Quantifying the Impact of Wind Turbine Wakes on Power Output at Offshore Wind Farms


ABSTRACT

There is an urgent need to develop and optimize tools for designing large wind farm arrays for deployment offshore. This research is focused on improving the understanding of, and modeling of, wind turbine wakes in order to make more accurate power output predictions for large offshore wind farms. Detailed data ensembles of power losses due to wakes at the large wind farms at Nysted and Horns Rev are presented and analyzed. Differences in turbine spacing (10.5 versus 7 rotor diameters) are not differentiable in wake-related power losses from the two wind farms. This is partly due to the high variability in the data despite careful data screening. A number of ensemble averages are simulated with a range of wind farm and computational fluid dynamics models and compared to observed wake losses. All models were able to capture wake width to some degree, and some models also captured the decrease of power output moving through the wind farm. Root-mean-square errors indicate a generally better model performance for higher wind speeds (10 rather than 6 m s\(^{-1}\)) and for direct down the row flow than for oblique angles. Despite this progress, wake modeling of large wind farms is still subject to an unacceptably high degree of uncertainty.

1. Introduction

Although offshore wind energy now composes less than 2% of the total capacity of 94 GW of wind energy installed worldwide (International Energy Agency 2008), that fraction is anticipated to see substantial increases in the future. There are plans to install 150 GW in the waters of the European Union by 2030 with targets for individual countries of, for example, 20–25 GW in Germany, 33 GW in the United Kingdom, and 4.6 GW in Denmark (Wagner 2009). This scale of offshore installation necessitates development and evaluation of improved tools with which to model interactions between individual turbines, the atmosphere, and neighboring turbines to accurately predict wind farm power output and thus optimize wind farm design. A major focus of these efforts is centered on improved quantification of the role of wind turbine spacing on power losses due to wind turbine wakes. If the effect of turbine spacing on power losses and fatigue load due to wakes can be quantified, then this can be used to optimize wind farm layouts and thereby reduce costs. Barriers to quantifying the impact of spacing derive from difficulties in accurately measuring wake effects related to limited data availability and the complexity of interactions between wake behavior and atmospheric state. For an isolated wind turbine the wind speed deficit in a single wake immediately downstream of a turbine is primarily a function of the turbine thrust coefficient (Fig. 1), which is strongly related to the incident wind speed at hub height, while the wake width and height are determined by the turbine characteristics (hub height, rotor diameter, etc). As the wake moves downstream of the turbine, its expansion is a function of the ambient turbulence, turbine-generated turbulence, wind speed, wind direction,
atmospheric stability, and the point at which the wake impacts the ground. Previous research has highlighted modeling and evaluated wind speed or power reduction in individual wind turbine wakes both onshore (Crespo et al. 1999) and offshore (Barthelmie et al. 2006). Considerable effort has also been expended into developing models for turbulence generation in wind farms (Frandsen and Thøgersen 1999; Quarton and Ainslie 1990). However, wakes in multiturbine wind farms are also subject to downward and lateral merging with wakes from neighboring turbines. If the wind farm is sufficiently large, added turbulence from the turbines may impact the boundary layer, particularly in the shallow low-turbulence boundary layers found in many offshore areas. In a general sense, it is intuitive that increasing spacing between wind turbines should decrease power losses due to wakes because wakes begin to recover at some distance downstream as momentum is drawn into the wake from the surrounding undisturbed flow. However, practical constraints such as cabling costs and available space dictate that the turbine spacing cannot be infinitely large. It is also apparent that if the wind farm is sufficiently large there must be a limit to the energy that can be extracted. In most cases, turbine spacing at operational offshore wind farms is currently in the range of 4–12 rotor diameters ($D$) leading to estimated total wind farm wake losses of 10% for Middelgrunden (2.4 $D$, one line; Barthelmie et al. 2007b), 12.4% at Horns Rev (7 $D$ spacing; Sørensen et al. 2006), and 23% at Lillgrund (3.3–4.3 $D$ spacing; Dahlberg and Thor 2009).

Two types of models are used to estimate power losses due to wind turbine wakes. One is a wind farm model using a wake model that has been simplified or parameterized so that the description of wind farm wind resources and power losses due to wakes can be made relatively quickly. “Industry standard” models typically fall into this category, giving average results at individual turbine locations. The second is a computational fluid dynamics (CFD)-type model, which solves basic equations of the atmosphere and produces results on a fine mesh in space and time. Despite considerable progress, these models are currently too computing/time intensive to be used in most industry applications. Preliminary evaluation of wake models (either operated in stand-alone mode or as part of a wind farm model) based on individual case studies in the UpWind project Barthelmie et al. (2007d) suggested that standard wind farm models were underpredicting wakes (i.e., overpredicting power output), while CFD models were overpredicting wake losses. For total wind farm power output, Sørensen et al. (2006) suggested that one of the linearized models [the Wind Atlas Analysis and Application Program (WAsP); Mortensen et al. (2005)] applied with standard parameterizations gave the best performance of the models evaluated. Subsequent reevaluation of model parameterizations and constants has improved model performance, but the models continue to exhibit divergent behavior and agreement with the observations (which also exhibit a high degree of variability) remains elusive (Barthelmie et al. 2009). Some of the difficulty in evaluating models arises from what might be termed natural variability—for example, nonstationarity of wind speed and direction either across the wind farm or in time due to inhomogeneous fetch conditions (Barthelmie et al. 2007a) or atmospheric variability (e.g., frontal passages). Part of the difficulty also derives from issues pertaining to measurement uncertainty and model limitations, such as lacking parameterization for stability variations (Barthelmie et al. 2007c).

Here, we examine a new observational dataset derived from two large operational Danish wind farms designed to provide a more systematic basis for model evaluation and to assist in the physical understanding of wake behavior. The objectives of the analyses presented herein are

1. to determine if conditionally sampled observational data from two offshore wind farms with different layouts and turbine spacing can be used to quantify the relationship between wind turbine spacing and wake losses for realistic layout configurations,
2. to determine if the conditionally sampled data ensembles can be used to infer the downwind distance where lateral wake merging is observed, and
3. to provide a more comprehensive and robust evaluation of wind turbine wake models.
2. Development of the data ensembles

The observations used to develop the data ensembles are taken from the supervisory control and data acquisition (SCADA) systems from the large offshore wind farms at Nysted (Barthelmie et al. 2007c; Cleve et al. 2009) and Horns Rev (Jensen 2004; Table 1 and Fig. 2). Observations are from June 2004 to May 2006 at Nysted and for the calendar year 2005 at Horns Rev. Both wind farms lie in Danish waters with the prevailing winds and highest wind speeds from the southwest. Nysted is in the southern part of Denmark and has relatively short sea fetches in most directions (a minimum of 10 km to the north and less than 70 km to the south and west), whereas Horns Rev is off the western coast of Denmark in the North Sea and has a long sea fetch (more than 500 km) in the westerly directions (Fig. 2). The wind farms and wind turbines are of similar size, although turbines at Horns Rev are pitch regulated, those at Nysted are active stall regulated with two-speed generators. The turbine spacing at Horns Rev is 7 $D$ in both north–south and west–east directions, whereas at Nysted the turbine spacing is 5.8 $D$ in the north–south direction and 10.5 $D$ in the east–west direction (Fig. 3).

In the following, we have conditionally sampled the wind farm datasets to extract all 10-min periods where the incident flow is from west to east. Focusing on the west–east direction gives flow down the row in the prevailing wind direction and maximizes the wake signal. We, therefore, refer to turbines in north–south lines as “columns,” whereas those in west–east lines are “rows.” So, for example, following the turbine labeling schemes shown in Fig. 3, at Nysted there are nine wind turbines in each column (A1 ... A9, B1 ... B9, and so on to H1 ... H9) and eight wind turbines in each row, where the northernmost row contains turbines (A1, B1 ... H1) and the southernmost row turbines (A9, B9 ... H9). At Horns Rev there are eight wind turbines in each column (1 ... 8, 11 ... 19 and so on to 91 ... 98) and 10 wind turbines in each row where the northernmost row contains turbines (1, 11, 21, 31 ... 81, 91) and the southernmost row (8, 18, 28, 38 ... 88, 98).

![Fig. 2. Location of Horns Rev and Nysted wind farms in Denmark from which the data are presented.](http://journals.ametsoc.org/doi/abs/10.1175/2010JTECHA1398.1)
The data selection criteria and processing steps area follows:

(i) All measurements were validated comparing the power signal level to nacelle wind speed and mean pitch angle. All events like idling, start and stop sequences, and reduced power levels have been excluded. Direction is from wind turbine 7 for Horns Rev and A5 for Nysted (see Fig. 3). For the remaining turbines it is assumed that the turbines are not yawed but operating in line with the reference turbine.

(ii) The average power at each turbine is calculated for seven incident wind directions; for a wind direction where the flow is down an exact row (ER) including observations within $\pm 2.5^\circ$ ($278^\circ \pm 2.5^\circ$ at Nysted, $270^\circ \pm 2.5^\circ$ at Horns Rev, see Fig. 3), and then for mean wind directions of $+5^\circ$, $+10^\circ$, and $+15^\circ$ and $-5^\circ$, $-10^\circ$, and $-15^\circ$ from ER. Flow down an ER thus represents the likely maximum wake effect, while the wind directions that are slightly offset from ER assist in assessing the wake width. The choice of direction and the variability included are both important to the identification of wakes, and
the characterization of wake behavior. As shown in Barthelmie et al. (2009) for the single wake (i.e., at one turbine downstream), choosing the ER angle \( \pm 1^\circ \) includes only the wake centerline, \( \pm 5^\circ \) includes the wake center and about half the wake, extending to \( \pm 10^\circ \) includes most of the wake, and beyond \( \pm 15^\circ \) also includes nonwake conditions. This also illustrates the importance of accurate wind direction measurements to find the center of the wake precisely.

(iii) Data for each directional sector are then selected for three wind speed bands where the thrust coefficient is high and broadly consistent (see Fig. 1): 6 \( \pm 0.5 \), 8 \( \pm 0.5 \), and 10 \( \pm 0.5 \) m s\(^{-1}\). The wind speeds are the average for the free-stream turbines in the first column and are converted from the power curve in Fig. 1.

(iv) Data are selected only if the five upwind turbines are operational. Data are only included in the analysis if two simultaneous 10-min observation periods meet the data selection criteria from (i) and (ii). This criterion is applied in an attempt to select only stationary conditions. However, this also limits the number of observations in each category.

Power data from each turbine are then composited within each of the combined selection criteria (wind direction and free-stream wind speed) to compute an ensemble mean and standard deviation for each of the 21 conditions (i.e., seven directional classes and three wind speeds). Recall that these cases have been deliberately selected to show the maximum power loss due to wakes and are not representative of the whole dataset. Clearly, there is still variability in the atmospheric conditions (turbulence, stability, etc.) that is not accounted for by the ensemble means. As in most operational wind farms, there are no detailed upwind and downwind wind and turbulence profiles through the atmosphere to characterize larger-scale atmospheric conditions to assist in explaining the relatively large variance. Downwind profiles are available at two masts at each wind farm but have proved difficult to relate to the wake measurements (Frandsen et al. 2007, 2009). The number of observations in each case is still fairly limited—this is mainly caused by the stationarity requirement. Over time, the creation of a larger dataset may allow additional binning, for example, using turbulence at hub height in the free stream.

To examine the single wake, the average power at each turbine is calculated for each wind speed and direction bin. The average power in each column of the wind farm is then calculated (turbines A1 . . . A9, B1 . . . B9, and so on to H1 . . . H9 at Nysted; and 1 . . . 8, 11 . . . 18, and so on to 91 . . . 98 at Horns Rev, see Fig. 3). The power in each column is normalized to the power in the first column (A1 . . . A9 at Nysted, 1 . . . 8 at Horns Rev).

3. Empirical analyses of wind turbine wakes

a. Single wakes and wake width

As discussed above, observations from the two wind farms have been selected using the same criteria to generate comparable data ensembles, and thus any differences should be largely reflective of the different spacing (7 \( \times \) 7 D) at Horns Rev and (5.8 \( \times \) 10.5 D) at Nysted, although there may also be some trade-off between the turbine spacing in the downwind and lateral directions. As shown in Fig. 1, there are differences in the thrust coefficients \( C_t \) for the turbines deployed at Horns Rev and Nysted. To evaluate whether this is likely to be a cause of any differences in the wind speed and power production at the wake centerline, one can use the axial induction factor \( a \). The axial induction factor \( a \) is defined as the ratio between the wind speed in the wake \( U_0 \) and the wind speed exactly downwind of the rotor \( U_1 \) (see, e.g., Manwell et al. 2002):

\[
a = \frac{U_0 - U_1}{U_0},
\]

and \( C_t \) can be related to \( a \) by

\[
C_t = 4a(1 - a).
\]

Thus, \( U_1 \) can be calculated by combining Eqs. (1) and (2) to give \( U_1 \) for a range of wind speeds for each turbine. For the range of wind speeds used here (between 5 and 11 m s\(^{-1}\)), the difference between wind speeds at the rotor for the two turbines is a maximum of 0.43 m s\(^{-1}\) at 7 m s\(^{-1}\), indicating this is not likely to be a major cause of differences in the wake behavior, although it should be considered in the context of uncertainty in the observations.

As described above, wind turbine wakes can be characterized using a range of different metrics, including wake width (i.e., the horizontal distance from the wake centerline to the edge of the region of velocity deficit), the wake depth (i.e., the velocity or power deficit at the wake centerline), and the total momentum deficit integrated over the entire wake. Herein, we focus principally on the former two, and define the wake width as the distance on each side of the centerline at which the power deficit is within \( \pm 5\% \) of the free-stream power.

Figure 4a shows the average normalized power by wind direction for the second column (B1 . . . B9 at Nysted, 11 . . . 18 at Horns Rev). This gives a general indication of the wake width and depth observed in the second column.
(i.e., for single wakes), albeit with relatively poor resolution (i.e., wind direction sectors with 5° resolution). With the exception of the point at ER158 at Nysted, there is good agreement between both the wake width and the wake depth derived from the data collected at Horns Rev and Nysted. The average difference between the normalized power for the remaining points for the directions shown is 0.03 (equivalent to about 4 W), and all points from one site are within the standard deviation of the other, suggesting that the single wake is very similar for both sites despite the different turbine spacing (10.5 D at Nysted and 7 D at Horns Rev). At the wind speeds shown (8 m s⁻¹), lateral wind shear results in ±2% inflow variation on the wind speed at Horns Rev and ±3% at Nysted, which cannot be verified from the measurements.

A likely source of the asymmetry in the observations at Nysted is the influence of land in direction sectors from 285° northward. Modeling using WASP (Mortensen et al. 2005) indicates wind speeds in these sectors are enhanced by about 2.8% due to a speed-up effect as flow moves from land to sea about 14-km distance from the northwest corner of the wind farm. The dashed line in Fig. 4a indicates the wake shape at Nysted assuming the value at ER+5° is equivalent to that at ER−5°.

The following approximation for the width of a single wake $D_w$ was proposed by Frandsen et al. (2006):

$$D_w = \left(\frac{b}{k + \alpha s} + \alpha s\right)^{1/k} D,$$

where $D$ is the rotor diameter; $s = x/D$, where $x$ is the downwind distance; and $\alpha$, $\beta$, and $k$ are parameters that have to be determined experimentally. Based on recommendations given in Frandsen et al. (2006), the experimental parameters are set as $\alpha = 0.05$, $\beta = 1.4$, and $k = 3$. The expansion of the single wake to the downwind distance of the second column calculated by Eq. (3) is 1.29 $\times$ D at Nysted (i.e., 107 m) and 1.26 $\times$ D (i.e., 101 m) at Horns Rev, which equates to wake angles of 14.2° at Nysted and 20.4° at Horns Rev. As shown in Fig. 4, the predictions from (3) applied to Horns Rev agree well with the empirical estimate derived from the observations (approximately 25°). However, there is a relatively large discrepancy between the predicted versus observed values at Nysted, where the approximation given in (3) leads to an underestimation of the wake width relative to the observations. These comparisons thus indicate the need for further research to determine the correct values for the empirical coefficients in (3), and further, that the differences in single-wake expansion at Horns Rev and Nysted due to the differences in turbine spacing are within the experimental uncertainty.

b. Lateral wake merging

In the case of a single wake, momentum can be drawn in to the wake to reduce the velocity deficit from above, below, and either side. However, within a relatively short downwind distance the wake impacts the ground, and then in a multiple-turbine array they will merge laterally with wakes from neighboring turbines. Once the wakes merge laterally, the sole momentum reservoir available to supply momentum to the wake is from above. It is postulated that this lateral merging will manifest as 1) an erosion in the single-wake integrity when viewed as depicted in Fig. 4 as a variation in power variation and 2) a variation in power variation due to the differences in turbine spacing.
deficit with horizontal direction, and 2) an inflection point in the power deficit decay when viewed as a horizontal transect through the wind farm (as in Fig. 5).

Assuming the single-wake width is 20° at the second column of turbines, the distance $D_w$ can be calculated from

$$D_w = X \tan \theta.$$  \hspace{1cm} (4)

For a distance of 7 $D$ (i.e., the turbine spacing at Horns Rev), $D_w$ is 2.55 $D$, while for 10.5 $D$ (i.e., the turbine spacing in the W–E direction at Nysted), $D_w = 3.82 D$. For wakes to merge laterally, $D_w$ must be 5.8 $D$ at Nysted and 7 $D$ at Horns Rev. Assuming the wake angle remains at 20°, this would occur at a downwind distance from the first turbine column of 15.9 $D$ at Nysted and 19.2 $D$ at Horns Rev. This is in broad agreement with the observations shown in Figs. 4b and 4c (and Fig. 5, see next section), which indicate wake merging is evident at turbine columns 2–3 downwind (i.e., the wake profiles cease to show a well-defined Gaussian shape), although it should be recalled that wakes are merging downwind as well as laterally.

An interesting feature in data from both wind farms is that at the ER there is relatively little variation in the normalized power at the center of the wake regardless of the column number. This indicates that the wake superposition moving downwind has little impact on the depth of the power deficit at the wake center, or alternatively, that the power deficit at the wake center is mainly driven by the nearest upstream turbine. It further suggests that wake meandering has little or no impact on the power deficit at the wake center. Moving away from the center of the wake and examining data for the slightly oblique angles across the wind farms, the power deficit tends to increase moving downwind through the wind farm, although the magnitude of the deficit change with distance is not uniform or symmetric. At Nysted, from column 5 (E1 ... E9) and farther east the power is always below 75% of the free stream, whereas at Horns Rev this does not occur until column 7 (71 ... 78). This suggests that wakes are merging laterally from the third or fourth row, assuming that this process is responsible for the general wind speed decrease outside of the apparent direct wake. This is further examined by considering the power deficit by row in the next section.

c. Power deficit by downwind distance

Despite differences in the wind farm layouts (Fig. 3), when the average normalized power is computed as a function of downwind distance for the various wind speeds and directions, there is a high degree of similarity in data from Horns Rev and Nysted (Fig. 5). As indicated in the previous section, for flow down an ER (i.e., wind direction of 278° at Nysted and 270° at Horns Rev) there appears to be little influence of lateral wake merging, and after the second turbine these observations reflect the center of the wake asymptote to approximately 60% of the free-stream power. The power deficit at the second turbine is considerably reduced from the free stream but at subsequent turbines the power reduction is very small. For ER−5° and ER+5° the pattern is symmetric at Horns Rev, whereas at Nysted there is deviation at ER+5°, as discussed in the previous section. In general, for these wind directions the power reduction to the second turbine is large, but the reduction in power continues for each subsequent turbine. At ER ± 10° and ER ± 15°, the pattern evolves to one of a smaller change in the power from the first to the second turbine (and sometimes the third turbine) and a more distinct general downward trend moving through the wind farm. By the last turbine of the wind farm (8 wind turbines at Nysted and 10 at Horns Rev), the power output has reached a similar level to that for ER. This may be a result of the lateral merging of wind turbine wakes as discussed above. The difference between the (ER and ER ± 5°) cases and the (ER ± 10° and ER ± 15°) cases cannot be due to downwind merging, changes in the turbulence/vertical exchange over the wind farm, or the impact of turbine wakes with the ground, because the downwind distance at the end of the wind farms is very similar in both direct and more oblique flow through the wind farm.

As shown above, the width of the single wake is about 10° on either side of the wake centerline over the distance between the turbines in each row (10.5 $D$ at Nysted and 7 $D$ at Horns Rev). Figure 4 illustrates that power output (and therefore wind speeds) in-between the second turbine wakes are close to the free stream, and this can also be true at the third turbine. This can be seen by a return to a normalized power of 1 close to the edges of the wake (ER ± 15°) for the second column. Deeper in the wind farm to turbine column 5 or 6, although the central wake shape may still be discerned, in-between the power output is also reduced. The question is whether this is due to lateral wake merging or a different effect, such as pressure variations or the general momentum reduction outside of the wake area. It is clear from previous work that this general reduction in power output does not occur in small offshore wind farms in Denmark (Barthelmie et al. 2007b, 2006).

Power observations for equivalent wind speeds of 10.0 ± 0.5 m s$^{-1}$ show similar results to those depicted in Fig. 5 for a free-stream wind speed of 8 m s$^{-1}$. For equivalent wind speeds of 6.0 ± 0.5 m s$^{-1}$ there are similar results at Horns Rev; however, at Nysted by ER ± 5°, there is more of a general decrease moving through the
FIG. 5. Comparison of normalized power as a function of distance into the wind farm based on observations at Horns Rev and Nysted for a wind speed of 8.0 ± 0.5 m s⁻¹ and directions as shown in the lower-left-hand corner of each panel (Nysted/Horns Rev) ± 2.5°. The ER is experienced for a wind direction of 278° at Nysted and 270° at Horns Rev. Each point represents the mean normalized power of a column of turbines shown by distance from the first column, whereas the bars denote uncertainty bounds as ±0.5 * standard deviation on either side of the mean.
wind farm, which is likely because of the topographic effects associated with land in the upwind direction. This emphasizes the need for careful data screening and demonstrates that the results are still highly dependent on how data are selected, and this leads to a high degree of uncertainty in the model comparisons. Also, the standard deviation of the observations of normalized power as a function of turbine column is relatively large, ranging from 0.13 to 0.43 at Nysted and 0.07 to 0.51 at Horns Rev for the 8 m s\(^{-1}\) case. This has implications for the model evaluation presented in section 4.

4. Modeling wind turbine wakes: Model evaluation

An overview of the main features of the models used in this analysis is given in Table 2 and further details can be found in Barthelmie et al. (2009). Most of these models were previously evaluated for smaller wind farms (Barthelmie et al. 2004), single wakes (Barthelmie et al. 2006), and case studies (Barthelmie et al. 2009). Here, we extend this research using the ensemble statistics developed as described in section 2 and show results for a larger range of models than was available in our earlier research. This is also the first direct comparison of wakes at Nysted and Horns Rev. Some of the models applied are industry standard models, for example, WAsP and WindFarmer, whereas others are primarily research models. One major difference between WAsP and the other models is that WAsP uses the “top-hat” (flat) formulation for the wake profile rather than a near-Gaussian shape that more realistically fits the observations. Because the total momentum deficit in the wake is conserved, the use of the top-hat wake profile gives a smaller velocity deficit at the center of the wake. It is worthy to note that WAsP is being used beyond its recommended limits to simulate wakes for the ensemble statistics. In the following, WAsP is employed with a standard offshore wake decay coefficient of 0.04 for cases in ±5° sectors, while the other models and observations are shown for ±2.5° sectors. In addition, for WindFarmer and Wakefarm this is the second time they have been applied to data from Horns Rev, and based on the first evaluation, model improvements have been undertaken. Results from the National Technical University of Athens (NTUA) CFD model are the first results from the full CFD code run for three central rows of the wind farm. If wind farms are far enough from the coast not to experience consistent gradients of wind speed over the area of the wind farm, the rows other than the external rows are expected to experience similar wake behavior. Therefore, to limit the computing resources required, three rows may be simulated and substituted for the remaining turbines (excluding the external row).

There are a number of issues in comparing model simulations and wind farm observations of power losses in wakes that were detailed in Barthelmie et al. (2009). Similarly, it is difficult to make exactly the same simulations with models of different types even after the main variables, such as thrust coefficient, wind speed at hub height, free-stream wind profile, etc., have been set. Examples of this are that it is not possible to run WAsP for extremely narrow wind speed and direction bins because WAsP relies on a Weibull fit to the wind speed observations. For CFD, one issue is to accurately determine the turbulence profile and to recall that, for narrow sectors, the wake is centered on the given direction and no directional variability is included. As shown in Table 2, there are also practical issues relating to computing resources. Running a full wind farm simulation in WAsP takes of the order of minutes, while in the NTUA model requires a time scale of days to make even one simulation limiting the number of runs performed.

To provide a quantitative evaluation of model performance versus the observations, the root-mean-square error (RMSE) of normalized power was calculated for each case as

\[
\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (P_{\text{observed}} - P_{\text{modeled}})^2}{n}},
\]

where \(P\) is the normalized power at each turbine (observed or modeled) where the power is normalized to the free-stream power, and \(n\) is the number of turbines (\(n = 72\) for Nysted, and \(n = 80\) for Horns Rev).

As discussed above, it is difficult to synthesize model performance in a single metric, particularly for such a large number of model applications. However, one metric that provides at least some measure of the degree of agreement between models and observations is the wind farm efficiency \(e\) defined as

\[
e = \frac{1}{n} \sum_{i=1}^{n} \frac{p_i}{p_0},
\]

where \(p_i\) is the power of each turbine \((i)\), \(p_0\) is the power of the free-stream turbine, and \(n\) is the number of turbines in the wind farm. This metric is used below to provide an overview of model performance.

a. Single wakes and wake width

Figure 6 provides an example of the normalized power observed and modeled for the second column of wind turbines at Nysted and Horns Rev for a free-stream wind speed of 8 ± 0.5 m s\(^{-1}\). All four models capture the wake
<table>
<thead>
<tr>
<th>Name</th>
<th>WAsP</th>
<th>WindFarmer</th>
<th>Wakefarm</th>
<th>NTUA</th>
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<tr>
<td>Company</td>
<td>Risø DTU</td>
<td>Garrad Hassan</td>
<td>ECN</td>
<td>NTUA</td>
</tr>
<tr>
<td>Type</td>
<td>Linearized wind farm model</td>
<td>Axis-symmetrical, parabolized CFD</td>
<td>Parabolized CFD</td>
<td>CFD, fully elliptic 3D turbulent</td>
</tr>
</tbody>
</table>

Near-wake model (<3 \(D\))
- No
- Initial profile based on parameterization by Ainslie (Ainslie 1988)
- Lookup table with pressure gradients calculated with free-vortex wake method
- Resolved within the CFD model, rotor disk as momentum sink (force evaluated from thrust coefficient)
- Near Gaussian
- Valid for the whole wake (in theory)

Far wake model
- Linear based on Jensen (1983)
- RANS, Eddy viscosity turbulence closure
- \(k-\varepsilon\) turbulence closure
- Resolved within the CFD model, \(k-\varepsilon\) turbulence closure
- Top hat
- Near Gaussian
- Dictated by model
- Dictated by model
- \(>2D\)
- Valid for the whole wake
- \(>3D\)
- Valid for the whole wake

Main wake expansion parameter
- Wake decay coefficient broadly related to roughness, 0.04 used here
- Turbulence
- Turbulent kinetic energy
- Free-stream atmospheric conditions, turbulence kinetic energy
- Multiple wakes included and resolved within the whole wind farm computational domain

Multiple-wake model
- Uses sum of squares based on Katic et al. (1986)
- Consecutive downstream modeling of individual wakes with additional empirical correction in large wind farms
- Consecutive downstream modeling of individual wakes with modified axial force coefficient
- Multiple wakes included and resolved within the whole wind farm computational domain

Domain information
- Uses a polar zooming grid, approximate domain size here 50 km by 50 km
- Per turbine wake one axis-symmetric, polar grid, aligned with wind direction
- Cartesian grid properly adjusted to include all wind turbine rotor disks as momentum sinks

Modification for atmospheric stability
- None, although the wake decay parameter could be modified
- Through atmospheric turbulence
- Free-stream modified using Monin-Obukhov scaling
- Free-stream profiles and turbulence kinetic energy and dissipation rate equations modified, neutral simulations here

Approximate computer time required for these simulations
- Minutes
- Minutes
- Hours
- Days

References
- Mortensen et al. (2005), Rathmann et al. (2006)
- Schlez and Neubert (2009)
- Schepers (2003)
- Magnusson et al. (1996), Politis et al. (2009)
width at both Horns Rev and Nysted to within the experimental uncertainty. WindFarmer and Wakefarm exhibit greater agreement with the observed wake depth than WAsP at Nysted, though both Wakefarm and WAsP underestimate the magnitude of the wake depth. At Horns Rev, all the models lie within the experimental uncertainty (i.e., ±0.5 standard deviation from the mean), in terms of the wake magnitude for all wind directions.

As shown in Fig. 6, when the models are applied to the entire wind farm and evaluated for the eighth column, a more complex situation is observed. Model simulations from WAsP are within the standard deviation of the observations for ER at Horns Rev, but otherwise at Horns Rev and Nysted the normalized power is overpredicted by WAsP. In other words, predicted wakes’ losses deep in the array are too small, particularly at Nysted. Results from Wakefarm and WindFarmer are mainly within ±0.5 standard deviation of the mean derived from the observations, indicating satisfactory performance. Simulations using the NTUA CFD model for Horns Rev indicate good performance in the second column and for both ER and ER ± 5° in the eighth column but underpredict wake losses for both ER ± 10° and ER ± 15°.

RMSE (m s⁻¹) for each model prediction versus observation of the mean turbine power by wind direction and wind speed class at Nysted and Horns Rev are given in Table 3. For a free-stream wind speed of 8 m s⁻¹ averaging over all wind directions considered yields an RMSE of normalized power (modeled versus observed) of 0.07, 0.06, and 0.15 for WindFarmer, Wakefarm and WAsP, based on data from Nysted. Comparable results for analyses of the Horns Rev wind farm are 0.07 for the CFD model from NTUA, 0.07 for WindFarmer, 0.08 for Wakefarm, and 0.12 for WAsP. In general, the RMSE summarized in Table 3 indicates that the models perform better (i.e., exhibit lower RMSE) for higher wind speeds (10 rather than 6 m s⁻¹) and for direct flow down the row (i.e., ER) than for oblique angles.

b. Power deficit by downwind distance

Average normalized power as a function of downwind distance for a free-stream wind speed of 8 ± 0.5 m s⁻¹ for the seven wind directions as derived from the various models and observations are shown in Fig. 7 for Nysted and Fig. 8 for Horns Rev. As shown, for all of the cases the models lie within the uncertainty bounds from the observations, due in part to the very large observational uncertainties.
variability. At Nysted, the Wakefarm and WindFarmer models appear to capture the shape of power deficit as a function of distance into the wind farm, with the exception of WindFarmer at an incident wind direction of 263°. At Horns Rev (Fig. 8), there is good agreement between models and measurements in most wind directions, except at 255° where the observed values appear to be much lower than their counterpart at 285°.

The CFD model applied at NTUA shows good performance at Horns Rev within the standard deviation of the observations, except beyond turbine 5 for ER ± 10° and ER ± 15° where wake losses are underpredicted. Overall the performance of the improved models WindFarmer and Wakefarm is very promising, whereas WAsP and NTUA require further modification to more accurately capture wake behavior deep inside the wind farm.

c. Wind farm efficiency

Wind farm efficiencies computed using Eq. (6) using the observations and each of the models are shown for each wind speed and wind direction in Fig. 9. With the exception of WAsP, the models do a good job predicting the wind farm efficiency at Nysted for both 8 and 10 m s⁻¹, with less agreement away from the wake center for 6 m s⁻¹. At Horns Rev, the results are more variable, but there is also more uncertainty in the data because there are fewer observations (1 yr of data rather than 2).

At 8 and 10 m s⁻¹, the results from the Wakefarm and WindFarmer models are consistent in the wake center and likely within data uncertainty for the other directions. WAsP seems to be performing well at 8 m s⁻¹, but when applied in the standard formulation as herein it gives very different results for the other wind speeds. The results from the CFD model applied by NTUA to Horns Rev for a wind speed of 8 m s⁻¹ are extremely promising, particularly given that this is the first application of CFD to multiple turbines in multiple rows.

5. Conclusions

Major issues in accurately predicting average power output from large offshore wind farms include the assessment of power losses due to wakes and to flow interactions with neighboring wind farms. Here, a systematic analysis is presented, which was undertaken to address whether conditionally sampled observational data from two offshore wind farms with different layouts and turbine spacing can be used to quantify the relationship between wind turbine spacing and wake losses for realistic layout configurations, to provide an evaluation of wind turbine wake models and to provide a further assessment of the entire wind farm array effects.

As expected, the two datasets show broad similarities in terms of wake depth and width, although the layout of the

<table>
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<tr>
<th>Nysted</th>
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<th>10 m s⁻¹</th>
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<td>WAsP</td>
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FIG. 7. Normalized power at Nysted for a free-stream wind speed of $8 \pm 0.5$ m s$^{-1}$: the comparison of models with observations. Error bars shown are $0.5 \times$ standard deviation of the observations. Note that observations from WindFarmer and Wakefarm simulations are $\pm 2.5$° from the center angle, but WASP simulations are $\pm 5$°. Each point represents the average normalized power of a column of turbines shown by distance from the first column.
FIG. 8. As in Fig. 7, but at Horns Rev. Note that observations from NTUA, WindFarmer, and Wakefarm simulations are ±2.5° from the center angle, but WAsP simulations are ±5°. Each point represents the average normalized power of a column of turbines shown by distance from the first column.
two wind farms in terms of turbine spacing is different. Indeed, the differences in turbine spacing (10.5 versus 7 D) are not differentiable in the data ensembles from the two wind farms despite careful data screening. Analysis of the observations suggests that the wake center is either preserved as it moves through the wind farm or is dictated by the nearest upstream wake. In-between the apparent direct wake, a general wind speed decrease is noted from about the third column (i.e., approximately 20–30 D downstream of the leading edge of the wind farm), which is ascribed here to lateral wake merging, although there are other potential causes.

Model simulations were evaluated comparing wake width and normalized power output by turbine moving through the wind farm for both the industry standard and research models. All the models were able to capture wake width to some degree, and some models also captured the decrease of power output moving through the wind farm. Root-mean-square errors indicate generally better model performance for higher wind speeds (10 rather
than 6 m s\(^{-1}\)) and for direct down the row flow than for oblique angles. The first application of CFD to the simulation of wakes in a multiturbine, multirow wind farm exhibits very promising results relative to the observations from Horns Rev.

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**REFERENCES**


