al. (2006)], the other is the presence of cross-polar radiation. The bias $\delta Z_{DR}$ depends (section 3) on $Z_{DR}$ as well as on $\beta$, the phase difference between the transmitted H, V copolar radiation; $\gamma$, the phase difference between copolar and cross-polar patterns; and the total differential phase $\Phi_{DP}$ along the propagation paths.

3. $Z_{DR}$ bias due to the cross-polar radiation pattern in the SHV mode

a. An expression for the bias

Consider a circularly symmetric center-fed parabolic reflector antenna and a uniform distribution of scatterers. Performance characteristics of such antennas for dual-polarization radars are discussed by Bringi and Chandrasekar (2001, section 6.2). These authors provide an error budget and formulas for biases applicable to the AHV mode. With similar simplification, but extending the analysis to cross-polar patterns that are different than the copolar pattern, we formulate herein equations for the $Z_{DR}$ bias incurred with the SHV mode, and in section 4 we present bias equations for the AHV mode.

The effects on $Z_{DR}$ will be quantified under the following conditions: 1) the intrinsic $Z_{DR}$ is produced by oblate scatterers having zero canting angles so the off-diagonal terms of the backscattering matrix are zero; 2) the amplitudes of the H and V copolar radiation fields at the antenna are matched, but there is a differential phase $\beta$ between the two; and 3) differential attenuation along the path of propagation can, for most observations at 10-cm wavelengths, be neglected, but $\Phi_{DP}$ cannot be ignored. To simplify the notation, $\Phi_{DP}$ is incorporated into the backscattering matrix $S$ (i.e., $\Phi_{DP}$ is merged with the scatterer’s backscatter differential phase). Furthermore, it is not necessary to include the resolution volume depth; thus, the function $F$ (Doviak and Zrnić 2006, section 8.5.2.2), weighting the polarimetric properties of a scatterer, is only proportional to the intensity and phase of the radiation pattern at the spherical angles $\theta$, $\phi$ relative to the beam axes.

With these conditions we write, for the SHV mode, the matrix equation,

$$\begin{bmatrix} \delta V_h \\ \delta V_v \end{bmatrix} = V = F^T S F E_i,$$

$$= \begin{bmatrix} F_{hh} & F_{hv} \\ F_{hv} & F_{vv} \end{bmatrix} \begin{bmatrix} s_{hh} & 0 \\ 0 & s_{vv} \end{bmatrix} \begin{bmatrix} F_{hh} & F_{hv} \\ F_{hv} & F_{vv} \end{bmatrix} \begin{bmatrix} 1 \\ e^{i\beta} \end{bmatrix},$$  \hspace{1cm} (3)

for the received H and V channel incremental voltages generated by the scatterer. The superscript “T” denotes the transpose matrix, and $E_i$ is the transmitted electric field at the H, V ports of the feed horn. In addition, $F_{hv}$ is proportional to the H radiated electric field if the V port
is energized, and vice versa for $F_{vh}$. Constants of proportionality, which would make this equation dimensionally correct, and the arguments of $F_{ij}$ and $s_{ij}$ are omitted to shorten the notation; these omissions have no effect whatsoever on our results. The pattern functions $F_{ij}$ are not normalized but contain the peak power gain $g_{ij}$ so that

$$F_{ij}(\theta, \phi) = \sqrt{g_{ij}} f_{ij}(\theta, \phi). \quad (4)$$

It is further stipulated that $F_{hh}$ is a real function (i.e., has zero reference phase), but $F_{hv}$, $F_{vh}$ are complex (i.e., $F_{hv}$ and $F_{vh}$ have phases $\gamma_{hv}$ and $\gamma_{vh}$ relative to the phase of copolar H). Wang and Chandrasekar (2006) also consider the phase differences between the copolar and cross-polar patterns. The effects of the phase difference due to differential phase shifts within the receiver are neglected as they have no bearing on the results reported herein.

Executing the matrix multiplication in (3), the following equation ensues:

$$\begin{align*}
\delta V_h &= |s_{hh} F_{hh} (F_{hh} + F_{hv} e^{i\beta}) + s_{vh} F_{vh} (F_{vh} + F_{hh} e^{i\beta})| \\
\delta V_v &= |s_{hh} F_{hv} (F_{hv} + F_{hh} e^{i\beta}) + s_{vh} F_{vh} (F_{vh} + F_{hh} e^{i\beta})|.
\end{align*} \quad (5)$$

Of interest are the powers from the ensemble of scatterers weighted by pattern functions. Thus, we will take the ensemble average, $\langle |\delta V_h|^2 \rangle$, and integrate it over the pattern functions to obtain the power received in the H channel:

$$P_h \sim \int_{\theta, \phi} \langle |\delta V_h|^2 \rangle \sin \theta \, d\theta \, d\phi = \int_{\Omega} \langle |s_{hh} F_{hh} (F_{hh} + F_{hv} e^{i\beta}) + s_{vh} F_{vh} (F_{vh} + F_{hh} e^{i\beta})|^2 \rangle \, d\Omega, \quad (6)$$

where the angle brackets indicate the ensemble average over the distribution of the scatterers’ attributes [Doviak and Zrnić (2006), Eq. (8.45)]. To shorten the notation, the integral over $\theta$ and $\phi$ is replaced with the integral over the solid angle $\Omega$. A very similar expression for $P_v$ follows from the second row of (5).

The integral in (6) can be expressed as the sum of three terms, of which the first only containing $s_{hh}$ is

$$\int_{\Omega} \langle |s_{hh} F_{hh} (F_{hh} + F_{hv} e^{i\beta})|^2 \rangle \, d\Omega = \langle |s_{hh}|^2 \rangle \int_{\Omega} F_{hh}^2 [F_{hh}^2 + 2F_{hh} \text{Re}(F_{hv} e^{i\beta}) + |F_{hv}|^2] \, d\Omega, \quad (7a)$$

wherein it is assumed that the ensemble averages of the backscattering second moments (e.g., $\langle |s_{hh}|^2 \rangle$) are spatially uniform over regions where the pattern functions are significant.

$$2 \int_{\Omega} \text{Re}[s_{hh} s_{hv}^* (F_{vh} F_{hv}^2 + F_{hh} F_{hv}^2 e^{i\beta} + F_{hh} F_{hv}^2 e^{-i\beta} + F_{hh} F_{hv}^2) \, d\Omega, \quad (7b)$$

and the third term, the magnitude squared of the second term in (6), is

$$\int_{\Omega} \langle |s_{vh} F_{vh} (F_{vh} + F_{hh} e^{i\beta})|^2 \rangle \, d\Omega = \langle |s_{vh}|^2 \rangle \int_{\Omega} |F_{vh}|^2 [F_{vh}^2 + 2F_{hh} \text{Re}(F_{hv} e^{-i\beta}) + F_{hh}^2] \, d\Omega. \quad (7c)$$

Next are listed the corresponding three terms composing the power $P_v$ of the vertically polarized weather signal:

$$2 \int_{\Omega} \text{Re}[s_{hh} s_{vh}^* (F_{hv} F_{vh}^2 + F_{hh} F_{hv}^2 e^{i\beta} + F_{hv} F_{hh}^3 e^{-i\beta} + F_{hh} F_{hv}^2) \, d\Omega, \quad (8b)$$

and

$$\langle |s_{hh}|^2 \rangle \int_{\Omega} |F_{hv}|^2 [F_{hh}^2 + 2F_{hh} \text{Re}(F_{hv} e^{i\beta}) + |F_{hv}|^2] \, d\Omega, \quad (8a)$$

$$\langle |s_{vh}|^2 \rangle \int_{\Omega} |F_{vh}|^2 [F_{hh}^2 + 2F_{hh} \text{Re}(F_{hv} e^{-i\beta}) + |F_{hv}|^2] \, d\Omega, \quad (8a)$$

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and in which the intrinsic differential reflectivity $Z_{\text{dr}}$ is

$$Z_{\text{dr}} = \frac{\langle |s_{\text{hh}}|^2 \rangle}{\langle |s_{\text{vv}}|^2 \rangle},$$

and the complex copolar correlation coefficient is

$$\rho_{hv} e^{j\phi_{hv}} = \frac{\langle s_{\text{hh}}^* s_{\text{vv}} \rangle}{\sqrt{\langle |s_{\text{hh}}|^2 \rangle \langle |s_{\text{vv}}|^2 \rangle}}.$$

The bias $\delta Z_{\text{DR}}$ (expressed in decibels) is computed from

$$\delta Z_{\text{DR}} = 10 \log(P_h/P_v) - Z_{\text{DR}}.$$

for specific values of the system parameters and polarimetric variables. Note that $\int_{\Omega} F_{\text{hh}}^4 d\Omega$ is much larger than any of the other terms in (9a) and (9b). Dividing these two equations by this term and taking the difference of the logarithmic functions produces the bias. Because the arguments of the logarithmic functions are close to 1, we use the first-order Taylor expansion and express the bias as

$$\delta Z_{\text{DR}} = 10(A_1 + A_2) \log(e),$$

where the term $A_1$ contains integrals of $F_{hv}$ to first order and $A_2$ contains the integrals of $F_{hv}$ to second order. Explicitly,

$$A_1 = \frac{2 \int_{\Omega} F_{\text{hh}}^3 \text{Re}\{F_{hv} e^{j\beta} - F_{hv} e^{-j\beta} + \rho_{hv} e^{j\phi_{hv}} (Z_{\text{dr}}^2 F_{hv} e^{-j\beta} - Z_{\text{dr}}^2 F_{hv} e^{j\beta})]\} d\Omega}{\int_{\Omega} F_{\text{hh}}^4 d\Omega}$$

and

$$A_2 = \frac{\int_{\Omega} F_{\text{hh}}^2 \left( |F_{hv}|^2 - |F_{hv}|^2 + Z_{\text{dr}}^2 |F_{hv}|^2 - Z_{\text{dr}} |F_{hv}|^2 + 2Z_{\text{dr}}^2 (F_{hv} e^{j\beta} - F_{hv} e^{-j\beta})\right)\} d\Omega}{\int_{\Omega} F_{\text{hh}}^4 d\Omega}.$$

\subsection*{b. Types of cross-polar radiation patterns}

For simplicity, we consider two types of ideal patterns. One has a prominent cross-polar peak collocated with the peak of the copolar pattern. A second type has a quad of equal cross-polar peaks located diagonally to the H and E principal planes; this is typical of a center-fed parabolic reflector (Fradin 1961). For parabolic reflectors with offset feeds, the number of cross-polar peaks is reduced to two (e.g., Durić et al. 2008). Contours of the 3-dB levels of the main lobes for these pattern types are sketched in Fig. 2. Measurements of the cross-polar pattern require good antenna range and have not been a priority of the radar community. Hence, there are few available from which it appears that the cross-polar patterns can be a combination of the first two types.

Prior to quantifying the bias, a brief discussion of radiation patterns follows starting with the one measured
on the KOUN radar. This pattern is examined for obvious practical reasons, which are to quantify its effects on the KOUN polarimetric radar and to anticipate the performance of the forthcoming dual-polarization WSR-88D radars. Measurements of the cross-polar radiation field indicate an H cross-polar pattern with a peak of about 30 dB below the V copolar peak, centered on the copolar beam axis. We have also examined cross-polar patterns measured by A. Canada (Paramax 1992) on another WSR-88D reflector. These measurements also show a cross-polar main lobe coaxial with the copolar lobe, and the ratio of the cross-polar peak to the copolar peak is about the same as that measured on the KOUN. Although the WSR-88D antennas have cross-polar patterns that are likely a combination of the two types shown in Fig. 2, cross-polar peaks coaxial with the copolar beam are the most significant contributor to $Z_{DR}$ bias; thus, we will first focus on that pattern.

The cause of cross-polar peaks along the beam axis of the feed has not been established, but it is known that the concentricity and circularity of the horn components on the order of a few thousandths of a wavelength are necessary to substantially reduce spurious emissions (Potter 1963). Measurements of the feed horn cross-polar radiation pattern suggest that the principal contributor to the coaxial cross-polar peak could be the cross-polar pattern of the feed horn illuminating the reflector (Doviak and Zrnić 1998). For a well-designed and fabricated polarimetric feed horn [with a deep and broad cross-polar null, e.g., Deguchi et al. (2008)], and an ideal parabolic reflector, the cross-polar radiation from the reflector should vanish along the principal planes. In that case the only prominent peaks of the cross-polar pattern are the ones inherent to the geometry (parabolic) of the reflector (see the appendix). Nonetheless, the feed and its supporting structures could convert copolar to cross-polar radiation, contributing to an on-axis cross-polar field peak.

On the other hand, because cross-polar radiation is weak, and because low-elevation angles are required for far-field pattern measurements of large-diameter antennas, accurate cross-polar patterns are difficult to obtain. For example, at low-elevation angles, copolar radiation incident on the terrain can be converted to cross-polar radiation upon scatter. Thus, the lack of a well-defined on-axis null could be an artifact of the site where patterns are measured. For example, cross-polar patterns of the WSR-88D reflector at a wavelength of 5 cm [section 3d(1)] indicate that on-axis cross-polar radiation due to scatter from the feed and its supporting structures can be more than 40 dB below the copolar peak.

Although there can be many causes of on-axis cross-polar peak radiation, there is no need to specify the cause to develop the expressions for the $Z_{DR}$ bias.

c. $Z_{DR}$ bias due to coaxial copolar and cross-polar pattern peaks

Expression $A_1$ (12b) contains terms to first order in $F_{hv}, F_{vh}$ that are much larger than the second-order terms in $A_2$ (12c); hence, $A_2$ can be ignored so that the bias (12a) can be written as

$$
\delta Z_{DR} = 20 \log(e) \left\{ \frac{W_{hv} [\cos(\theta + \gamma_{hv})] - \rho_{hv} Z_{dr}^{1/2} \cos(\Phi_{DP} + \beta - \gamma_{hv})}{-W_{vh} [\cos(\theta - \gamma_{vh})] - \rho_{hv} Z_{dr}^{-1/2} \cos(\Phi_{DP} + \beta + \gamma_{vh})} \right\},
$$

(13)
where \( \gamma_{hv} \) and \( \gamma_{vh} \) are the phases of the cross-polar radiation relative to the copolar phase and

\[
W_{hv} = \int_{\Omega} F_{hh} |F_{hv}| \, d\Omega / \int_{\Omega} F_{hh} \, d\Omega \quad (14a)
\]

and

\[
W_{vh} = \int_{\Omega} F_{hh} |F_{vh}| \, d\Omega / \int_{\Omega} F_{hh} \, d\Omega \quad (14b)
\]

are the antenna’s bias weighting factors that quantify the contribution of the integrated product of the copolar and cross-polar fields to \( Z_{DR} \) biases. From hereon the integration domain \( \Omega \) will be omitted in the integral symbol to compact the equations.

Let us first consider the case \( f_{hv} = f_{vh} \). This case applies to cross-polar radiation produced by the reflector (section 3d) and also to nonorthogonal feed ports [section 3c(2)]. Thus, defining \( W_{hv} = W_{vh} = W \) and \( \gamma_{hv} = \gamma_{vh} = \gamma \) and substituting these into (13) produces

\[
\delta Z_{DR} \approx 20 \log(e) W \left\{ -2 \sin(\beta) \sin(\gamma) + \rho_{hv} \left[ Z_{dr}^{-1/2} \cos(\Phi_{DP} + \beta + \gamma) - Z_{dr}^{1/2} \cos(\Phi_{DP} + \beta - \gamma) \right] \right\} \, (dB). \quad (15)
\]

This equation indicates that the maximum bounds on \( \delta Z_{DR} \) are

\[
\delta Z_{DR} \approx \pm 20 \log(e) W \left[ 2 + \rho_{hv} \left( Z_{dr}^{-1/2} + Z_{dr}^{1/2} \right) \right]. \quad (16a)
\]

These bounds occur if \( \beta = \pm 90^\circ \), \( \gamma = \pm 90^\circ \), and \( \Phi_{DP} = 0^\circ \) (i.e., bias is always positive) or \( \gamma = \pm 90^\circ \) and \( \Phi_{DP} = 180^\circ \) (i.e., bias is always negative). Thus, depending on the particular values of the phases \( (\beta, \gamma, \text{and} \Phi_{DP}) \) the bias can take any value between the boundaries given by (16a). Because for rain \( (Z_{dr}^{-1/2} + Z_{dr}^{1/2}) \approx 2 \), and \( \rho_{hv} \approx 1 \), the largest positive or negative bias is

\[
\delta Z_{DR} \approx \pm 80 W \log(e) = \pm 35 W \, (dB). \quad (16b)
\]

These large biases can be incurred if the transmitted wave is circularly polarized, and the cross-polar and copolar fields are in phase quadrature.

From (15) it can be deduced that the narrowest span of bias occurs if \( \beta = 0^\circ \) or \( 180^\circ \), and \( \gamma = 180^\circ \) or \( 0^\circ \). Then, the bias is contained within the maximum bounds (i.e., for \( \Phi_{DP} = 0, \pi \))

\[
\delta Z_{DR} \approx \pm 20 W \log(e) \rho_{hv} (Z_{dr}^{1/2} - Z_{dr}^{-1/2}). \quad (16c)
\]

To achieve this narrow span of bias, the transmitted field should be slanted linear at either \( \pm 45^\circ \) while the cross-polar field pattern (within the main lobe) should be in or out of phase with respect to the phase of the copolar pattern. Changing the transmitted phase \( \beta \) is simple but setting it to a desirable value is not trivial. On the other hand, the phase difference between cross-polar and copolar main lobes is typically the intrinsic property of the antenna.

Suppose that the phase difference \( \beta \) is set to \( 0^\circ \) or \( 180^\circ \) (by design), but the cross-polar field is in phase quadrature with the copolar field (i.e., \( \gamma = 90^\circ \)). Under these conditions, \( \delta Z_{DR} \) is contained within the intermediate bounds:

\[
\delta Z_{DR} \approx \pm 20 W \log(e) \rho_{hv} (Z_{dr}^{1/2} + Z_{dr}^{-1/2}) \approx \pm 17.4 W. \quad (16d)
\]

The three bias boundaries [i.e., (16a), (16c), and (16d)] normalized by \( W \) are plotted in Fig. 3.

In summary, Fig. 3 indicates that the largest span of bias (top curve) is incurred if \( \beta = \pm 90^\circ \) (i.e., circularly polarized transmitted field) and \( \gamma = 90^\circ \). Change in any one of these would therefore reduce the bias below the boundary. For slant linear polarization (e.g., \( \beta = 0^\circ \)), the worst case of positive bias is the middle curve (16d). This boundary and the highest boundary are essentially independent of \( Z_{DR} \). For the case \( \beta = 0^\circ \) and \( \gamma = 180^\circ \), the maximum positive bias is the lowest curve (its mirror image about the abscissa represents the negative bias). In the region of \( Z_{DR} \) typical for rain, these curves are linear.
We shall use Fig. 3 to determine the bias for some possible values of the antenna parameters. Assume axially symmetric Gaussian radiation patterns so that \( |f_y(\theta)|^2 = \exp[-\theta^2/(4\sigma_y^2)] \) describes the one-way power pattern (Doviak and Zrnić 2006, section 5.3). Then,

\[
W_{hv} = \frac{4\theta_1^2}{\theta_1^2 + 3\theta_1^2 \sigma_{hv}^2} \quad \text{and} \quad (17a)
\]

\[
W_{vh} = \frac{4\theta_1^2 \sigma_{vh}^2}{\theta_1^2 + 3\theta_1^2 \sigma_{vh}^2} \quad \text{where the one-way 3-dB beamwidths of the copolar and cross-polar power patterns are } \theta_1 \text{ and } \theta_4. \text{ Because the power into the H or V port must equal the corresponding copolar and cross-polar power radiated, the gains and beamwidths must obey the constraint } \sigma_{hh}^2 + \sigma_{vh}^2 = \sigma_{hh}^2 + \sigma_{hh}^2 = 1 \text{, where } \sigma_{hh}, \sigma_{vh} \text{ are the second central moments of the respective two-way power patterns.}
\]

Following derivations similar to the ones for \( P_h \) and \( P_v \) in Eqs. (9a) and (9b), but assuming uniformly distributed spherical scatterers and a horizontally polarized field, we obtain the linear depolarization ratio that the antenna would measure:

\[
L_{dr} = \frac{\int_{\Omega} |F_{hh}F_{hv} + F_{hh}F_{vh}|^2 d\Omega}{\int_{\Omega} |F_{hh} + F_{hv}|^2 d\Omega}, \quad (17c)
\]

which in the worst case \( F_{hv} = F_{vh} \) reduces to

\[
L_{dr} = \frac{4\int_{\Omega} F_{hh}^2 F_{hv}^2 d\Omega}{\int_{\Omega} F_{hh}^2 d\Omega}. \quad (17d)
\]

If the peak of the cross-polar pattern is 40 dB below the copolar peak, and beamwidths are equal as some data (Fig. 5) suggest, \( W = 0.01 \), and \( L_{DR} = -34 \text{ dB} \). From Fig. 3 we find that the maximum positive bias is about 0.35 dB (i.e., for \( \Phi_{DP} = 0 \); otherwise, it is less). This bias would drop to about 0.18 dB if the transmitted H and V fields are in phase; this would produce a maximum rainrate error of less than 25% (Fig. 1). Further reduction is possible only if the on-axis cross-polar radiation is less, as the data suggest (section 4), or if copolar and cross-polar patterns are in phase (or 180° out of phase).

1) \( Z_{DR} \) BIAS DUE TO A ROTATED HORN

It will be assumed that rotation of the horn in the polarization plane is the only mechanism causing cross coupling. That is, the cross-polar radiation with a properly oriented horn is negligible (i.e., the intrinsic \( F_{hv} = F_{vh} = 0 \)). Computing the bias in this case can be done by introducing the rotation matrix in Eq. (3). Multiplying the rotation matrix by the \( F \) matrix, we obtain the effective matrix \( F^{(e)} \):

\[
F^{(e)} = \begin{bmatrix} F_{hh} \cos \alpha & -F_{hh} \sin \alpha \\
F_{hh} \sin \alpha & F_{hh} \cos \alpha \end{bmatrix}, \quad (18)
\]

where \( \alpha \) is the rotation angle with a positive sign counterclockwise. In this case the terms \( F_{hv}^{(e)} = -F_{hh} \sin \alpha \), \( F_{vh}^{(e)} = F_{hh} \sin \alpha \), etc. replace the terms \( F_{hv}, F_{vh}, \) etc. in the equations for the weighting functions. Thus, introducing the terms from (18) into (12) and carrying forward the computations, the following approximate formula for the bias is obtained:

\[
\delta Z_{DR} = 20W_{rot} \log(e)[-2 \cos \beta + \rho_{hv}(Z_{dr}^{1/2} + Z_{dr}^{1/2} \cos(\Phi_{DP} + \beta))], \quad (19)
\]

where now the bias-weighting factor \( W_{rot} = \tan(\alpha) \). For small angular rotations, this result agrees with that obtained by Doviak et al. (2000).

Feed horn rotation can be set to tolerances of the order of 0.1° (Doviak and Zrnić 1998, section II.6.7) at which \( \tan(0.1°) = 0.0017 \), and the maximum bias (top graph in Fig. 3) is about 0.06 dB. Hence, we conclude that on well-designed antennas, horn rotation should not be a factor.

2) BIAS DUE TO NONORTHOGONALITY OF THE H AND V PORTS

Let us assume that the H, V ports are separated by an angle \( \chi < \pi/2 \) and the horn is rotated about its axis to null one of the cross-polar fields. For example, if the cross-polar V field produced by excitation of the H port had an on-axis null (i.e., \( F_{vh} = 0 \)), the copolar H field would be \( F_{hh} \). But if the V port is then excited, the cross-polar H would be \( -F_{hh} \sin \alpha \), where \( \alpha = (\pi/2) - \chi \) (positive counterclockwise) and the copolar V would be \( F_{vh} \). Thus, the matrix \( F^{(e)} \) becomes

\[
F^{(e)} = \begin{bmatrix} F_{hh} & -F_{hh} \sin \alpha \\
0 & F_{hh} \cos \alpha \end{bmatrix}, \quad (20)
\]

and by substituting the terms from (20) into (13) and simplifying, the following bias equation is obtained:

\[
\delta Z_{DR} = 20W_{ort} \log(e)[-\cos \beta + \rho_{hv} Z_{dr}^{1/2} \cos(\Phi_{DP} + \beta)]. \quad (21)
\]

In (21), \( W_{ort} = \sin \alpha \) and, as with (19), the bias peaks at \( \beta = -90° \) and \( \Phi_{DP} = 90° \); “ort” is orthogonality. At the same \( \alpha \) tolerance as that for the rotated horn, the bias is insignificant.
d. Z\textsubscript{DR} bias due to a four-lobed cross-polar radiation pattern

Cross-polar radiation patterns with nulls along the principal planes and a distinct equal-amplitude principal peak near the copolar peak in each of the quadrants (Fig. 2, bottom) are the subject of this section. This type pattern is inherent in a center-fed parabolic reflector illuminated with linearly polarized radiation (Fradin 1961, section VII.2). For an example, the reader is referred to Chandrasekar and Keeler (1993, Fig. 11). Offset parabolic reflectors (e.g., the Scanning Polarimetric Imaging Radiometer, SPIRA; Đurić et al. 2008) produce cross-polar patterns with two principal peaks near the copolar peak. Cross-polarized peaks, inherent to the center-fed parabolic reflector, can be substantially reduced if a circular horn is used to illuminate the reflector (Fradin 1961, section VII.3). The general procedure used in section 3a to compute reflector (Fradin 1961, section VII.3). The general procedure used in section 3a to compute cross-polar patterns with two principal peaks near the copolar peak. Cross-polarized peaks, inherent to the center-fed parabolic reflector, can be substantially reduced if a circular horn is used to illuminate the reflector (Fradin 1961, section VII.3). The general procedure used in section 3a to compute Z\textsubscript{DR} biases also applies to this case. Nonetheless, to obtain analytical solutions, further simplification and assumptions are required.

The electric field pattern \( f_{hv}(\theta, \phi) \) is assumed to be axially symmetric about its peak, but the electric field at each peak alternates in sign as one circles from one peak to the next around the copolar beam axis; thus, the copolar and cross-polar fields are in phase or antiphase, and \(|F_{hv}| = |F_{v}|\) (see the appendix). Therefore, the terms \( F_{hh}^n F_{hv}^n \) in (7) integrate to zero for any \( k \) if the exponent \( n \) is odd and if there is an even number of peaks; that is, the first- and third-order terms in \( F_{hv} \) vanish. Hence, \( A_1 = 0 \), so that \( A_2 \) from (12c) produces the bias:

\[
\delta Z_{\text{DR}} = -10 \log(e) [Z_{\text{dr}} - Z_{\text{dr}}^{-1} + 4P_{hv}(Z_{\text{dr}}^{1/2} - Z_{\text{dr}}^{-1/2})] \\
\times \cos(\Phi_{\text{DP}}) \left[ \frac{\int F_{hh}^2 |F_{hv}|^2 d\Omega}{\int F_{hh}^4 d\Omega} \right].
\]

Let us apply (22) to a cross-polar pattern having four peaks. Assume a Gaussian shape for the copolar lobe and the following offset Gaussian shape,

\[
|F_{hv}(\theta, \phi)|^2 = g_{hv} f_{hv}(\theta, \phi)^2 \\
= g_{hv} \exp \left[ -\frac{(\theta - \theta_1)^2 + (\phi - \phi_1)^2}{4\sigma_{hv}^2} \right],
\]

for each of the cross-polar lobes. Here, \( \theta_1 \) and \( \phi_1 \) are angular locations of the cross-polar radiation peaks. Then, we define

\[
W_4 = \frac{\int F_{hh}^2 |F_{hv}|^2 d\Omega}{\int F_{hh}^4 d\Omega} = 4 \frac{2g_{hv}\theta_1^2 e^{-4g_{hv}^2\ln(2)\Phi_{dp}}}{g_{hh}(\theta_1^2 + \phi_1^2)} (23)
\]

as the antenna’s bias-weighting factor for a four-lobed cross-polar radiation pattern. The 3-dB width of the one-way copolar power pattern is \( \theta_1 \), whereas the 3-dB one-way width of each cross-polar lobe is \( \theta_{1/2} \). Fradin’s equations (see the appendix) are used to compute the locations of the cross-polar peaks for a center-fed parabolic reflector. In the case of a WSR-88D reflector, the cross-polar pattern peaks should be about 1° away from the copolar beam axis. Because of the strong on-axis cross-polar lobe, it was difficult to discern the cross-polar peaks due to the KOUN reflector. But for a reflector having the same specifications, \( \Phi_{dp} \) measured (i.e., about 0.5°; Fig. 5) at a wavelength of 5 cm indicates good agreement with \( \theta_{1/2} \) calculated (i.e., 0.47°) from theory [i.e., Eq. (A3)].

For rain \( Z_{dr} > 1 \), and from (22), it follows that the largest bias is negative if \( \Phi_{dp} = 0° \). Under this condition (i.e., \( \Phi_{dp} = 0° \)) and for \( P_{hv} = 1 \), \( \delta Z_{\text{DR}} \) normalized with \( W_4 \) is plotted in Fig. 4. Note that the maximum negative bias grows almost linearly with differential reflectivity (i.e., \( \delta Z_{\text{DR}}/W_4 \approx -6.15 Z_{\text{DR}} \)) in the range of 0–3 dB. Let us now examine a specific polarimetric weather radar example.

Assume the ratio of gains \( (g_{hh}/g_{hv}) = 0.001 \), then according to (17d), \( L_{\text{DR}} = -24 \text{ dB} \), \( W_4 \) in (23) equals \( g_{hv}/g_{hh} \), and the maximum negative bias (i.e., at \( \Phi_{dp} = 0° \)) obtained from Fig. 4 is about \( -0.0062 \) Z\textsubscript{DR} (dB), a negligible amount. Note that for this idealized case the
$L_{\text{DR}} = -24 \text{ dB}$ yet the bias in $Z_{\text{DR}}$ is much smaller than for the case of an antenna having an on-axis cross-polar lobe and $L_{\text{DR}} = -34 \text{ dB}$.

The primary reason for the significantly better performance of this type of cross-polar radiation pattern is that the four symmetrically located pattern peaks alternate sign so that there is cancellation of some cross-polar contribution. Another reason is that the bias-weighting factor is proportional to the integral of the square of the normalized cross-polar radiation, whereas it is proportional to the integral of the normalized radiation if the cross-polar pattern is coaxial with the copolar pattern. Furthermore, the displacement of these peaks from the copolar beam axis causes the cross product of the copolar pattern with the cross-polar pattern to be smaller than in the case where the peaks are coincident.

**AN EXAMPLE**

In Fig. 5, we show two cross-polar patterns measured on the University of Oklahoma’s (OU) Polarimetric Radar for Innovations in Meteorology and Engineering (PRIME). This antenna reflector is a replica of the WSR-88D reflector, but has four feed-support struts as opposed to three and is illuminated with 5-cm wavelength radiation.

Some cross-polar radiation patterns can be represented as being the sum of a centered pattern (Fig. 2, top) with the quad pattern (Fig. 2, bottom). The exact computation of the bias is straightforward, although tedious. Significant simplification is possible by noting that the dominant factor is the first-order (in powers of $F_{\text{hv}}$) term $\int F_{\text{hh}}^2 F_{\text{hv}} \, d\Omega$ for a coaxial cross-polar peak, and the second-order term $\int F_{\text{hh}}^2 F_{\text{vh}}^2 \, d\Omega$ for off-set cross-polar peaks.

The two-way copolar power pattern and the two normalized cross products (i.e., $F_{\text{hh}}^2 F_{\text{hv}}$ and $F_{\text{hh}}^2 |F_{\text{vh}}|^2$) in the three principal planes are presented in Figs. 6a–c. It is clear from Fig. 6 that the cross-polar pattern peak

**FIG. 5.** Cross-polar pattern functions $|F_{\text{hh}}(\theta)|^2 / F_{\text{hh}}(0)$ (thick curve) and $|F_{\text{vh}}(\theta)|^2 / F_{\text{hh}}(0)$ (thin curve) along the 45° diagonal of the OU PRIME antenna (frequency is 5.625 GHz).

**FIG. 6.** (a) The two-way pattern $F_{\text{hh}}^2(\theta)$ (i.e., the solid thin curve), the normalized product $F_{\text{hh}}^3(\theta)/F_{\text{hh}}(0)$ (i.e., the solid thick curve), and the normalized product $F_{\text{hh}}^2(\theta)/F_{\text{hh}}(0)$ (i.e., the dotted curve) in the E plane. (b) As in (a), but with measurements made in the H plane. (c) As in (a), but in the 45° plane.
collocated with copolar beam axis contributes most to the bias-weighting factor. Because the normalized term $F_{hh}^3 |F_{vh}|^2 / F_{hh}^4(0)$ has almost the same angular width as $F_{hh}^2$ (Fig. 6a), the antenna’s bias-weighting factor, $W_{vh}$ in (17b), can be approximated with $\sqrt{g_{vh}/g_{hh}} (0.01$ in this case). Furthermore, if $|F_{hv}| = |F_{vh}|$, $W_{hv} = W_{vh} = W = 0.01$. With this value the maximum positive bias (i.e., if $\gamma_{hv} = 90^\circ$, and $\beta = -90^\circ$) can be read from the top curve in Fig. 3. It is about 0.35 dB. This is significant but unlikely to happen as it requires a juxtaposition of $\gamma = 90^\circ$, $\beta = -90^\circ$, and $\Phi_{DP} = 0^\circ$.

The maximum negative bias contributed by the four cross-polar peaks is computed using Eqs. (22) and (23), the ratio $(g_{hv}/g_{hh}) \approx 3.16 \times 10^{-4}$ (i.e., about -35 dB from Fig. 5), and $\theta_p = \theta_r$. This maximum negative bias is approximately $-0.002 Z_{DR}$, which is insignificant (positive biases are smaller yet).

4. Bias in the alternate horizontal–vertical (AHV) mode

Consider next the AHV mode and apply the same formalism starting with (3). For computing $P_h$, set the lower element in the right-most matrix to zero, and for $P_v$ set the upper element to zero. Then, after performing the multiplications, it can be shown that all first- and third-order terms in $F_{hv}$ and $F_{vh}$ are absent; ignoring the fourth-order term the powers can be expressed as

$$\begin{align*}
\frac{P_h}{\langle |s_{vh}|^2 \rangle} &\sim Z_{dr} \int F_{hh}^4 d\Omega + 2\rho_{hv} \sqrt{Z_{dr}} \cos(\Phi_{DP} + 2\gamma_{hv}) \\
&\times \int F_{hh}^2 |F_{vh}|^2 d\Omega \quad \text{and} \quad (25a)
\end{align*}$$

$$\begin{align*}
\frac{P_v}{\langle |s_{vh}|^2 \rangle} &\sim \int F_{hh}^2 d\Omega + 2\rho_{hv} \sqrt{Z_{dr}} \cos(\Phi_{DP} - 2\gamma_{hv}) \\
&\times \int F_{hh}^2 |F_{hv}|^2 d\Omega. \quad (25b)
\end{align*}$$

Thus, the bias $\delta Z_{DR}^{(A)}$ for the AHV mode is

$$\begin{align*}
\delta Z_{DR}^{(A)} &\sim 20 \log(e) \rho_{hv} \{W_{vh}^{(A)} Z_{dr}^{-1/2} \cos(\Phi_{DP} + 2\gamma_{hv}) \\
&- W_{hv}^{(A)} Z_{dr}^{1/2} \cos(\Phi_{DP} - 2\gamma_{hv})\} (dB), \quad (26)
\end{align*}$$

where

$$W_{vh}^{(A)} = \int \frac{F_{hh}^2 |F_{vh}|^2 d\Omega}{F_{hh}^4 d\Omega} \quad \text{and} \quad (27a)$$

We can see that $\delta Z_{DR}^{(A)}$ is independent of the polarization basis (i.e., independent of the angle $\beta$ between the transmitted H and V fields). If slant linear polarization is transmitted (i.e., $\beta = 0^\circ$) for the SHV mode and $F_{hv} = F_{vh}$, (15) is identical in form to (26), with the significant difference being the bias-weighting factors.

For a well-designed and fabricated polarimetric feed horn, and an ideal parabolic reflector, the cross-polar radiation should vanish along the principal planes; thus, there should be a null on axis. The radiation patterns seen in Fig. 5 suggest that the on-axis cross-polar radiation is well below the copolar peak (i.e., $-40$ dB lower) whereas the cross-polar peak measured at the KOUN antenna is about $-32$ dB (Doviak and Zrnić 1998, Figs. II.9). Note that both of these measurements were made at the same manufacturer’s site. It might be that the smaller copolar beamwidth of the 5-cm OU PRIME mitigates reflection from the terrain that could have contributed to the on-axis cross-polar radiation peak or the coupling on the OU PRIME antenna might be less than on the KOUN antenna.

Pattern measurements (Baron Services 2009) for an upgraded WSR-88D antenna (one of which is reproduced in Fig. 7) were made by Seavey Engineering on their antenna range in Massachusetts. These measurements also show a well-defined cross-polar null on axis. For this range, there is a shallow valley between the test antenna and a source in the far field. Thus, there is likely less conversion of copolar radiation, by the terrain, to coaxial cross-polar radiation. We hypothesize that this might be the reason why, for the same reflector and the same wavelength, there is a null (Fig. 7) whereas for the KOUN radar there is a strong cross-polar peak on axis. But, the feed on the upgraded antenna is different from the feed on the KOUN and could be the major cause of the differences in measurements. The large on-axis cross-polar peak seen in the E-plane plot is an obvious artifact; the pattern is offset by 1° from where it should be and the value of the on-axis radiation must be the same no matter along which plane the measurement is made.

It should also be noted that for an antenna of similar design (i.e., center-fed parabolic reflector), Bringi and Chandrasekar (2001, Fig. 6.15) report principal plane cross-polar radiation, measured at another manufacturer’s test range, is everywhere below $-45$ dB. If the coaxial cross-polar radiation is $-45$ dB below the copolar peak, and slant 45° polarized radiation is transmitted, the differential reflectivity bias is less than 0.1 dB.
Let us assume that we have a center-fed antenna in which the on-axis radiation lobe is negligible. Under this condition let us compare the $Z_{DR}$ biases using the SHV and AHV modes. Thus, assuming four equal cross-polar lobes offset from the beam axis, we use (22) and (26) for this comparison. In this case the antenna’s bias factor $W_4$ is the same for both modes. For rain it is safe to set $r_{hv} = 1$, and note $Z_{dr} > 1$, and thus we can write $Z_{dr} = 1 + \Delta$, and assume that $\Delta < 1$. Under these conditions it can be shown that (22) reduces to

$$\frac{\delta Z_{DR}^{(S)}}{W_4} = -20(Z_{dr} - 1) \log(e)[1 + 2 \cos \Phi_{DP}], \quad (28a)$$

which is the normalized bias for the SHV mode, whereas for the AHV mode, (26) becomes

$$\frac{\delta Z_{DR}^{(A)}}{W_4} = -20(Z_{dr} - 1) \log(e) \cos \Phi_{DP}. \quad (28b)$$

Comparing these two, the SHV $Z_{DR}$ bias is about 3 times larger than that for the AHV mode. Nevertheless, assuming that $W_4 \approx 3.16 \times 10^{-4}$ (i.e., $F_{hv}$ peak at least $-35$ dB below the copolar peak; Fig. 5), the SHV bias is approximately 0.002 $Z_{DR}$, which is still insignificant.

5. Summary and conclusions

Herein we investigated the bias in the differential reflectivity measurement. The cross-polar radiation introduces a bias that depends on several parameters including differential reflectivity itself. For accurate rainfall measurement this bias should be smaller than about 10% of $Z_{DR}$ (in decibels, Fig. 1).

With this in mind we set out to quantify the bias caused by cross-polar radiation. We examine two models of cross-polar patterns. One has a cross-polar main lobe centered on the copolar main lobe; the other has four lobes of equal magnitude that are displaced symmetrically about the beam axis. Use of customary approximations (i.e., radiation lobes having Gaussian shape) and a uniform distribution of horizontally oriented scatterers leads to simple analytic equations for the antenna’s differential reflectivity’s bias-weighting factors $W_{hv}$ and $W_{vh}$ (i.e., the spatial integral of the normalized products of copolar and cross-polar radiation patterns).

A small linear depolarization ratio is a sufficient but not a necessary condition to have acceptable bias of $Z_{DR}$ in the case of the SHV polarimetric mode. Both the shape and value of the cross-polar pattern are crucial. Antennas having cross-polar nulls along the principal planes, but multiple lobes associated with the reflector, cause significantly less bias than those having a single cross-polar lobe centered on the copolar beam axis. Thus, four-lobe cross-polar patterns producing the same $L_{DR}$ as single-lobe patterns are much more forgiving. The absolute upper bound of the $Z_{DR}$ bias in the case of a single-lobe pattern with $L_{DR}$ of $-40$ dB is 0.17 dB and it is $5.4 \times 10^{-3} Z_{DR}$ (dB) for the four-lobe pattern in which each lobe has the same gain as a single cross-polar lobe.

The worst bias in the case of a single-lobe cross-polar pattern can be reduced by about a factor of 2 if the transmitted wave is slant linearly at $\pm 45^\circ$ (Fig. 3, middle curve). Then, the level of cross-polar peak radiation must be at least 45 dB below the copolar peak to keep the $Z_{DR}$ bias under 0.1 dB.

If the cross-polar radiation has an on-axis null, the only significant cross-polar radiation peaks are the four equal-gain lobes due to the reflector (section 3d); furthermore, if the gains of these lobes are equal, then they need to be below $-21$ dB to ensure that the $Z_{DR}$ bias is less than 0.1 dB (at $Z_{DR} \leq 2$ dB). Measurements on some antennas (Fig. 5) suggest these gains can be well below $-30$ dB.
In agreement with previous investigations, it turns out that $Z_{\text{DR}}$ bias is not an issue for polarimetric radars utilizing the alternate (AHV) mode. For the simultaneous (SHV) mode, bias in $Z_{\text{DR}}$ is larger, but the maximum bias can be less than 0.1 dB if the transmitted wave is slanted 45° (section 4) and the cross-polar pattern is a quad of equal lobes; this would produce a worst-case rain-rate error of less than 15% (Fig. 1).

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APPENDIX

Cross-Polar Radiation Induced by the Parabolic Reflector

Fradin (1961, section 7.2) shows that the copolar and cross-polar fields in the aperture of a center-fed parabolic reflector illuminated with the field of a vertical (i.e., $y$ directed) dipole are given by

$$E_y = -A \frac{4 f^2 + p^2 \cos 2\phi}{(4 f^2 + p^2)^2} \quad \text{and} \quad (A1)$$

$$E_x = -A \frac{p^2 \sin 2\phi}{(4 f^2 + p^2)^2}, \quad (A2)$$

where the horizontal $x$ direction is also in the aperture, $f$ is the focal length of the parabolic reflector (for the WSR-88D, $f = 0.375D$, where $D$ is the antenna diameter), $A$ is a complex constant (dependent on $f$, $D$, the dipole moment, and the wavelength), $p = \sqrt{x^2 + y^2}$ is the radial distance from the $z$ axis to any point in the aperture plane, and $\phi$ is the angle measured from the $x$ axis.

Using these equations, it is easily seen that the cross-polar field has nulls along the principal axes (i.e., $x$ and $y$), and each quarter sector of the aperture is a source of cross-polar radiation having alternating phases. Thus, the far-field pattern should have, in the absence of spar and feed blockages and reflector surface perturbations, nulls along the principal planes. Furthermore, for the WSR-88D antenna, (A2) shows the peak of the aperture’s cross-polar field is on the periphery of the aperture and along diagonals at $\pm 45°$. We shall treat each sector as a source of radiation emanating from a phase center located at the center of gravity of the cross-polar aperture function [i.e., (A2)] in each sector. Because the aperture distribution in each sector is symmetrical about the $\phi = 45°$ diagonals, the four phase centers lie along these diagonals. Using (A2), we compute the phase centers to be at the radial distance $p_{\phi} = 0.71 D/2$. The cross-polar radiation has a peak at an angle $\theta_p$, measured from the copolar beam axis (i.e., $z$ axis), where radiation from each of the four sectors is constructively added. The sectors on either side of the diagonals always add in phase, but the sectors along the diagonal add in phase at

$$\theta_p = \sin^{-1} \left( \frac{\lambda}{2 \rho} \right). \quad (A3)$$

For the KOUN parameters, $\lambda = 0.11 m$ and $D = 8.53 m$, and thus $\theta_p$ can be computed to be 1.04°. This assumes that an electric dipole illuminates the reflector, whereas the actual feed suppresses illumination on the edge of the reflector so that the phase center would be at a slightly smaller radius. Thus, the cross-polar peak should appear at an angle $\theta_p$ larger than 1.04°.

We conclude that there are four principal lobes of cross-polar radiation, one each along the azimuthal directions $\phi = \pm 45°$ and $\phi = \pm 135°$, and at an angular displacement given by (A3). Such cross-polar radiation lobes are seen in the pattern measurements presented by Bringi and Chandrasekar (2001, Fig. 6.15), as well as the data in Figs. 5 and 7.

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