Modeling Nonlinear MEMS Beams and the Chaotic Duffing Oscillator in SPICE

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Redstone Arsenal, AL 35898

OAK RIDGE INSTITUTE FOR SCIENCE AND EDUCATION
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MOTIVATION

Model **nonlinear** behavior of **electromechanical systems** and their **supporting electronics** using **only using SPICE**

OUTLINE

- **Stress and Strain in Materials**
- **Silicon as a Linear Material**
- **Linear Resonant Microstructures**
- **SPICE Limitations**
  - Supporting Electronics for MEMS
  - Linear MEMS Model
  - Need for Other Software
- **Nonlinear MEMS Beams (BAW Example)**
- **Nonlinear Modeling**
  - Conventional
  - Extension in SPICE
- **SPICE Simulation** (Nonlinear MEMS with Supporting Electronics)
  - **Large Amplitude Behavior**
  - **Chaotic Transients**
  - **Bifurcation**
- **Conclusion**
- **Future Work**

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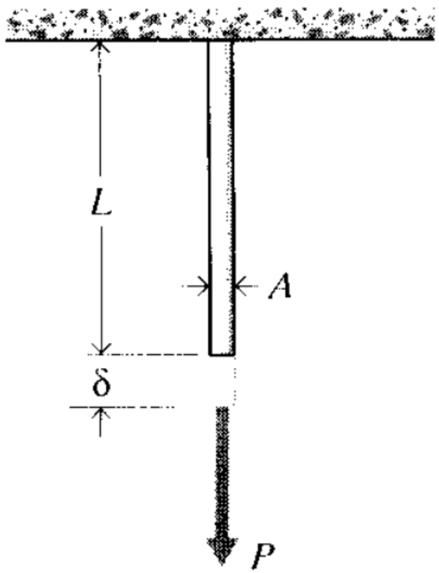
STRESS AND STRAIN IN MATERIALS



Roylance, David. "Mechanical properties of materials." *Massachusetts Institute of Technology* (2008): 51-78.

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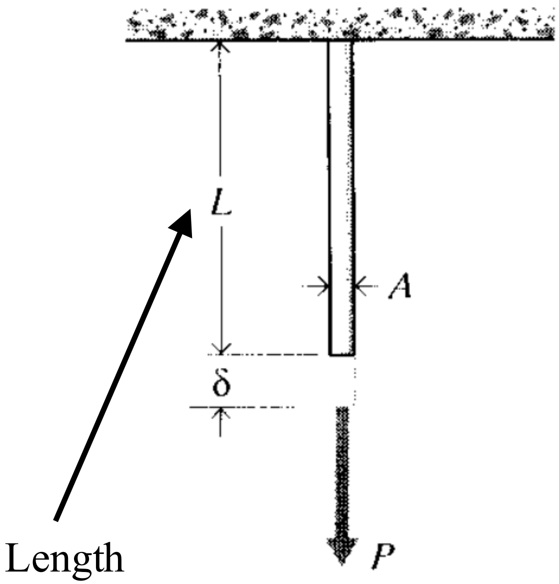

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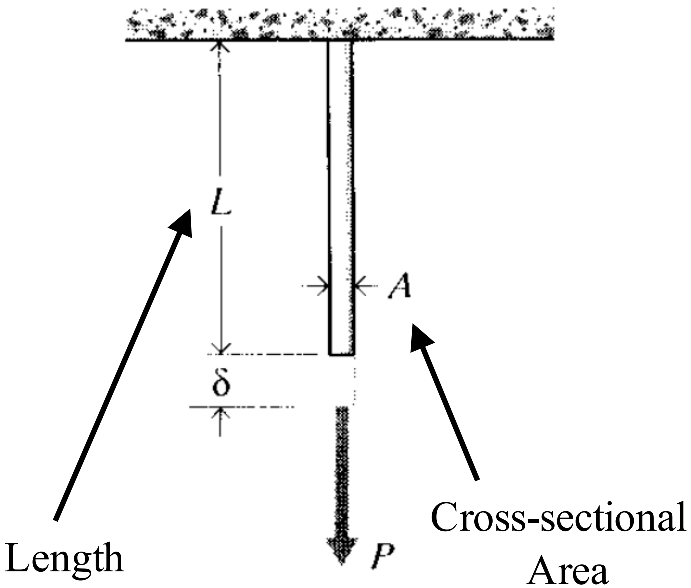

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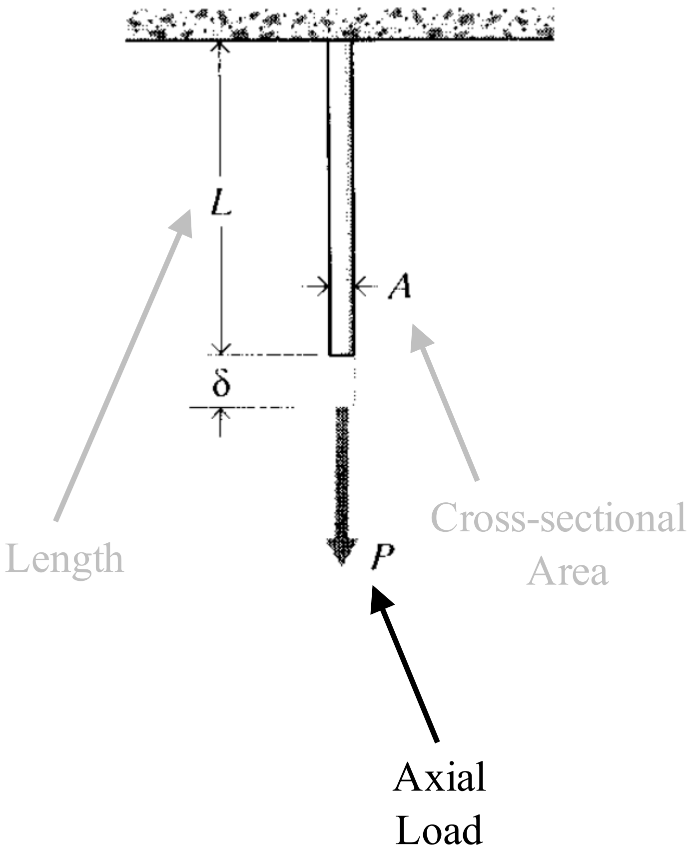

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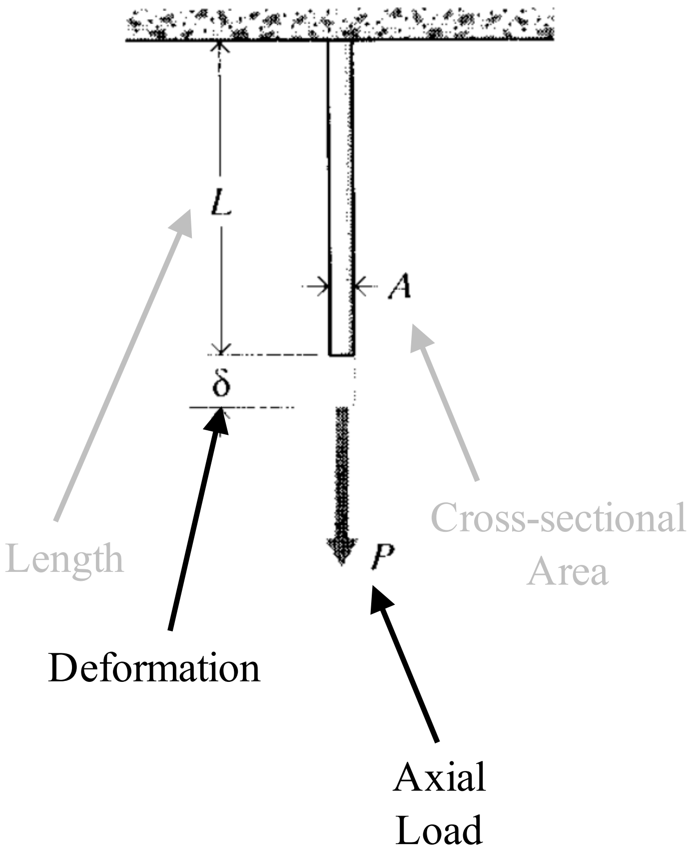

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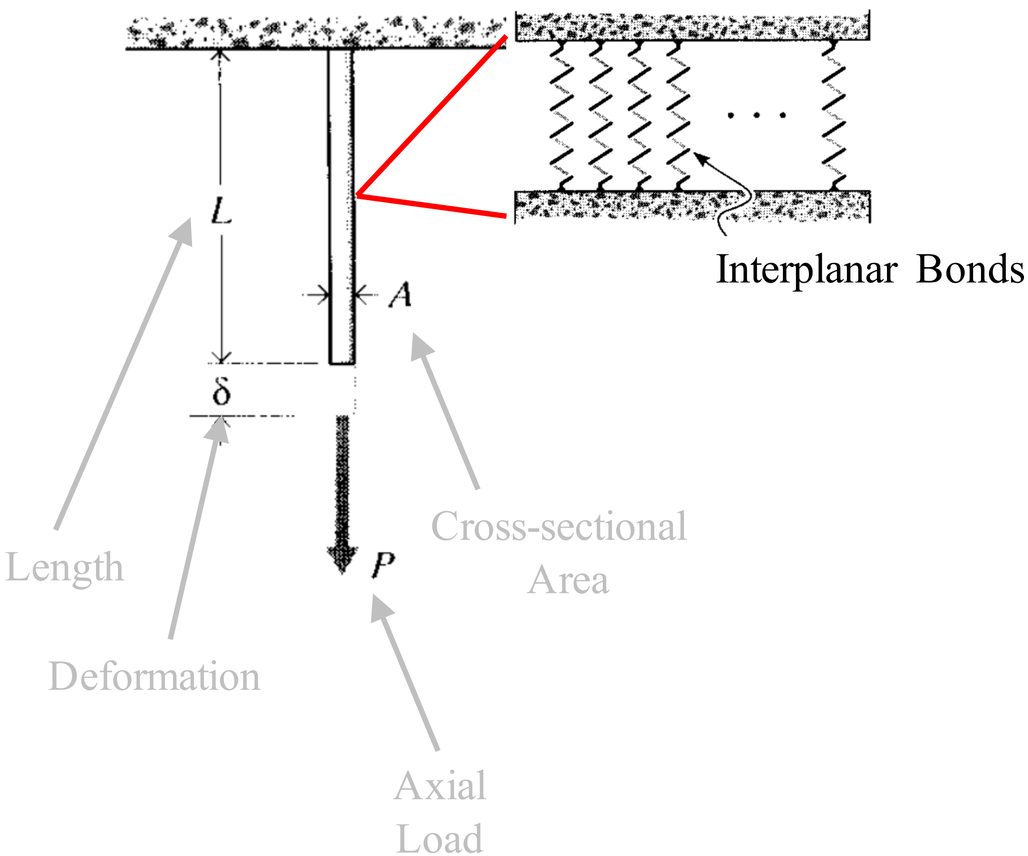

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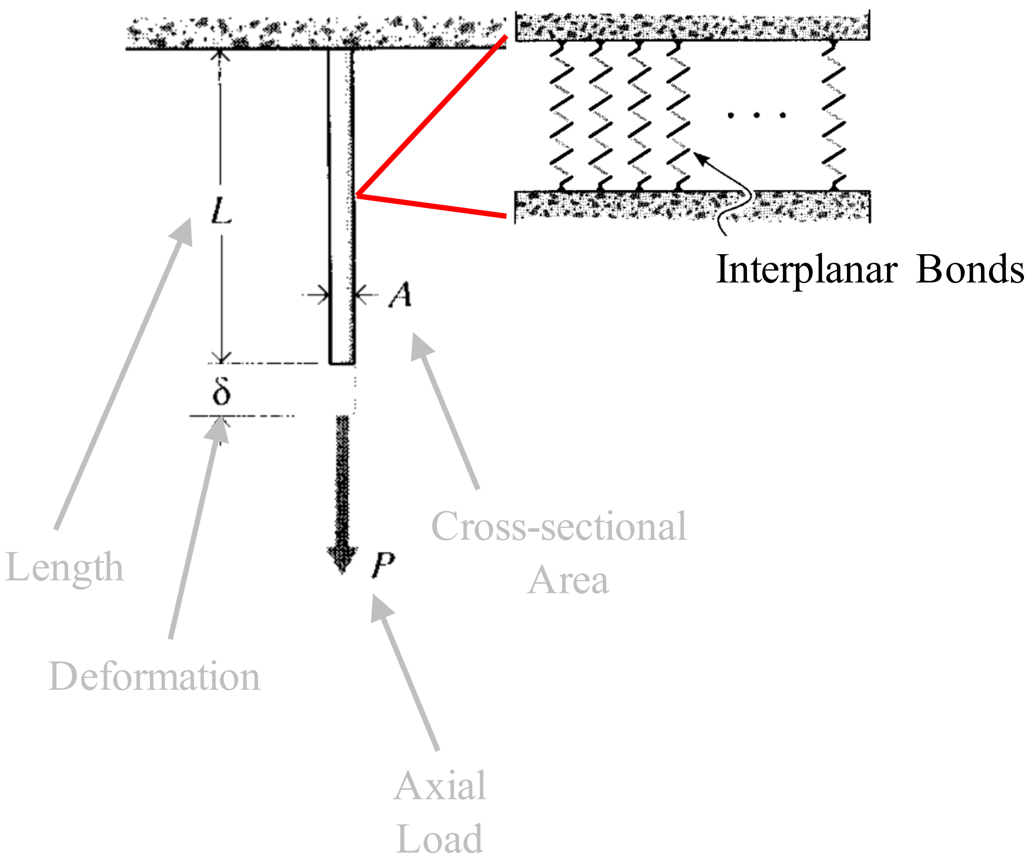

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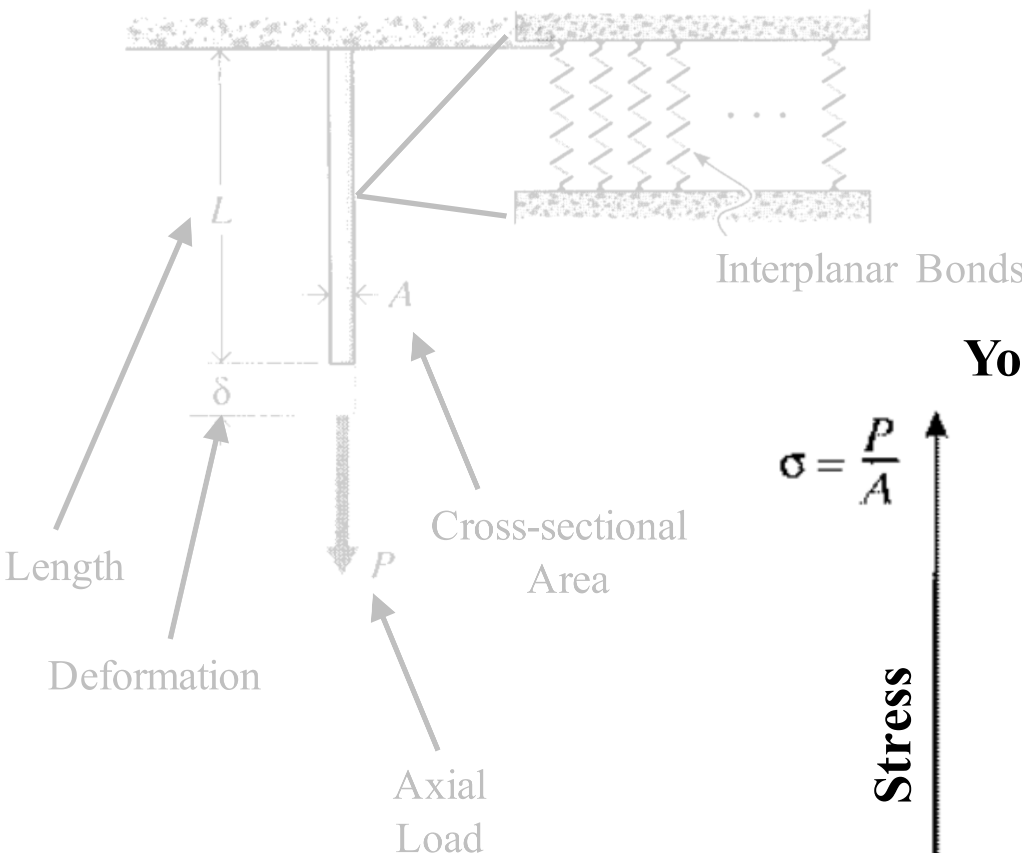
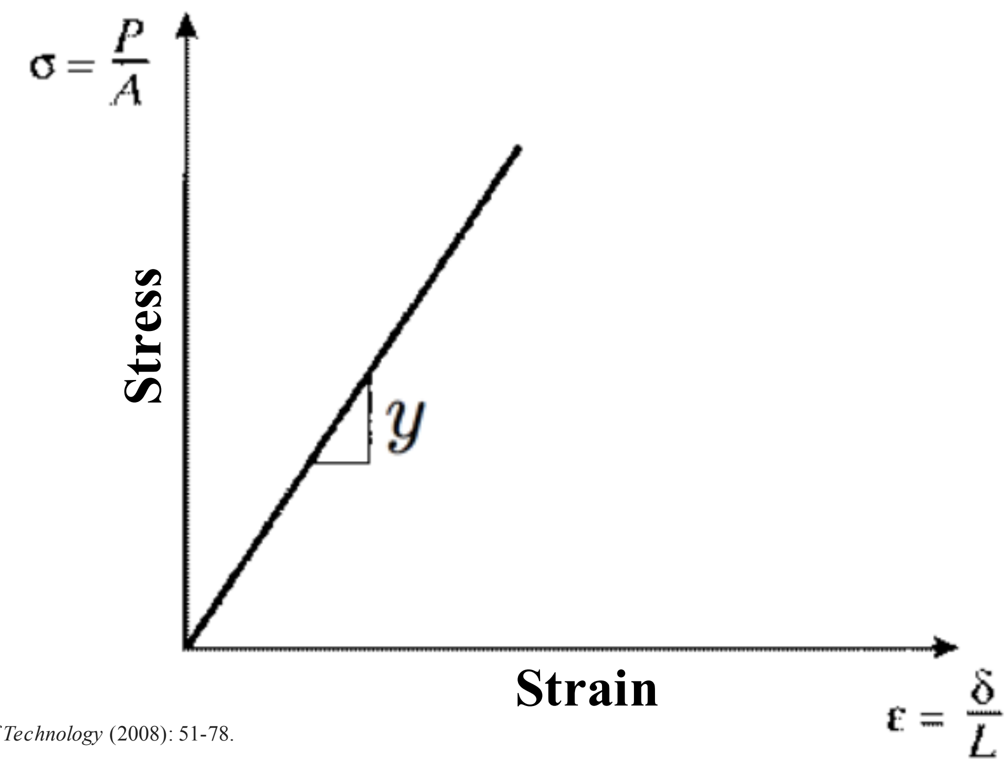


$$P = k\delta$$

Hooke's Law gives the relation

Young's Modulus: Stiffness in Tension



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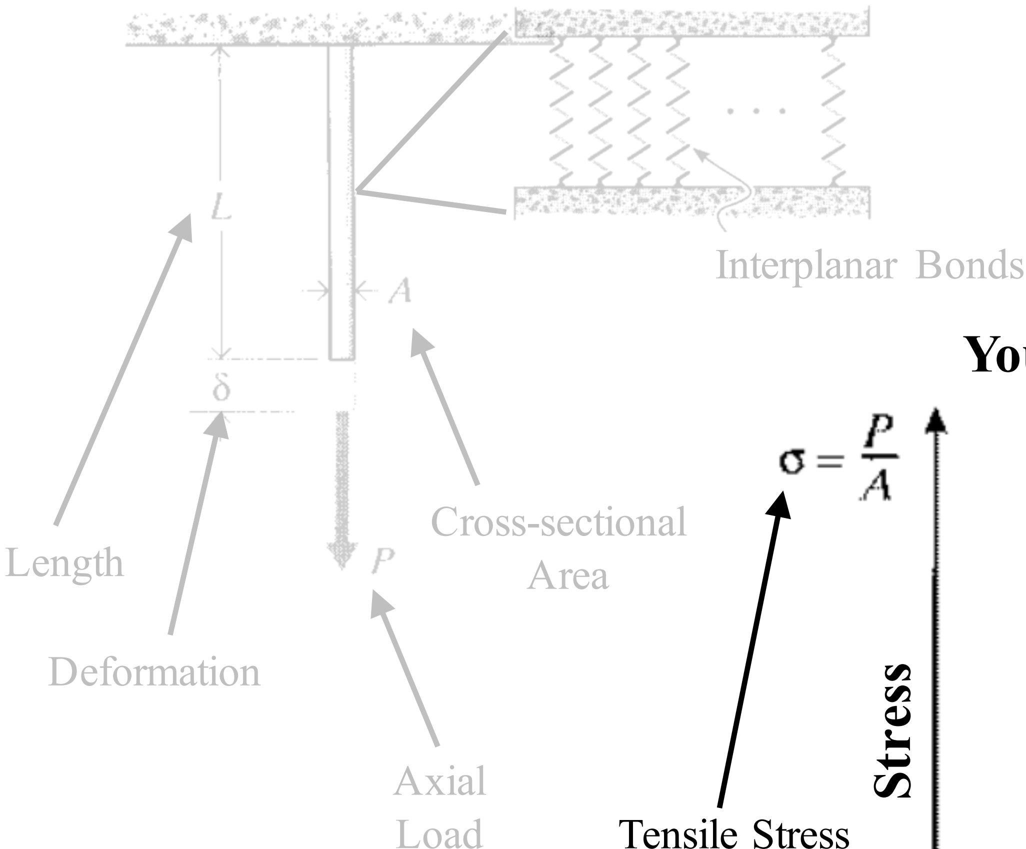

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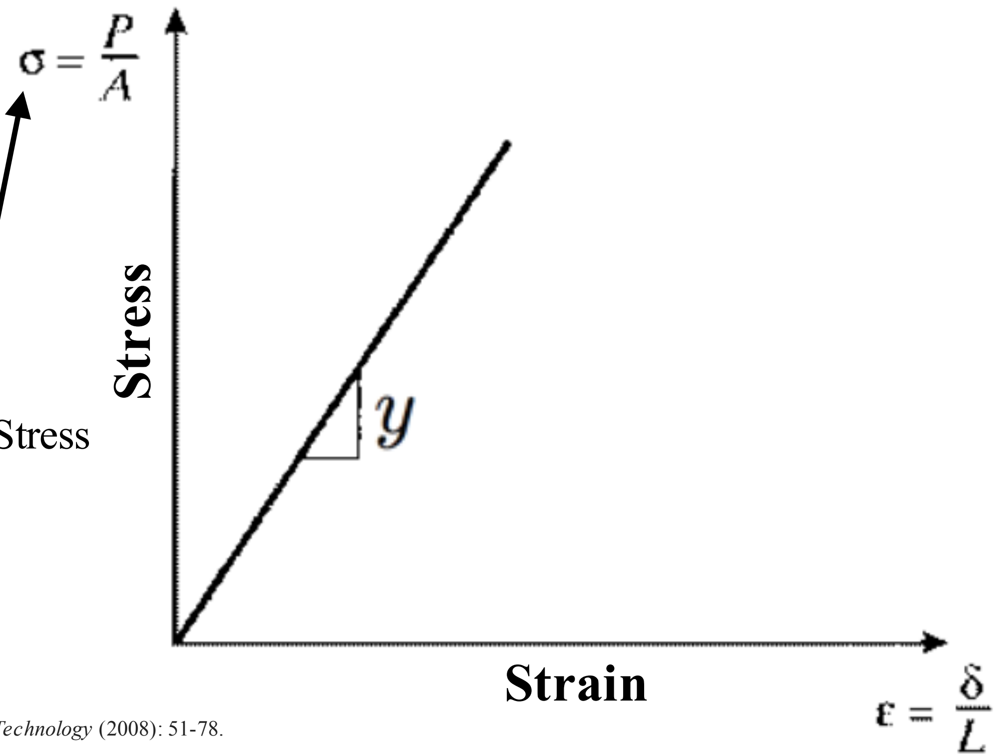

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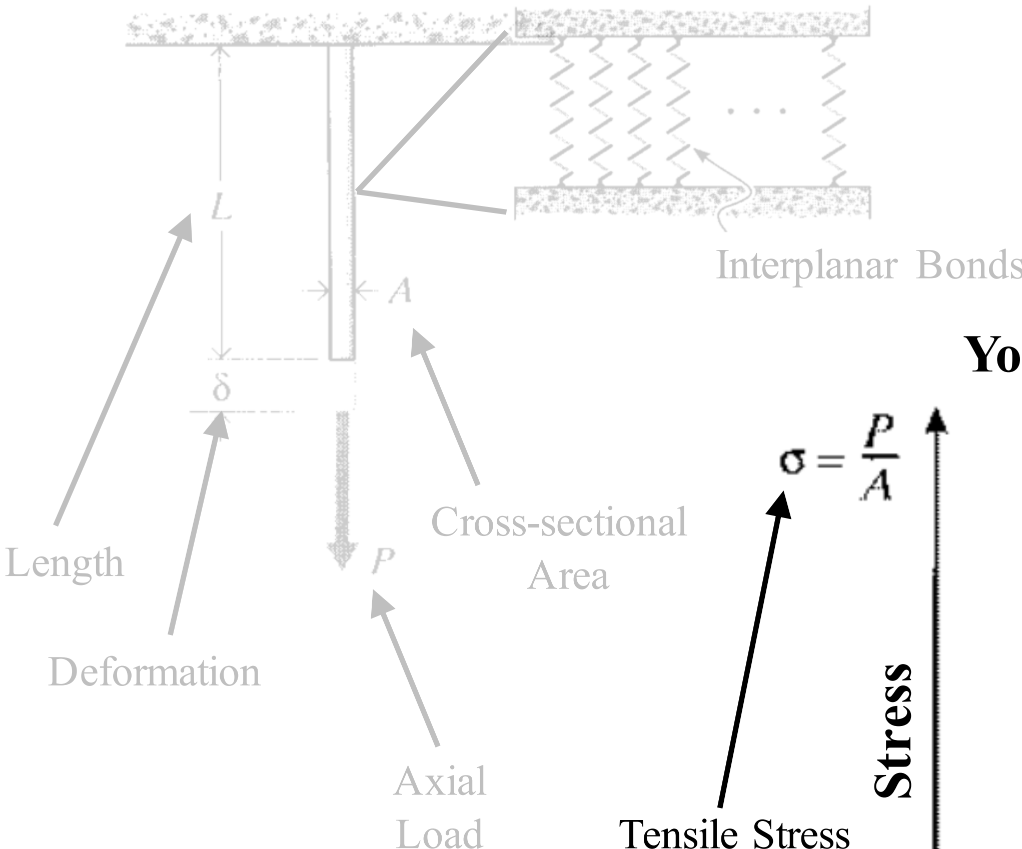

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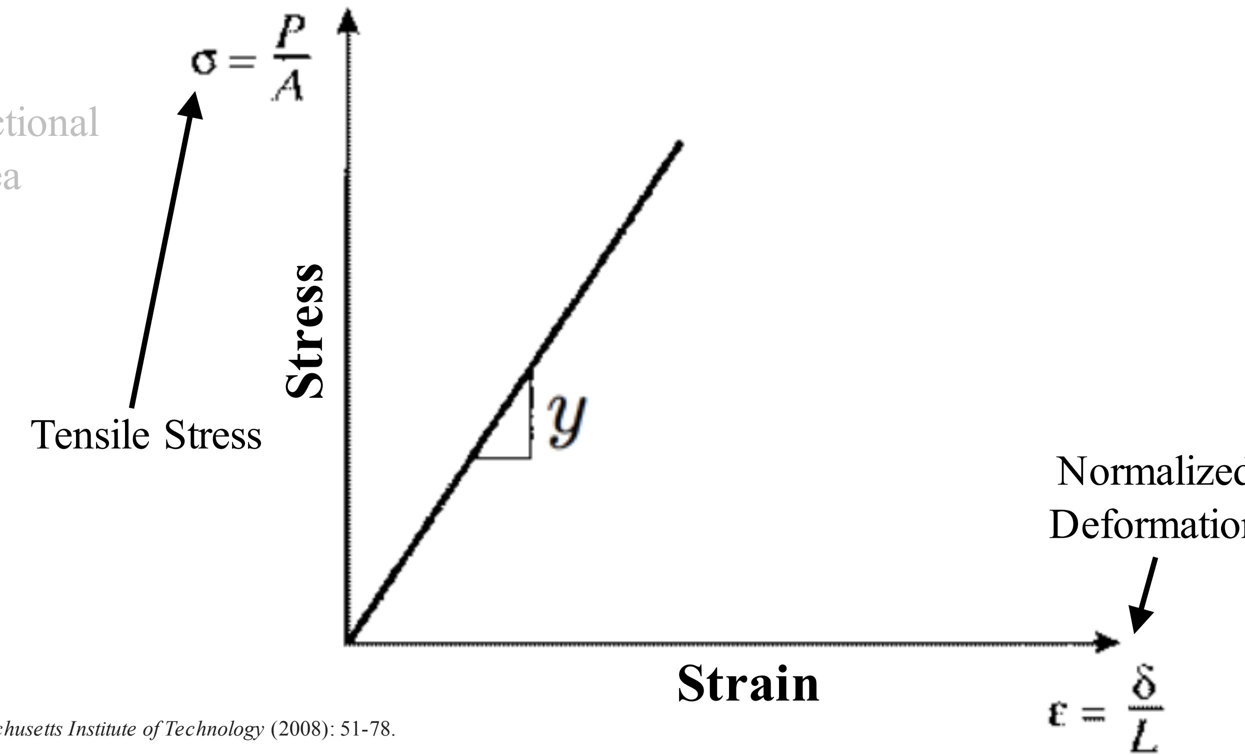

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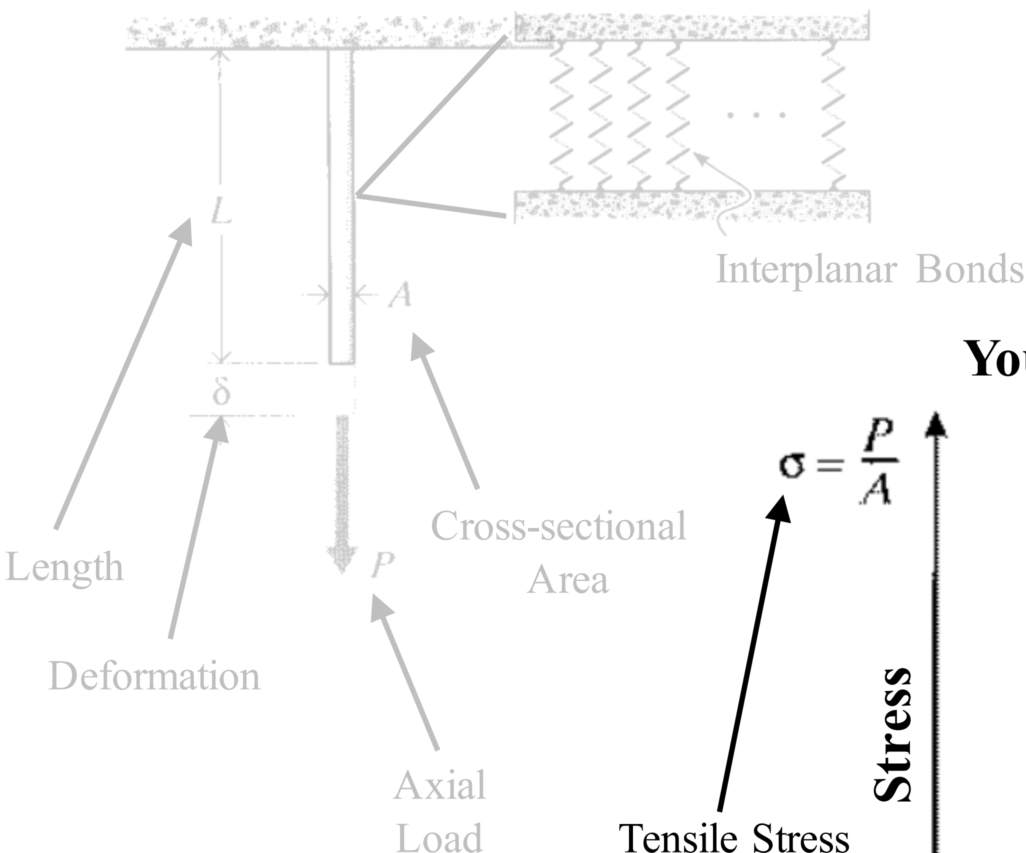

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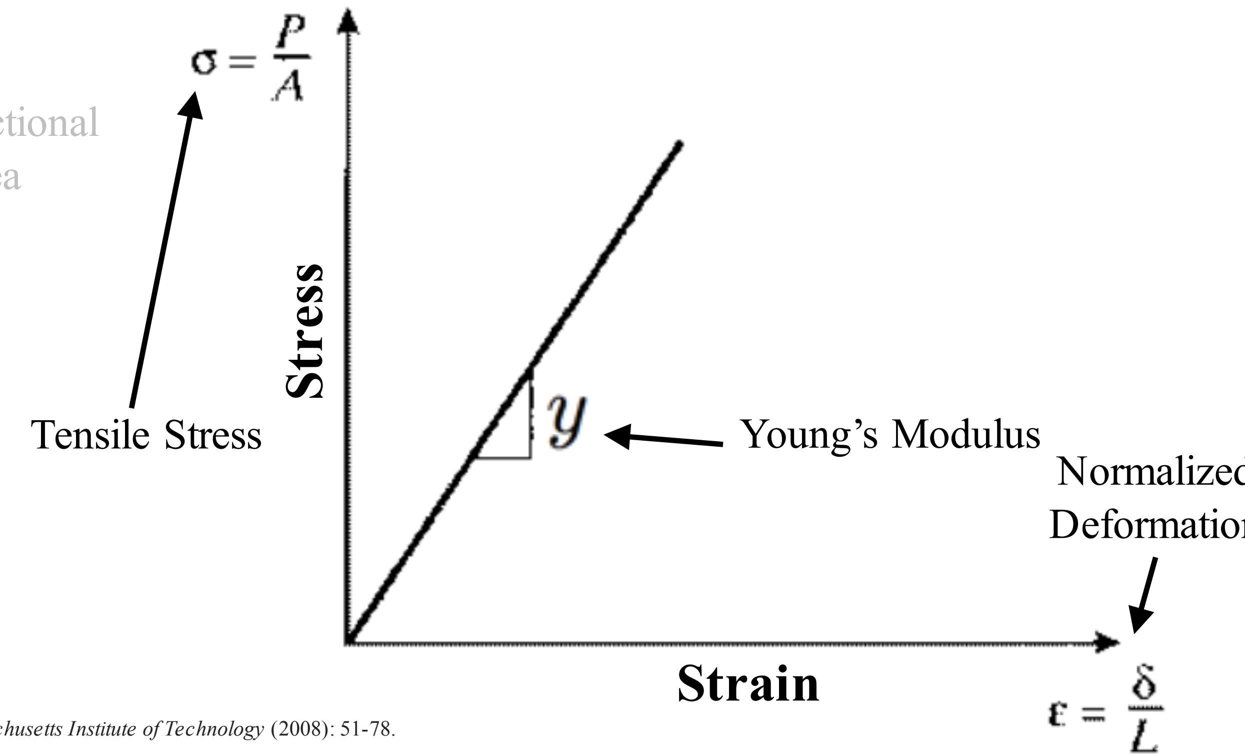

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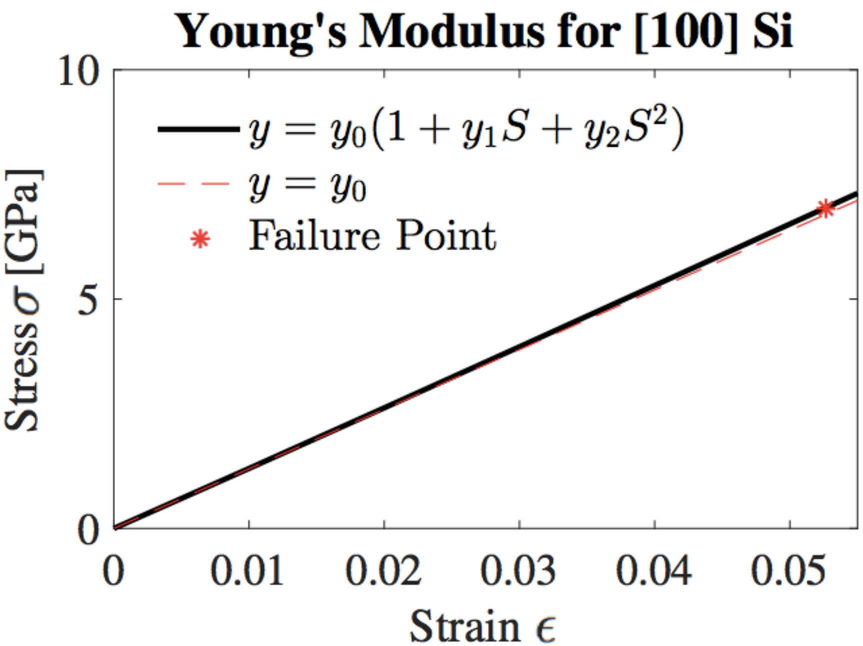
SILICON AS A LINEAR MATERIAL



$$Y_0 = 130 \text{ GPa}$$
$$Y_1 = 0.65$$
$$Y_2 = -4.6$$

Madou, Marc. "Fundamentals of microfabrication. 1997." *CRC Press LLC. Capítulo* 1 (2003): 1-50.

V.Kaajakari,T.Mattila,A.Lipsanen,andA.Oja,“Nonlinearmechanical effects in silicon longitudinal mode beam resonators,” *Sensors and Actuators A: Physical*, vol. 120, no. 1, pp. 64–70, 2005.

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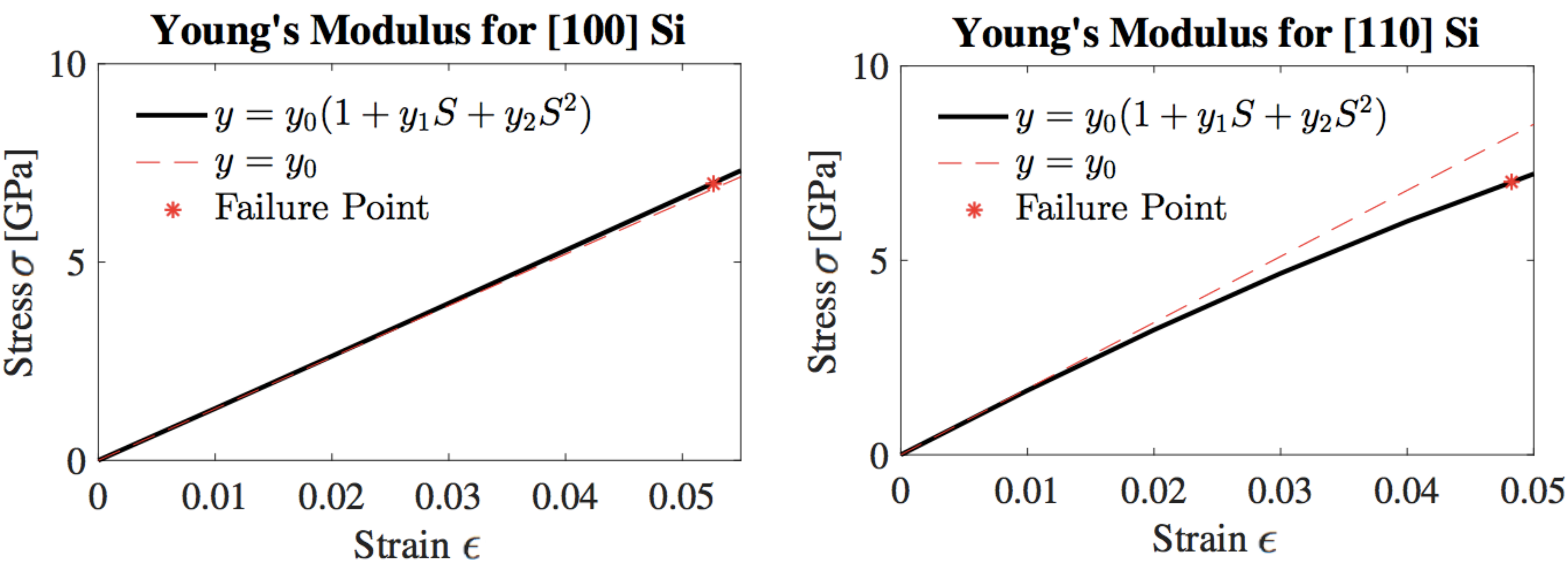

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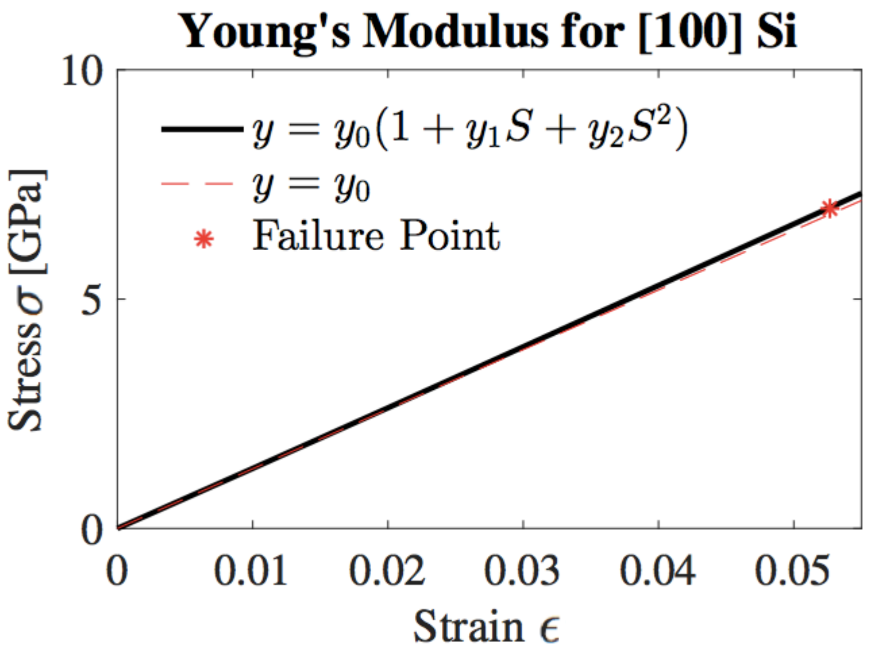

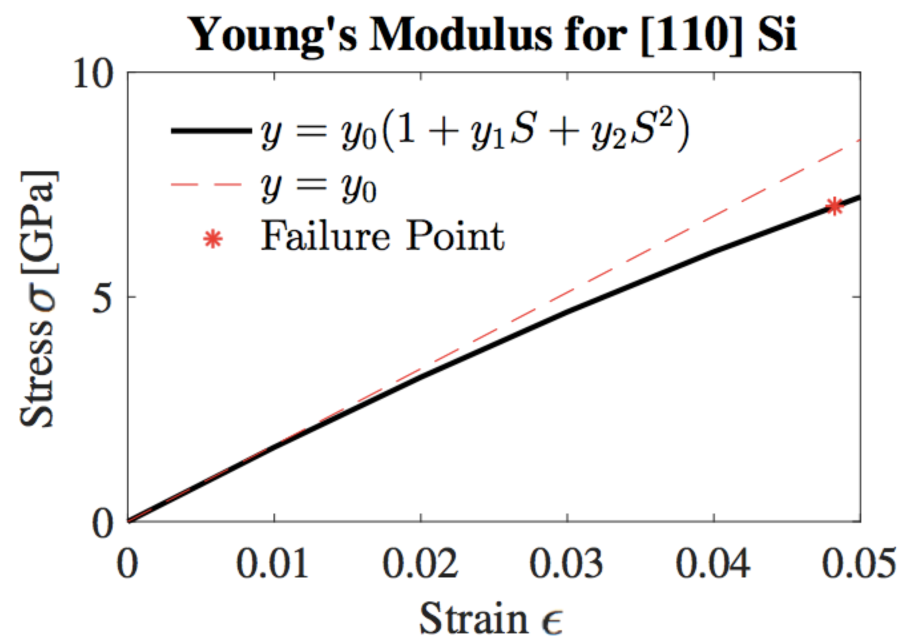

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$$y = \frac{\sigma}{\epsilon} = y_0(1 + y_1 S + y_2 S^2)$$

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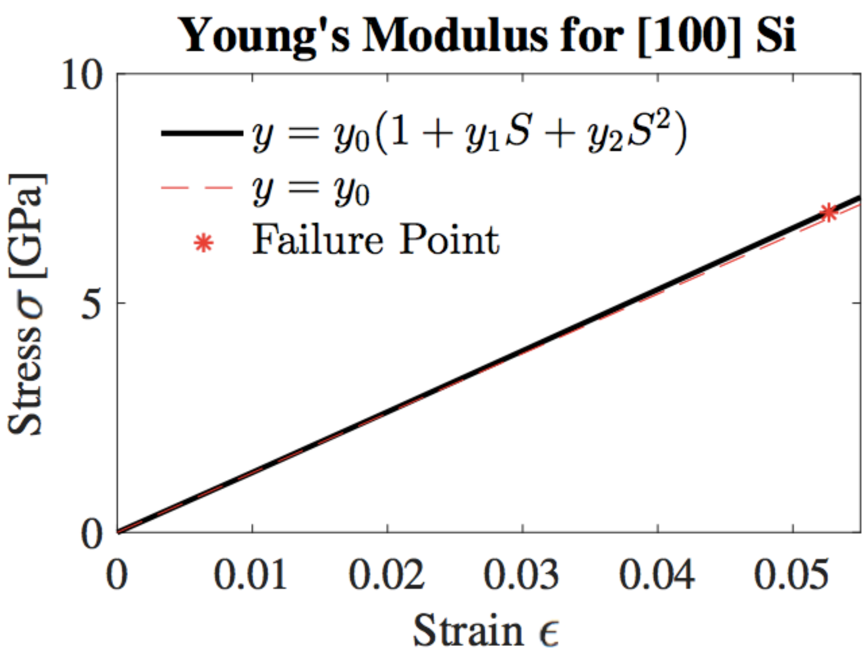

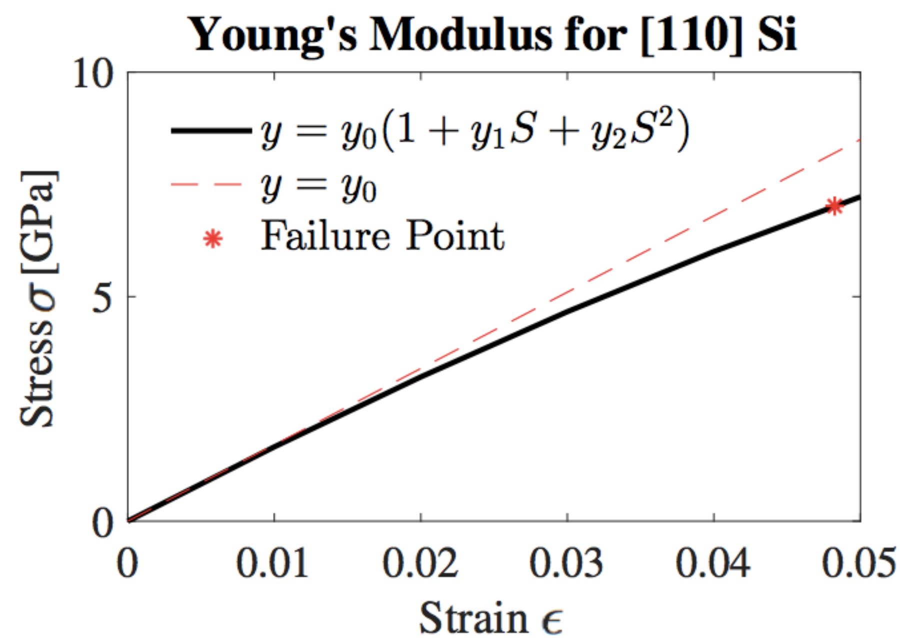

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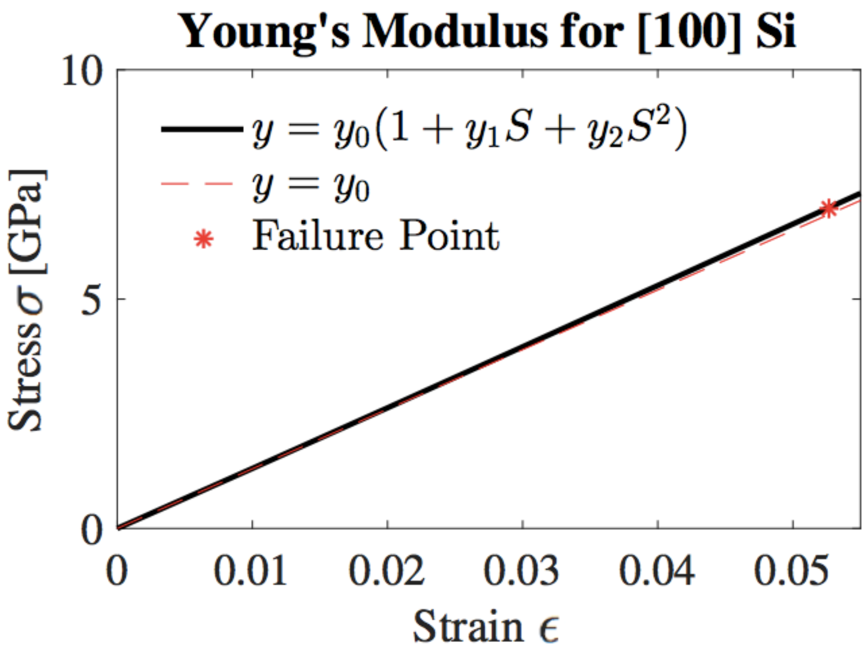

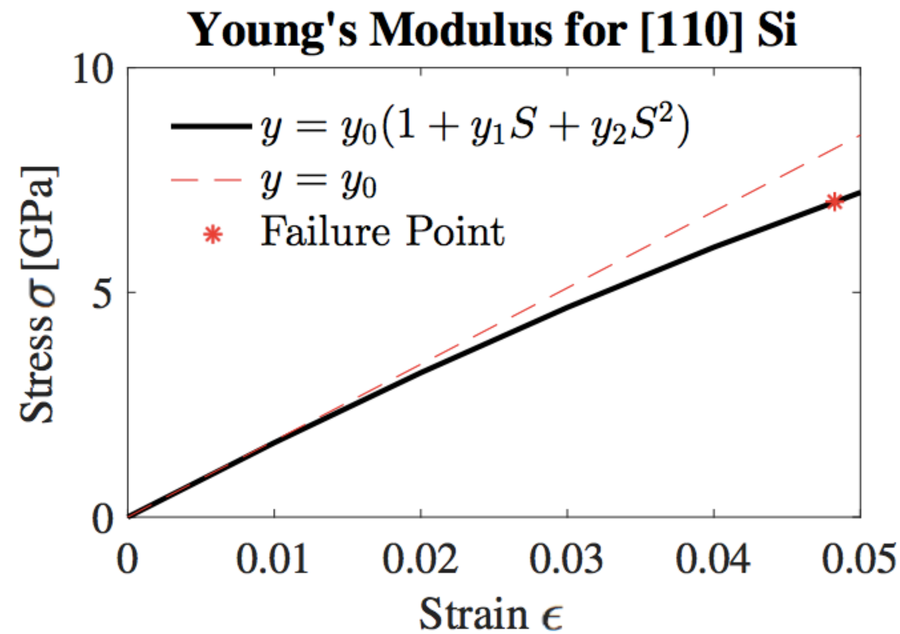

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Nonlinear Terms

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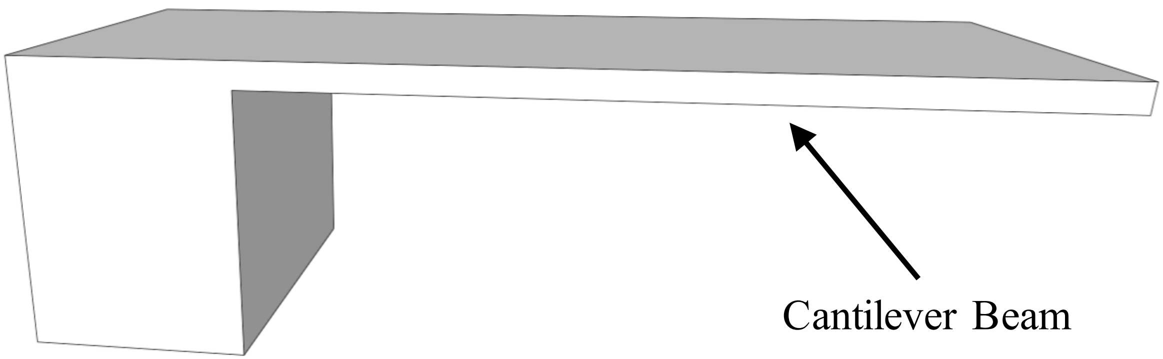

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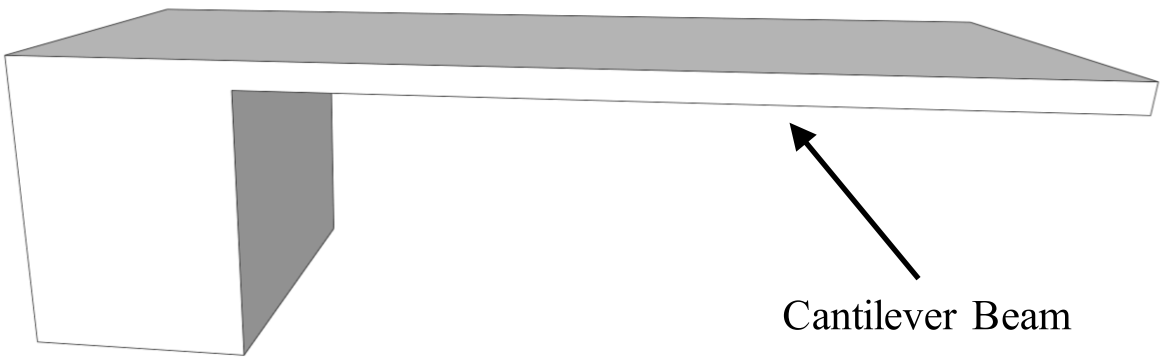

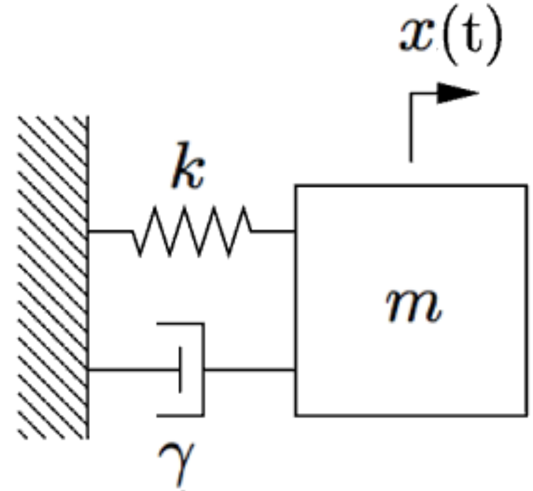

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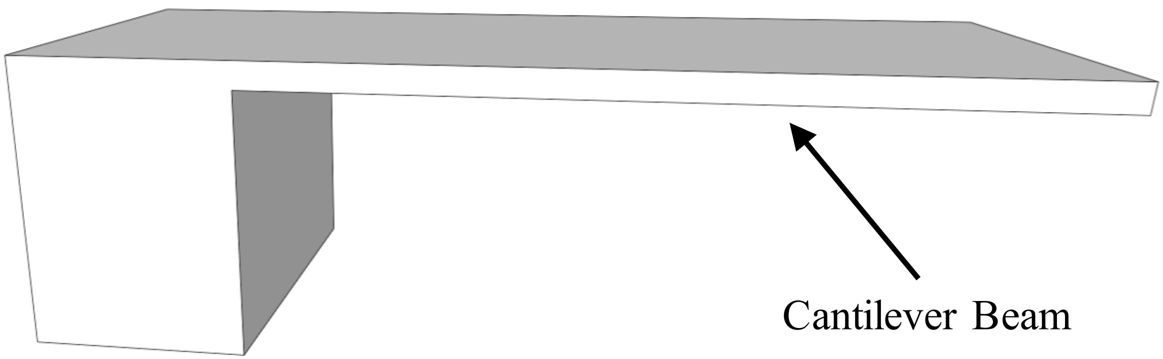

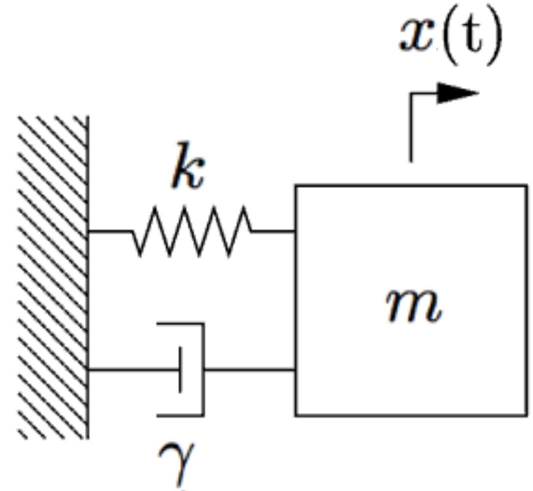

$$m\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+kx=F\cos\omega t$$

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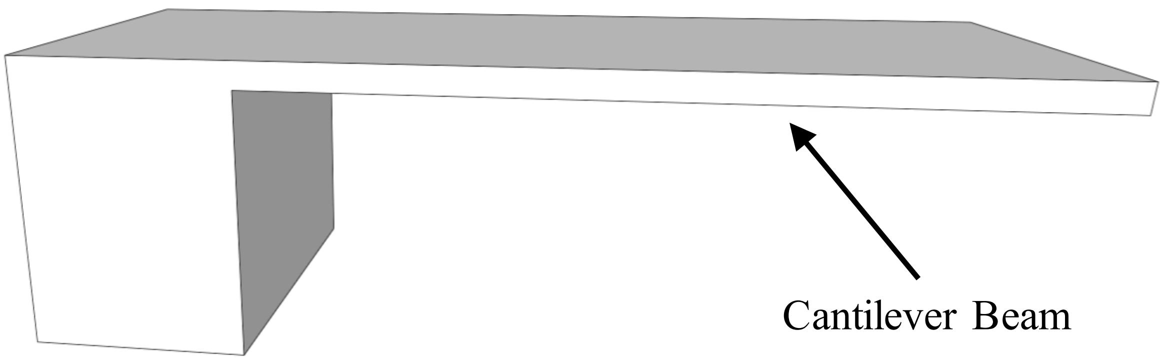

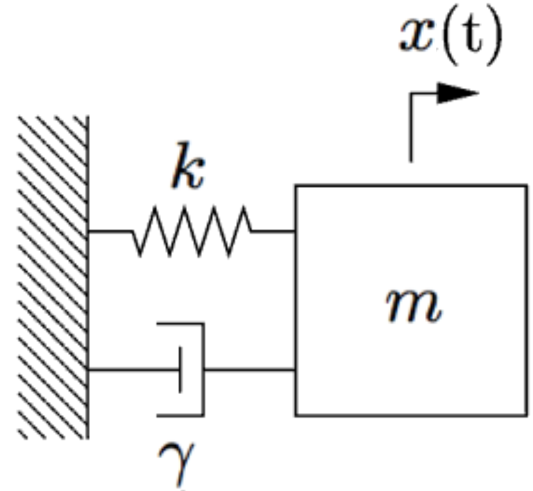

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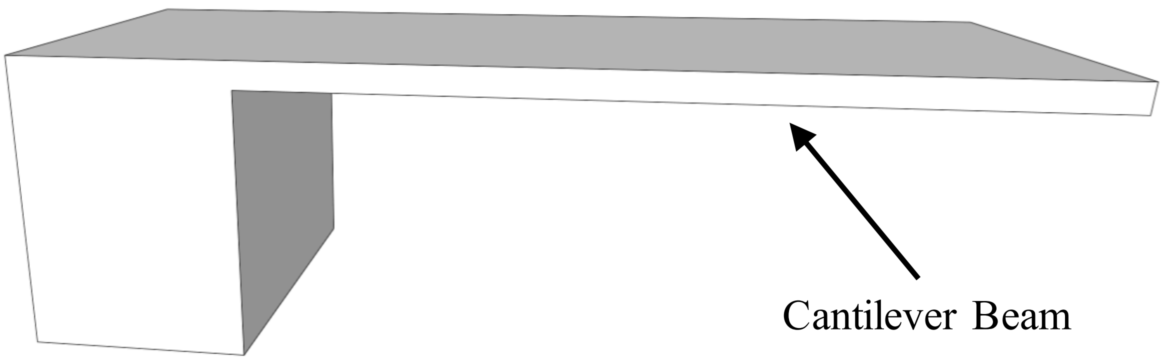
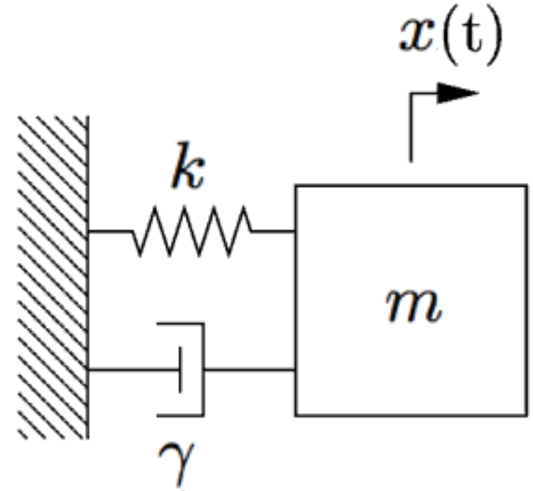
Mass

LINEAR RESONANCE IN MICRO STRUCTURES

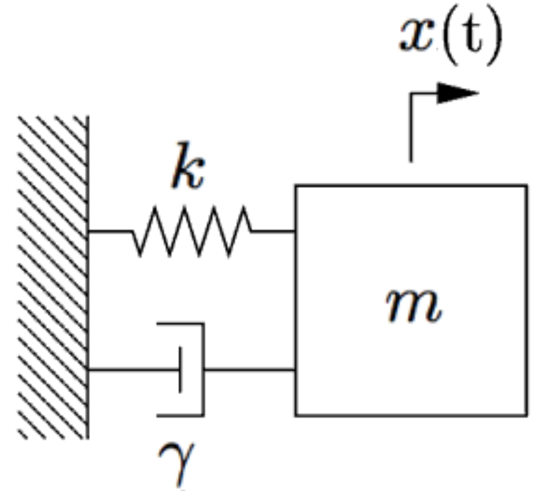




$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = F\cos\omega t$$

Mass   Damping coefficient

LINEAR RESONANCE IN MICRO STRUCTURES



$$m\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+kx=F\cos\omega t$$

Mass   Damping coefficient   Spring constant

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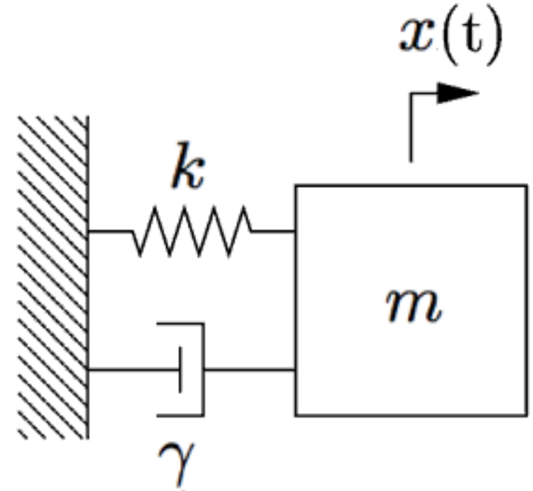

Temporal displacement

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = F\cos\omega t$$

Mass Damping coefficient Spring constant

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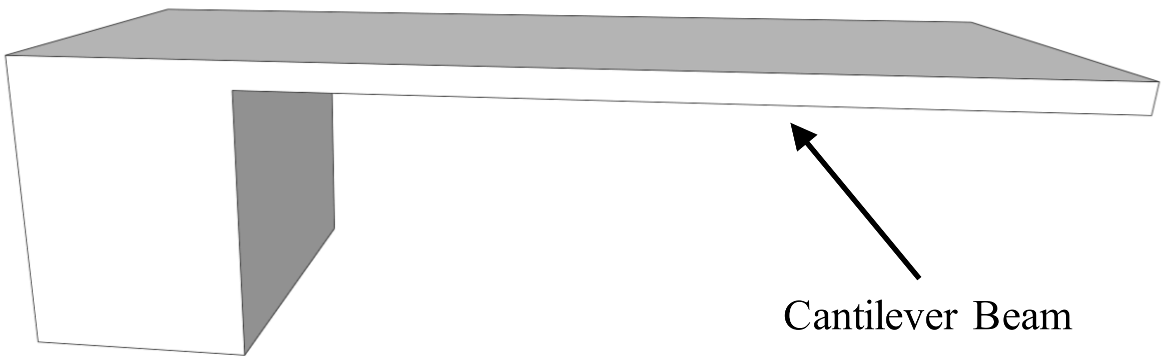
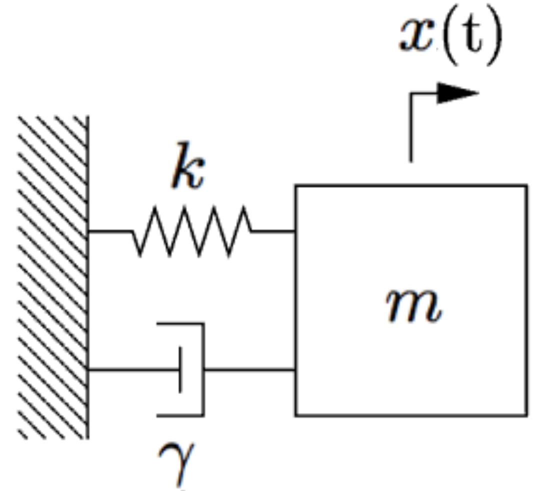




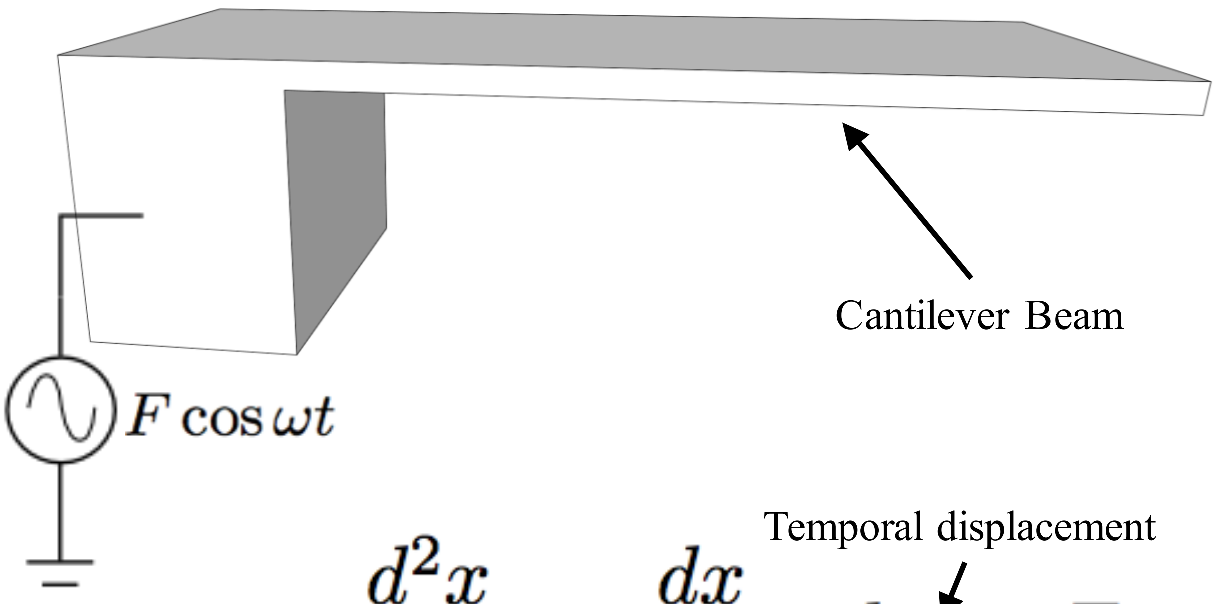
Temporal displacement

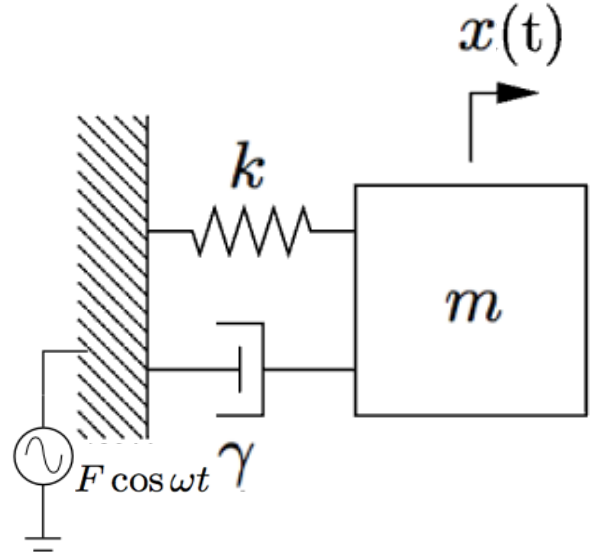
$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = F\cos\omega t$$

Forcing Function **(ZERO IF NOT DRIVEN)**

Mass   Damping coefficient   Spring constant

LINEAR RESONANCE IN MICRO STRUCTURES





$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = \underbrace{F\cos\omega t}_{\text{Forcing Function}}$$

Temporal displacement

Mass — Damping coefficient — Spring constant — Forcing Function

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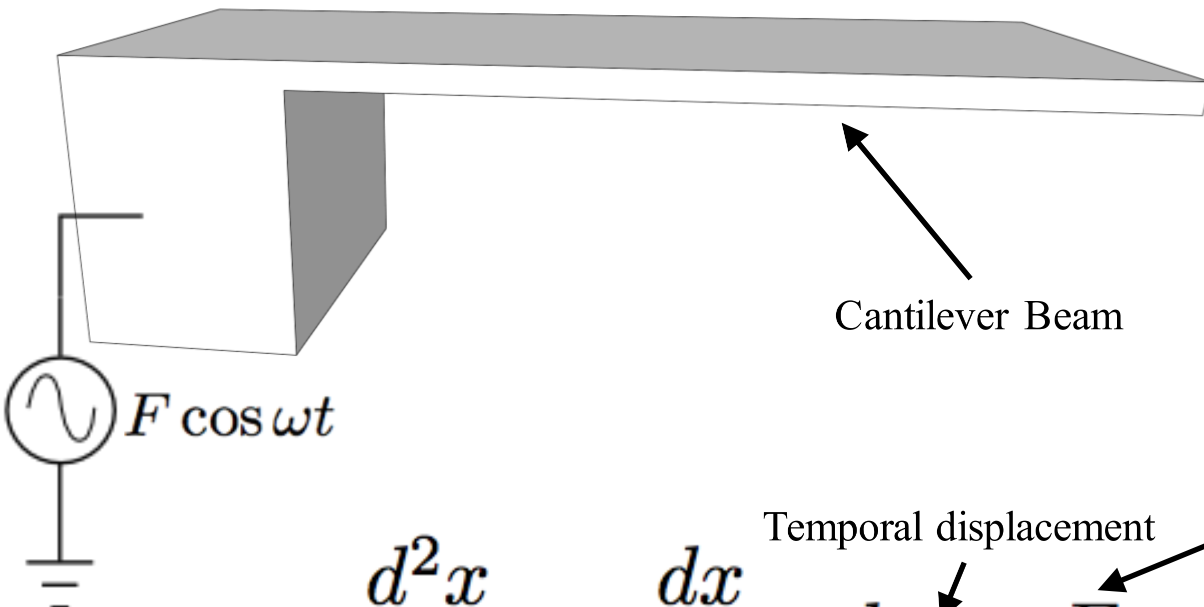

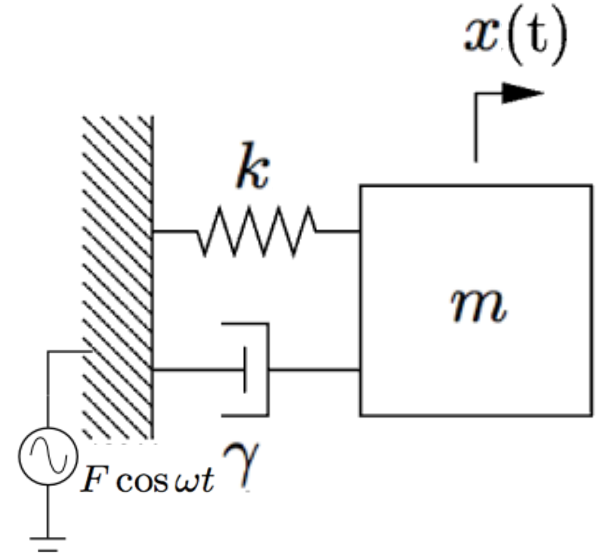

Temporal displacement

Forcing Amplitude

$$m\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+kx=F\cos\omega t$$

Forcing Function

Mass

Damping coefficient

Spring constant

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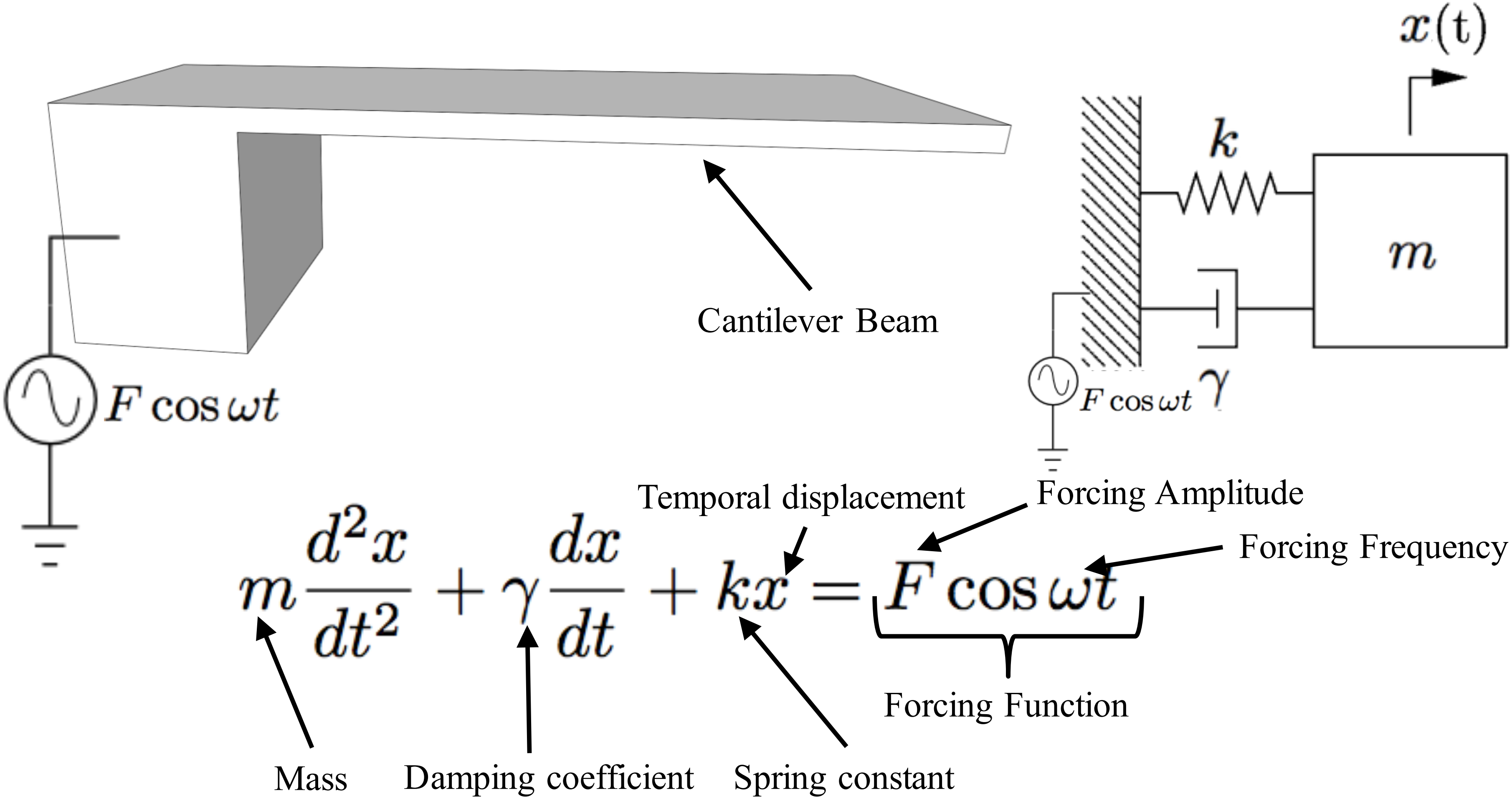

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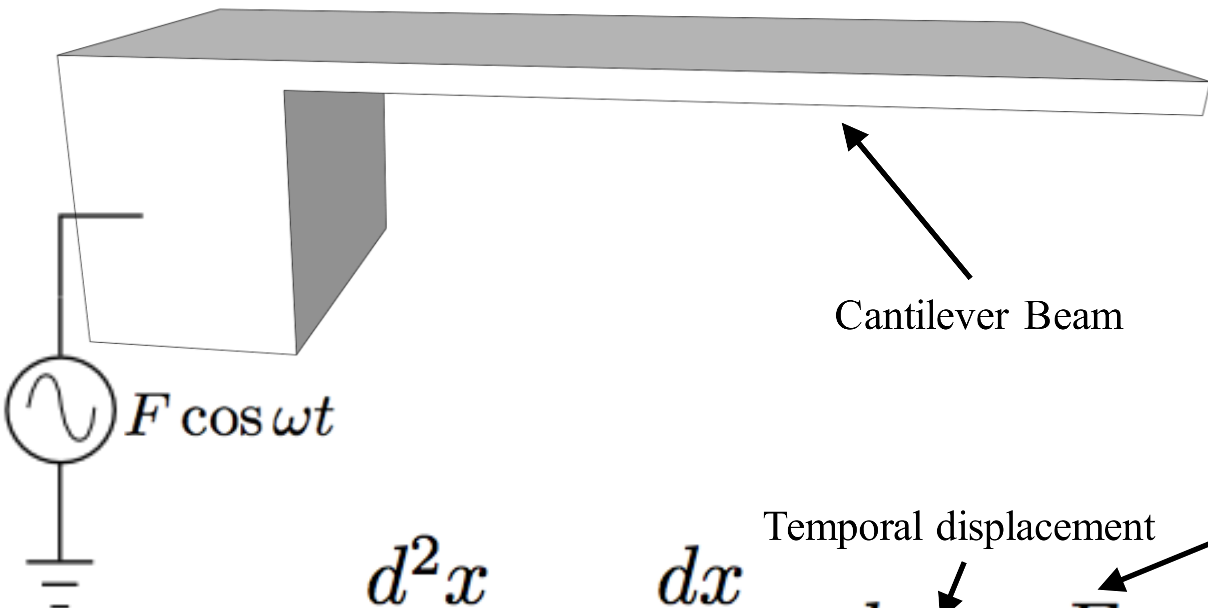
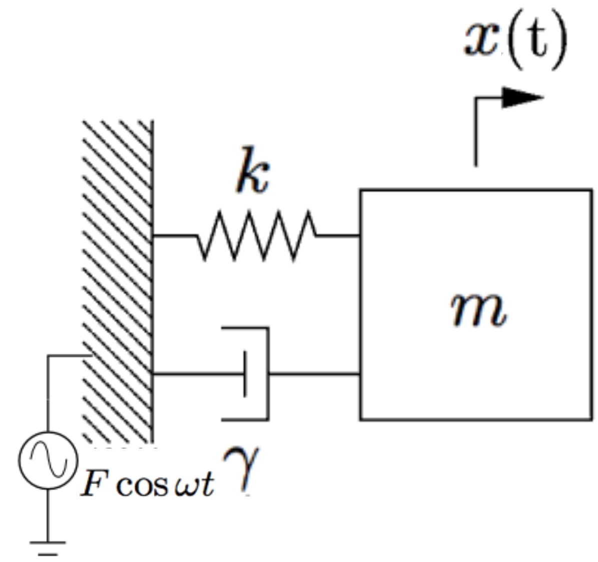




Temporal displacement

Forcing Amplitude

Forcing Frequency

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = F\cos\omega t$$

Forcing Function

Mass

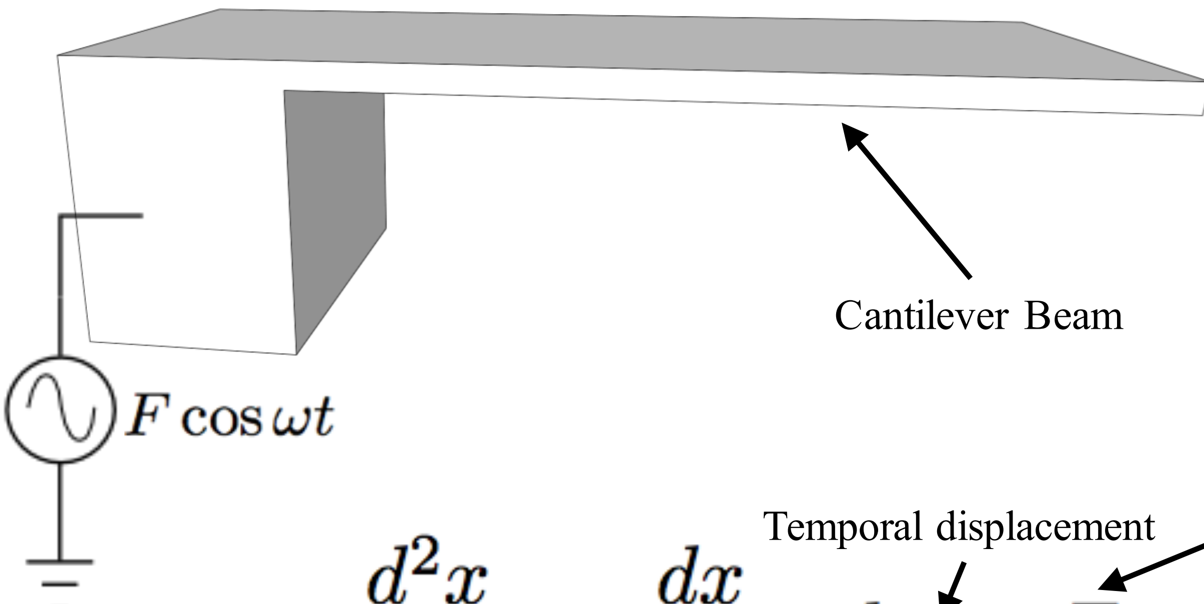
Damping coefficient

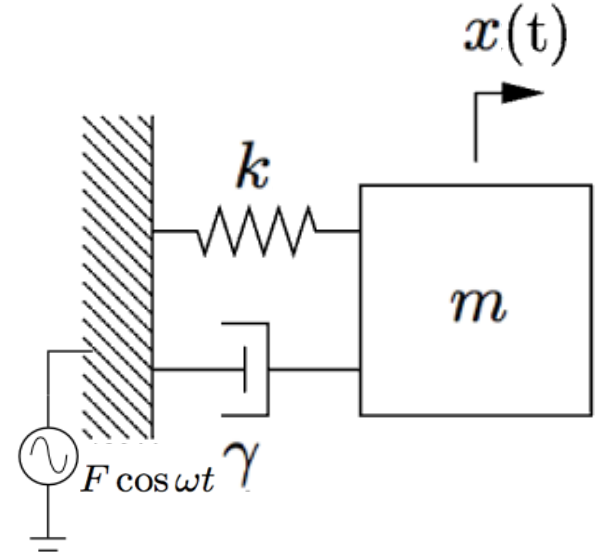
Spring constant

$$\omega_0 = \sqrt{\frac{k}{m}}$$

Resonance

LINEAR RESONANCE IN MICRO STRUCTURES





Temporal displacement — Forcing Amplitude — Forcing Frequency

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = F\cos\omega t$$

Forcing Function

Mass — Damping coefficient — Spring constant

$$\omega_0 = \sqrt{\frac{k}{m}}$$

Resonance

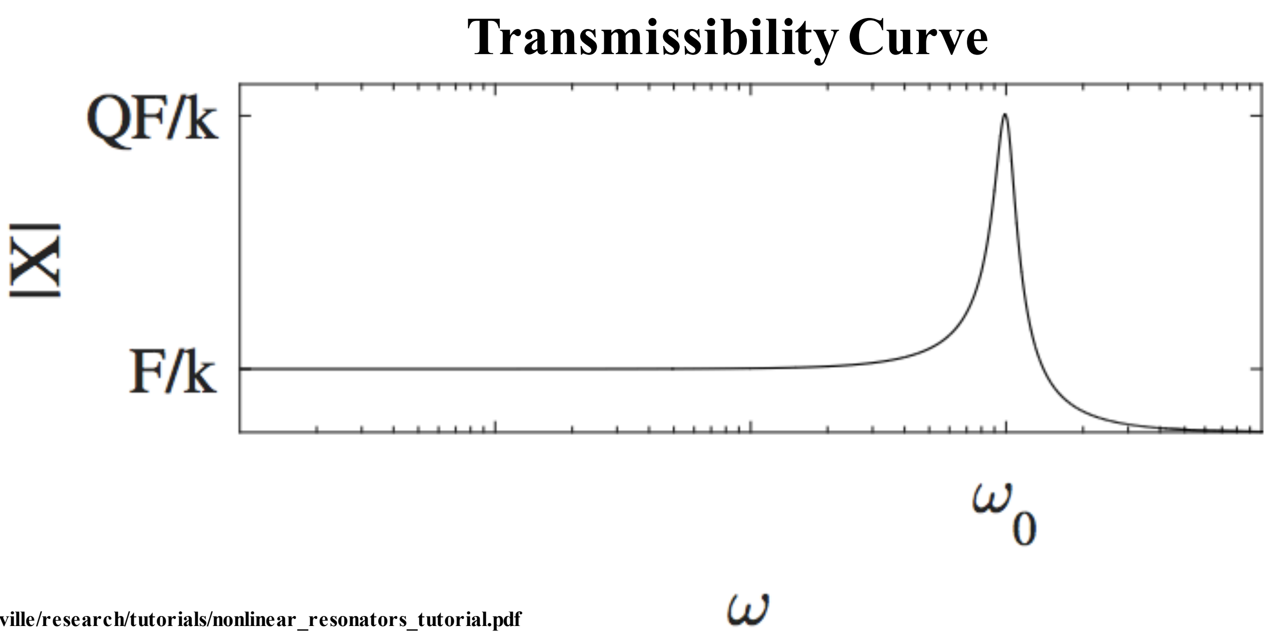
$$Q = \frac{\omega_0 m}{\gamma}$$

Quality Factor

LINEAR RESONANCE IN MICRO STRUCTURES

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = F\cos\omega t$$

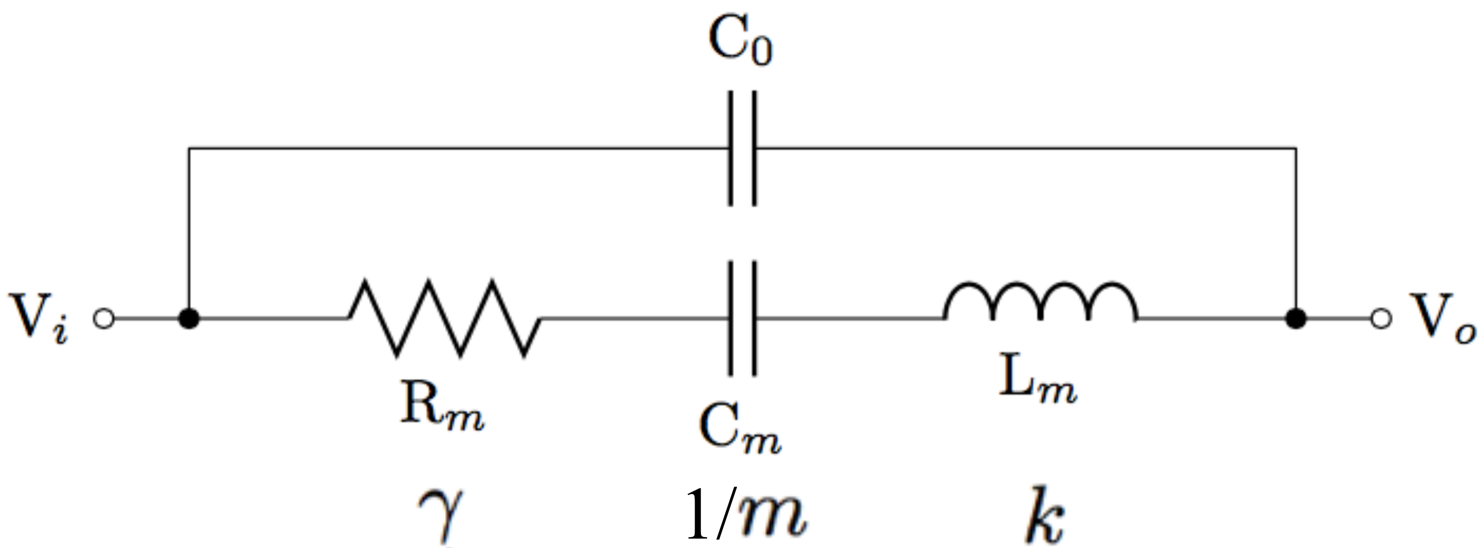
$$\omega_0 = \sqrt{\frac{k}{m}} \qquad Q = \frac{\omega_0 m}{\gamma} \qquad |X| = \frac{F}{m\sqrt{(\omega^2 - \omega_0^2)^2 + (\frac{\omega\omega_0}{Q})^2}}$$



http://www.kaajakari.net/~ville/research/tutorials/nonlinear_resonators_tutorial.pdf

ELECTRICAL EQUIVALENCE

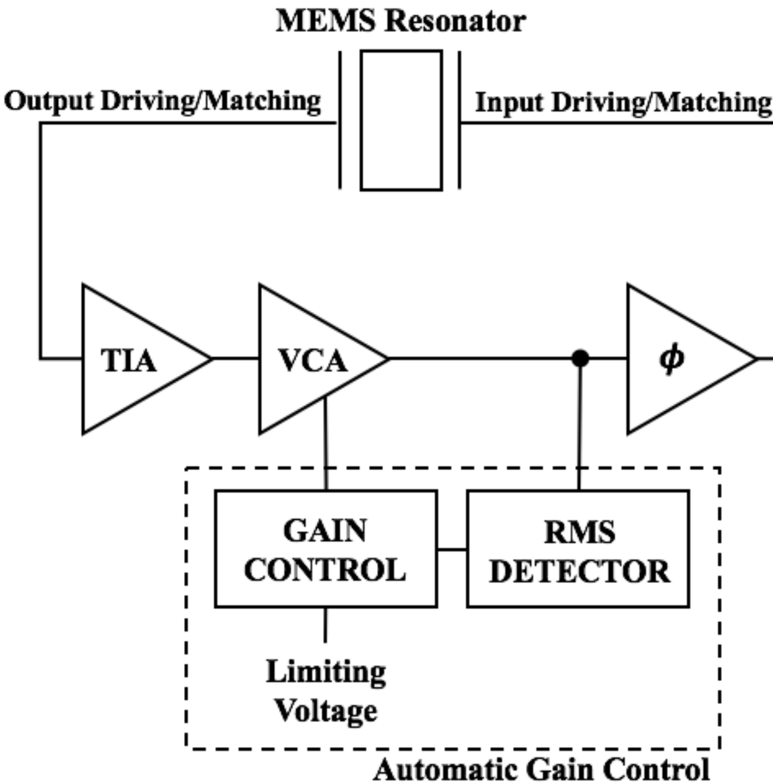
$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = F\cos\omega t$$



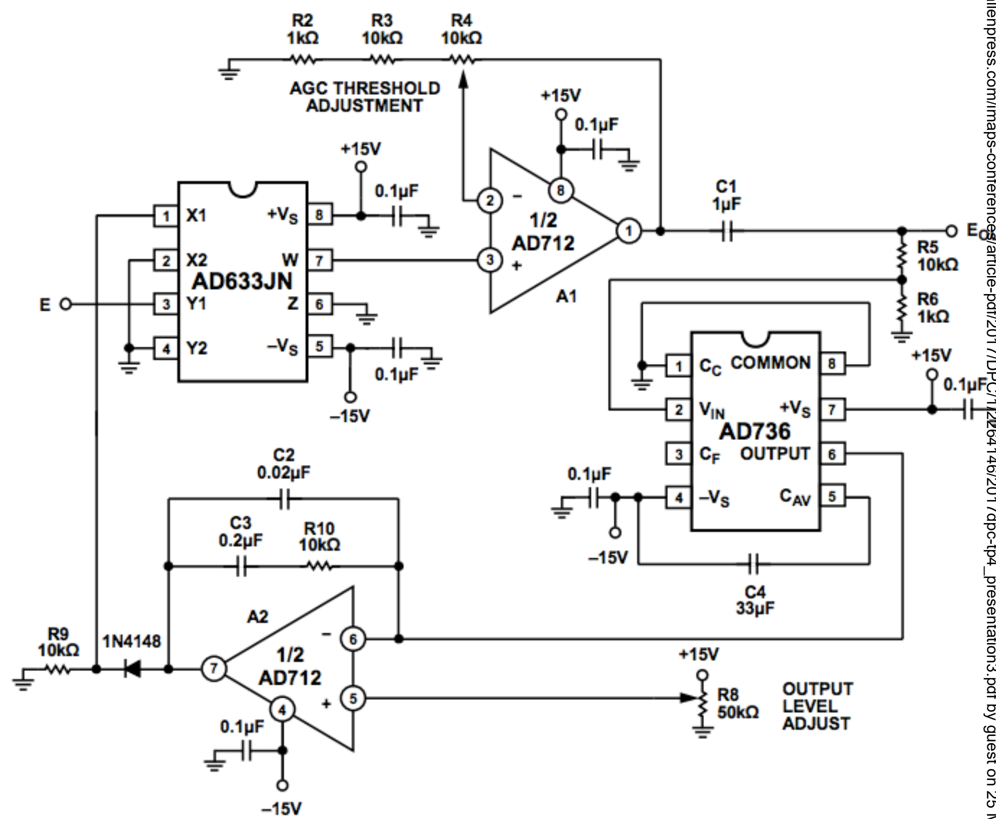
OUTLINE

- **Stress and Strain in Materials**
- **Silicon as a Linear Material**
- **Linear Resonant Microstructures**
- **SPICE Limitations**
  - Supporting Electronics for MEMS
  - Linear MEMS Model
  - Need for Other Software
- **Nonlinear MEMS Beams (BAW Example)**
- **Nonlinear Modeling**
  - Conventional
  - Extension in SPICE
- **SPICE Simulation** (Nonlinear MEMS with Supporting Electronics)
  - **Large Amplitude Behavior**
  - **Chaotic Transients**
  - **Bifurcation**
- **Conclusion**
- **Future Work**

SUPPORTING ELECTRONICS

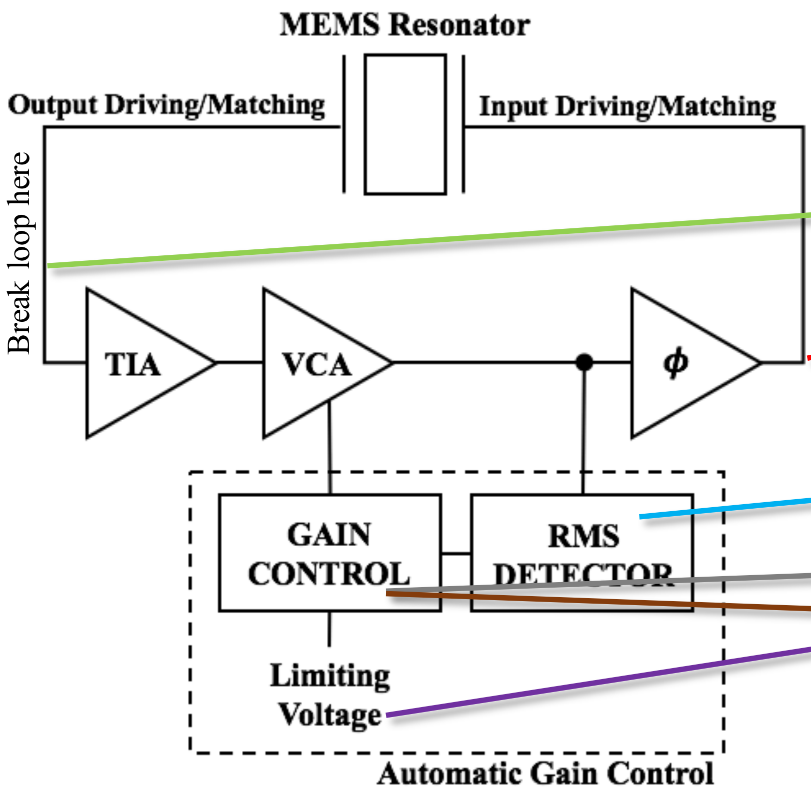


Typical AGC Circuit

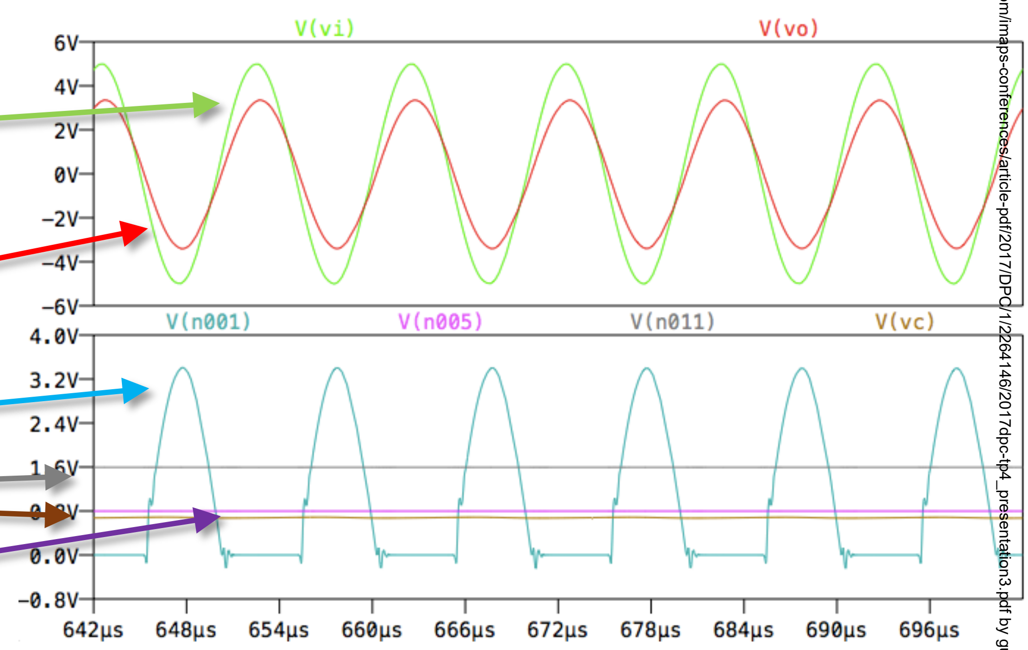


Devices, Analog. "AD633: Low Cost Analog Multiplier." *Analog Devices* (2000).

SUPPORTING ELECTRONICS



Attenuating Large Signal (Open Loop)



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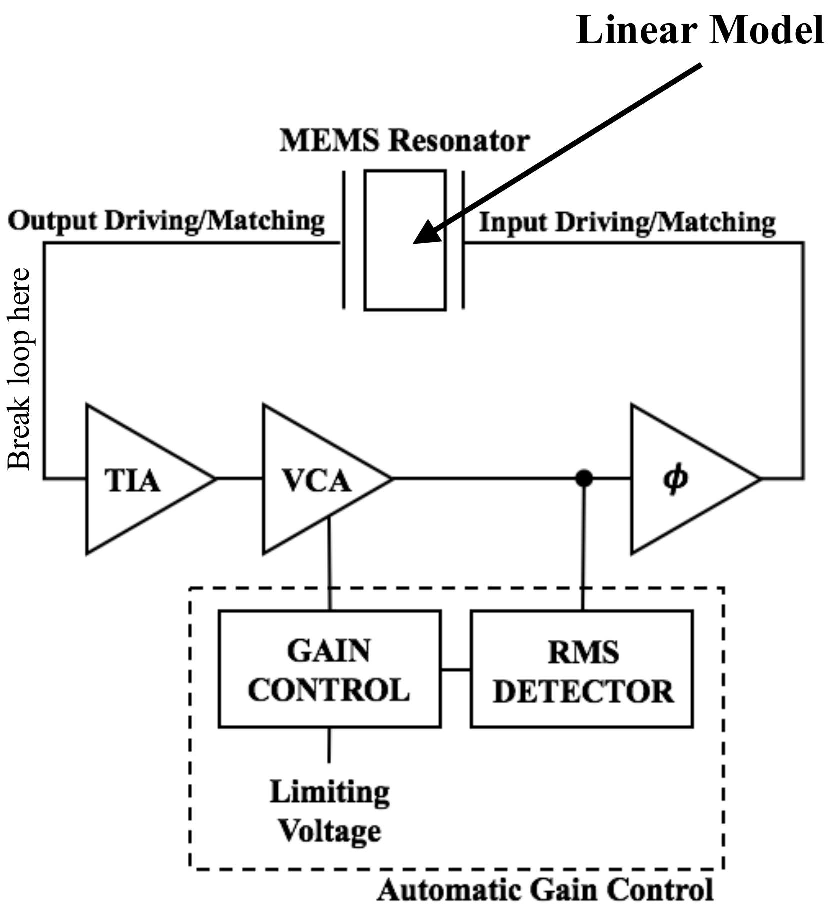

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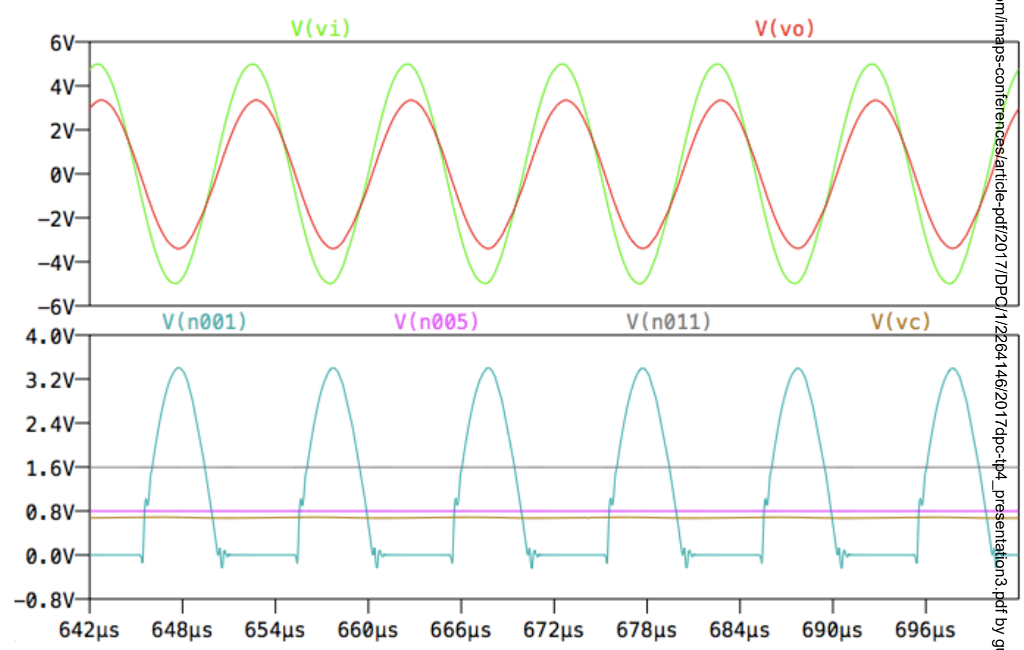


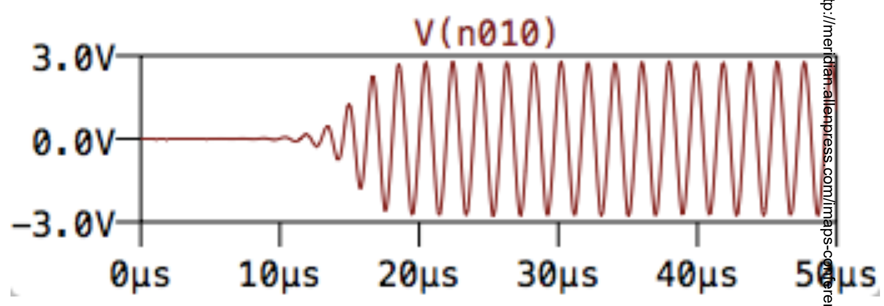
The Rub:

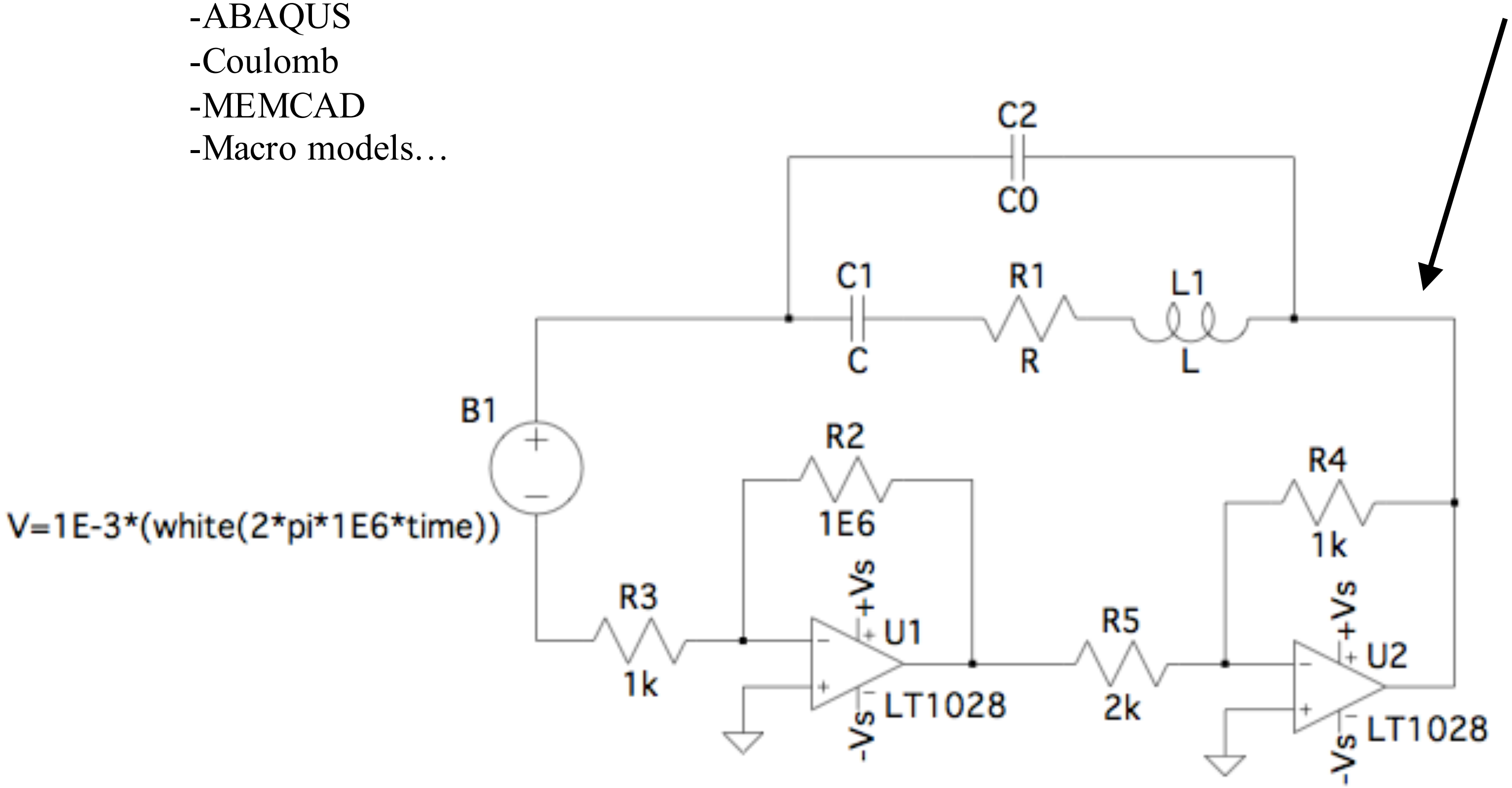
- Linear model is unrealistic for some conditions
- Linear behavior does not hold for transient events
- Supporting electronics can change things
- High quality factor devices amplify nonlinearities

SPICE SIMULATION

-Good for simple simulations
-Traditionally supports linear MEMS modeling (RLC)
-Usually needs to interface to other software

- -SIMULINK
- -MATLAB/SUGAR
- -ANSYS
- -ABAQUS
- -Coulomb
- -MEMCAD
- -Macro models…





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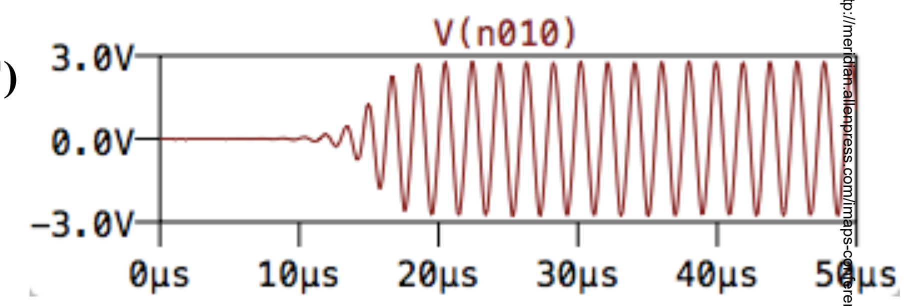

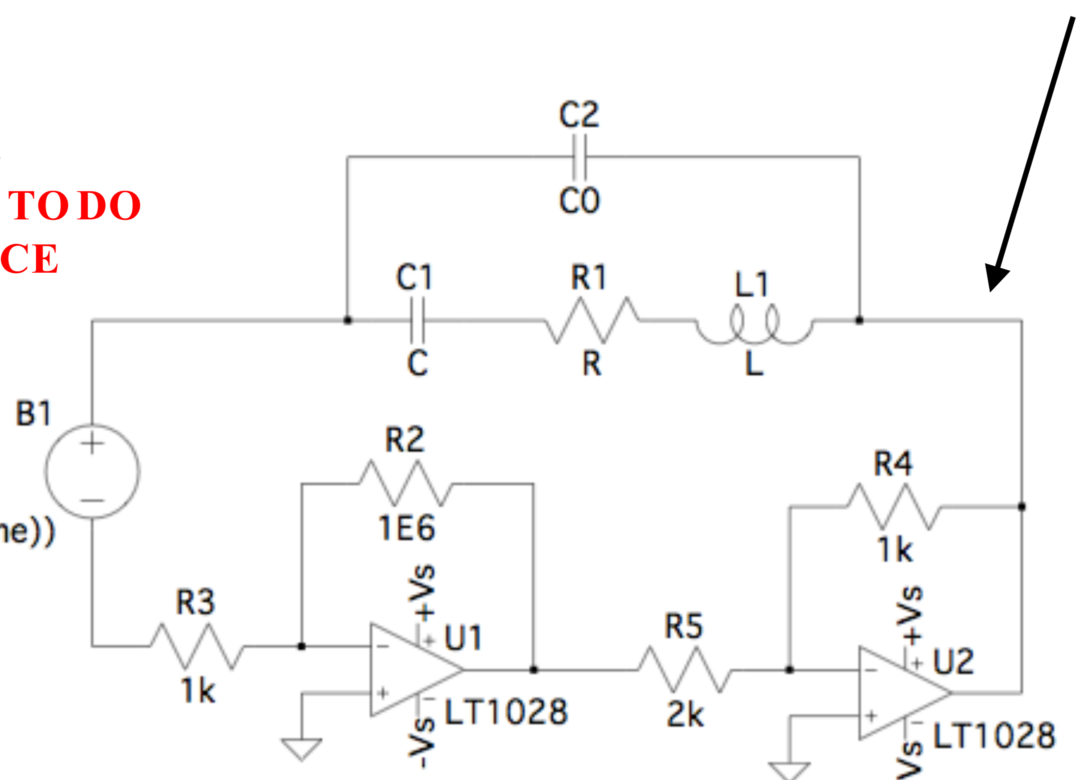

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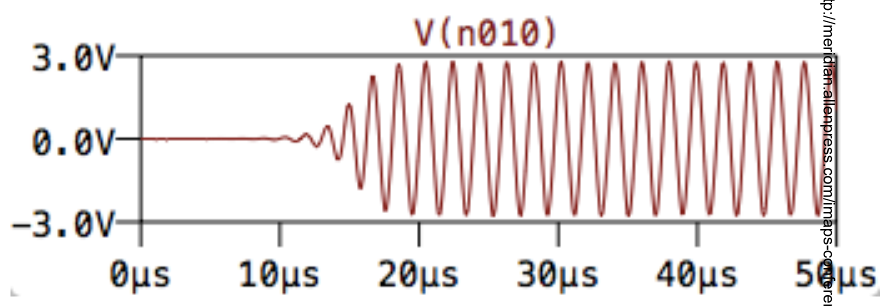

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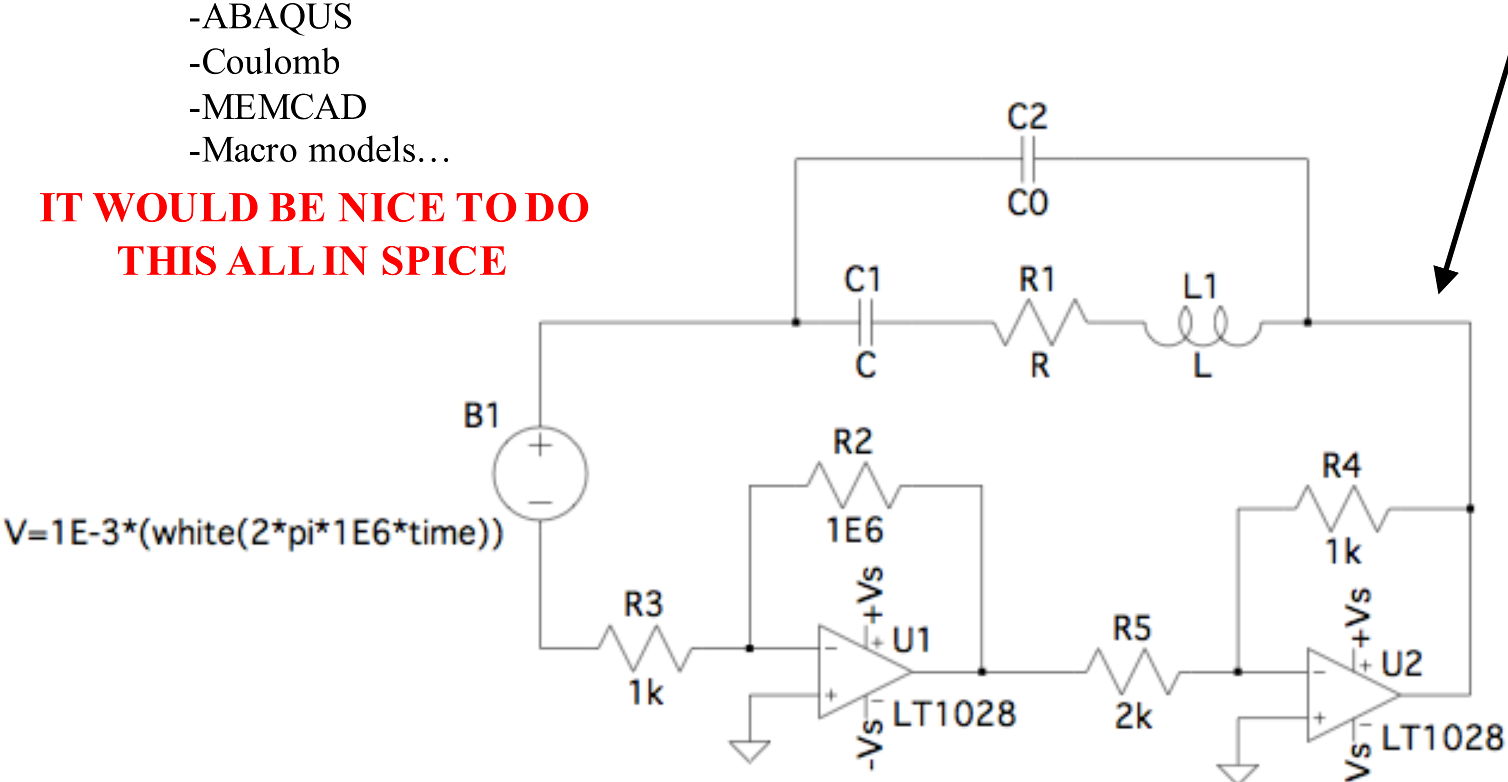

EXPAND OUR LINEAR MODEL TO ACCOUNT FOR NONLINEARITIES

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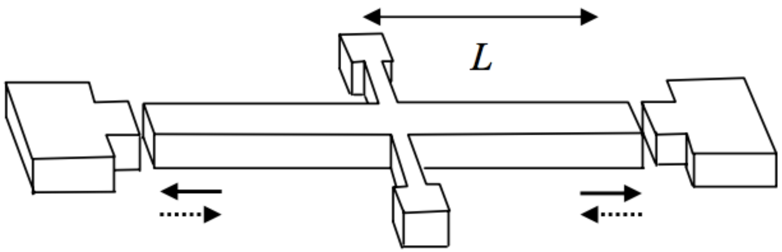
NONLINEAR MEMS BEAM (BAW RESONATOR)



Wave equation for longitudinal displacement

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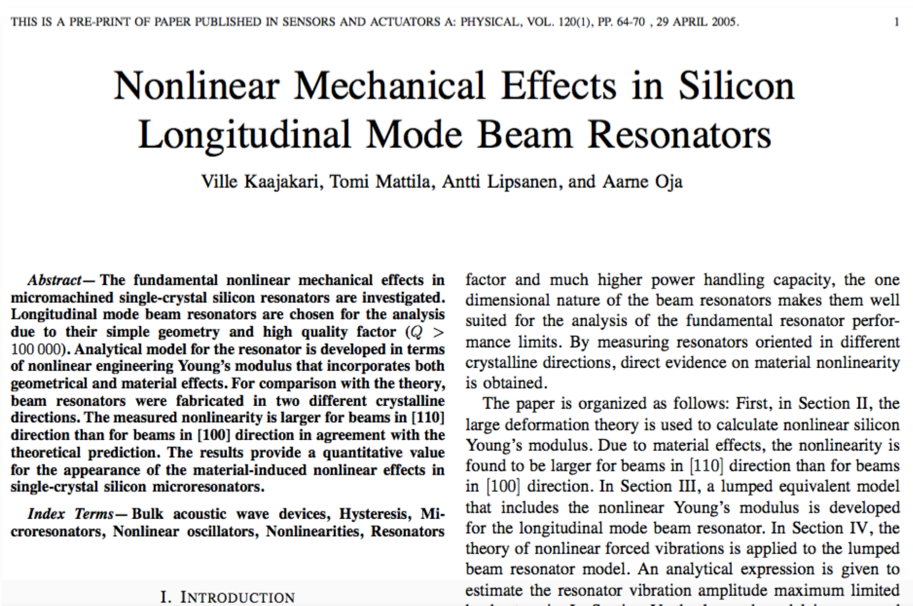
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Nonlinear Mechanical Effects in Silicon Longitudinal Mode Beam Resonators

Ville Kaajakari, Tomi Mattila, Antti Lipsanen, and Aarne Oja

***Abstract*— The fundamental nonlinear mechanical effects in micromachined single-crystal silicon resonators are investigated. Longitudinal mode beam resonators are chosen for the analysis due to their simple geometry and high quality factor ($Q >$ 100 000). Analytical model for the resonator is developed in terms of nonlinear engineering Young's modulus that incorporates both geometrical and material effects. For comparison with the theory, beam resonators were fabricated in two different crystalline directions. The measured nonlinearity is larger for beams in [110] direction than for beams in [100] direction in agreement with the theoretical prediction. The results provide a quantitative value for the appearance of the material-induced nonlinear effects in single-crystal silicon microresonators.**

***Index Terms*— Bulk acoustic wave devices, Hysteresis, Microresonators, Nonlinear oscillators, Nonlinearities, Resonators**

I. INTRODUCTION

factor and much higher power handling capacity, the one dimensional nature of the beam resonators makes them well suited for the analysis of the fundamental resonator performance limits. By measuring resonators oriented in different crystalline directions, direct evidence on material nonlinearity is obtained.

The paper is organized as follows: First, in Section II, the large deformation theory is used to calculate nonlinear silicon Young's modulus. Due to material effects, the nonlinearity is found to be larger for beams in [110] direction than for beams in [100] direction. In Section III, a lumped equivalent model that includes the nonlinear Young's modulus is developed for the longitudinal mode beam resonator. In Section IV, the theory of nonlinear forced vibrations is applied to the lumped beam resonator model. An analytical expression is given to estimate the resonator vibration amplitude maximum limited

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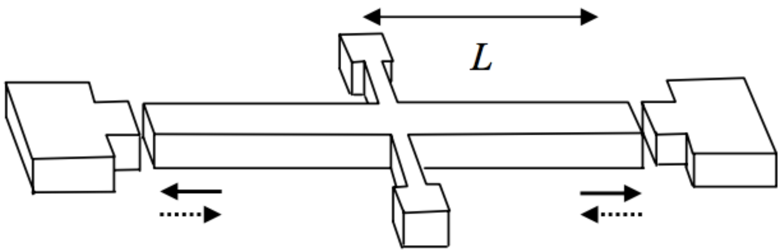

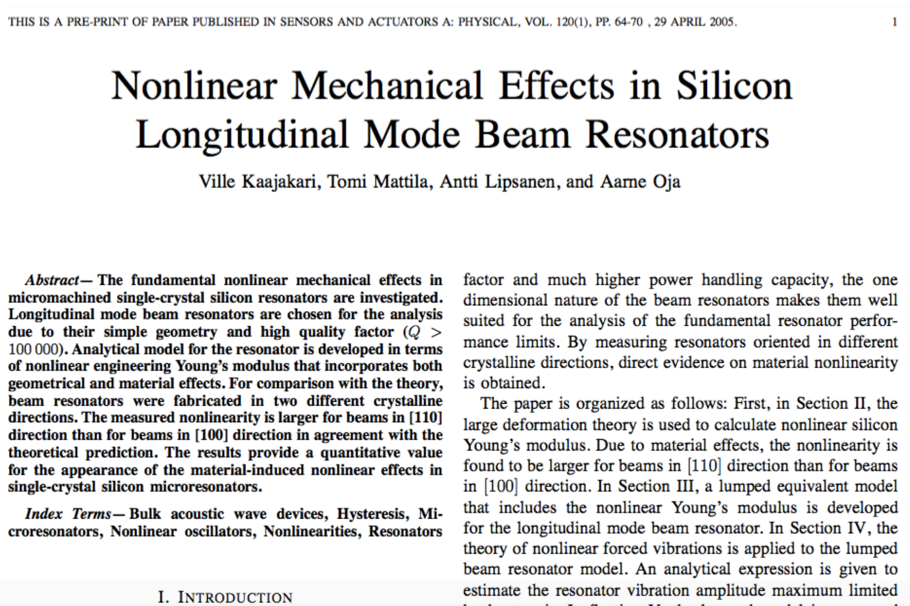
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Wave equation for longitudinal displacement

$$\rho A \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial z}\left(AY\frac{\partial u}{\partial z}\right)$$

Solution

$$u(z,t) = x(t)\sin \pi z/2L$$

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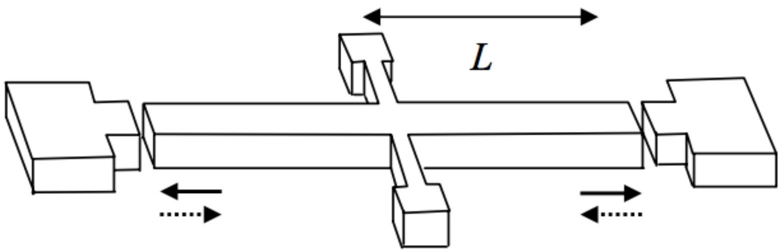

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Recall Young's Modulus with higher order terms

$$Y = \frac{\sigma}{\epsilon} = Y_0(1 + Y_1 S + Y_2 S^2)$$

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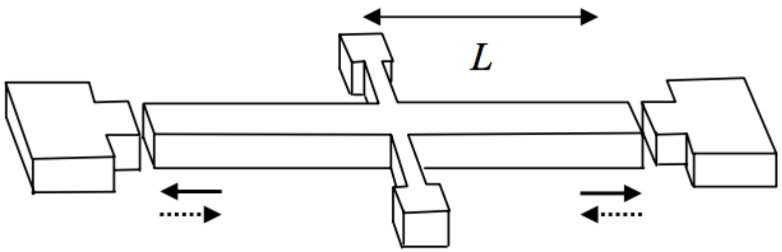

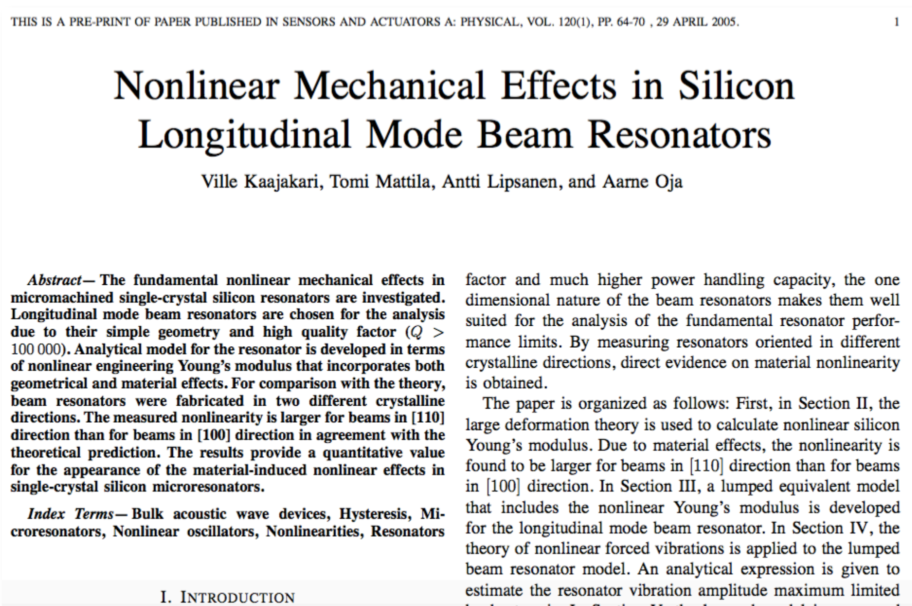

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Integrate with the solution and Young's Modulus when plugged into the wave equation

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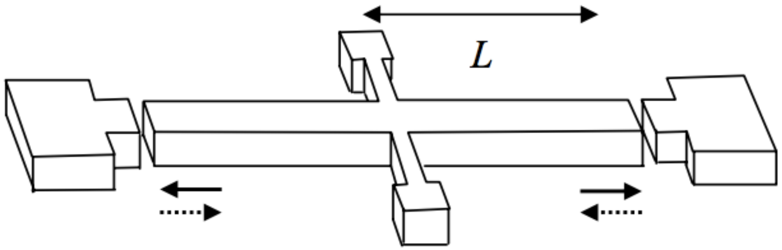

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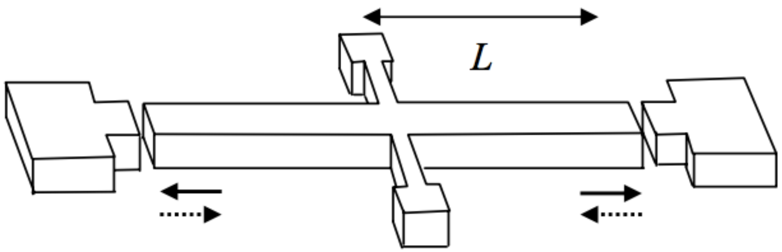

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$$m = \rho AL$$
$$k(x) = k_0(1 + k_1 x + k_2 x^2)$$
$$k_0 = \frac{\pi^2}{4}\frac{AY}{L}, k_1 = \frac{4Y_1}{3L} \text{ and } k_2 = \frac{3\pi^2 Y_2}{16L^2}$$

The result is:

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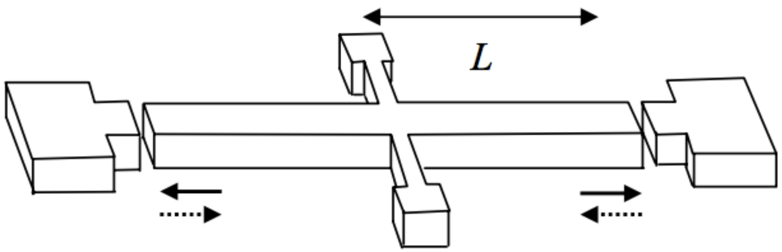

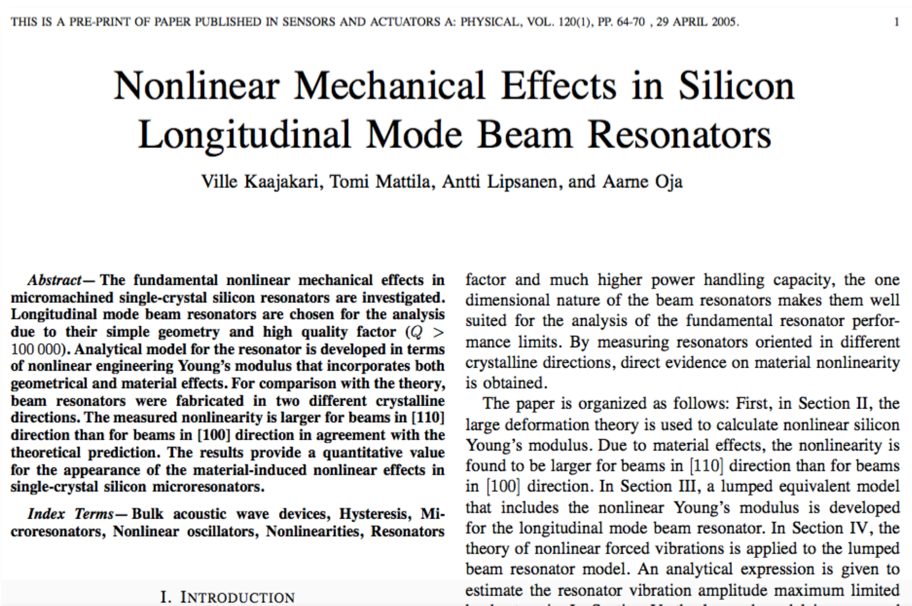
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Recall Young's Modulus with higher order terms

$$Y = \frac{\sigma}{\epsilon} = Y_0(1 + Y_1 S + Y_2 S^2)$$

Integrate with the solution and Young's Modulus when plugged into the wave equation

$$\rho AL\frac{\partial^2 x}{\partial t^2} = -\frac{\pi^2}{4}\frac{AY}{L}x\left(1 + \frac{4Y_1}{3L}x + \frac{3\pi^2 Y_2}{16L^2}x^2\right)$$

$$m = \rho AL$$

$$\begin{cases} k(x) = k_0(1 + k_1 x + k_2 x^2) \\ k_0 = \frac{\pi^2}{4}\frac{AY}{L}, k_1 = \frac{4Y_1}{3L} \text{ and } k_2 = \frac{3\pi^2 Y_2}{16L^2} \end{cases}$$

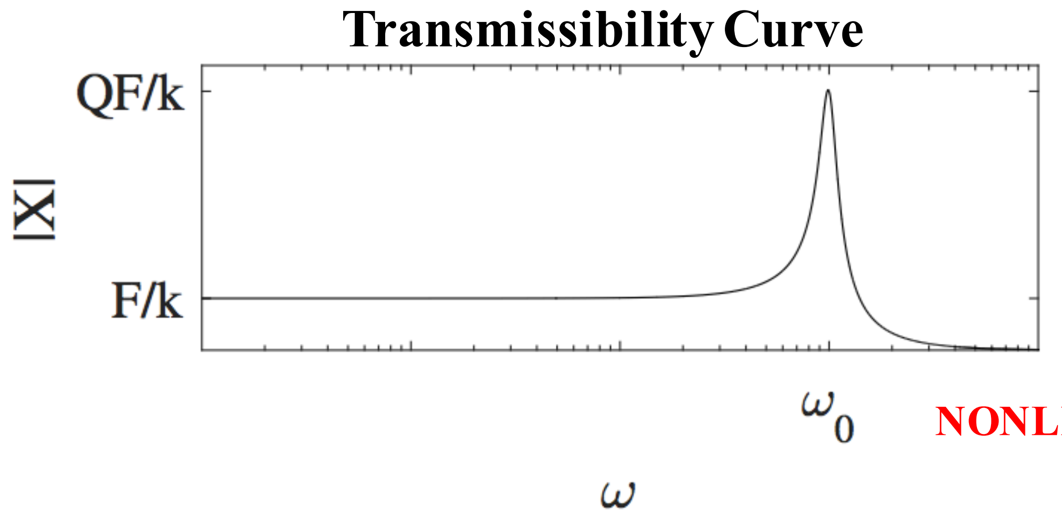
The result is:

$$m\ddot{x} + \gamma\dot{x} + k(x)x = F_\omega \cos\omega t$$

NO LUMPED ELEMENT MODEL FOR SPICE

V.Kaajakari,T.Mattila,A.Lipsanen,andA.Oja,"Nonlinearmechanical effects in silicon longitudinal mode beam resonators," ***Sensors and Actuators A: Physical*****, vol. 120, no. 1, pp. 64–70, 2005.**

EFFECTS OF NONLINEAR SPRING TERMS



$$\omega_0 = \sqrt{\frac{k}{m}} \qquad Q = \frac{\omega_0 m}{\gamma}$$

$$|X| = \frac{F}{m\sqrt{(\omega^2 - \omega_0^2)^2 + (\frac{\omega\omega_0}{Q})^2}}$$

NONLINEAR RESONANCE EFFECTS SET IN

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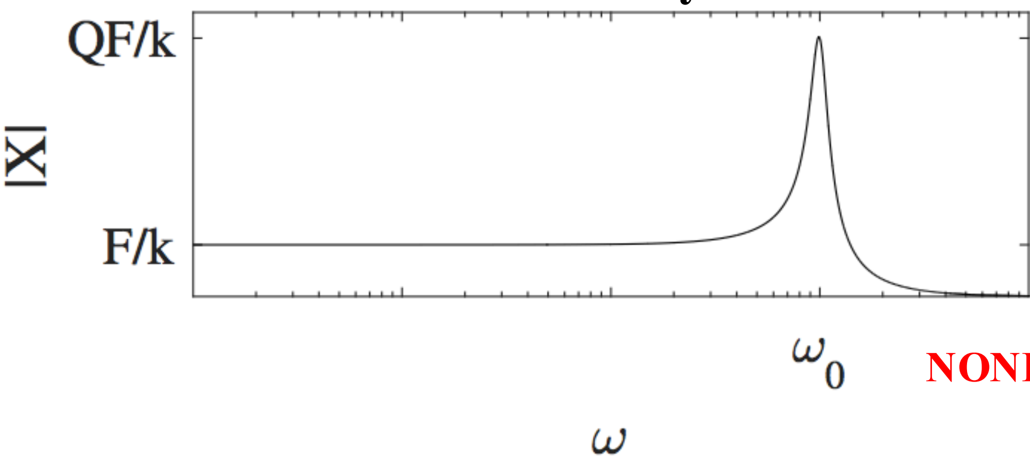
EFFECTS OF NONLINEAR SPRING TERMS

Transmissibility Curve



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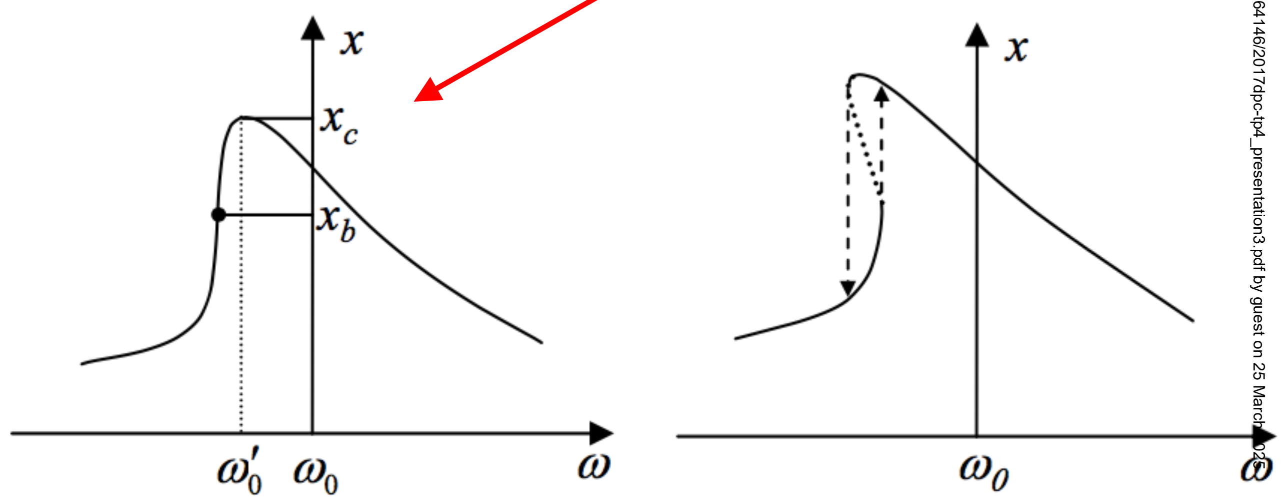

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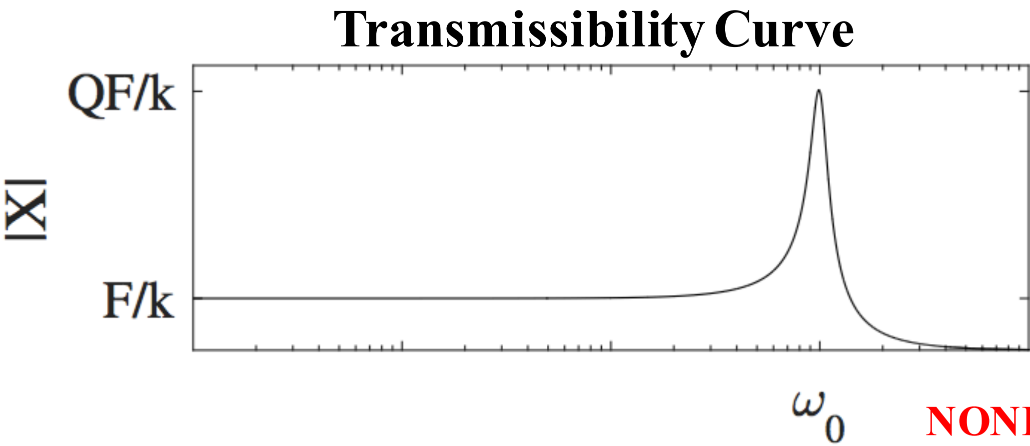

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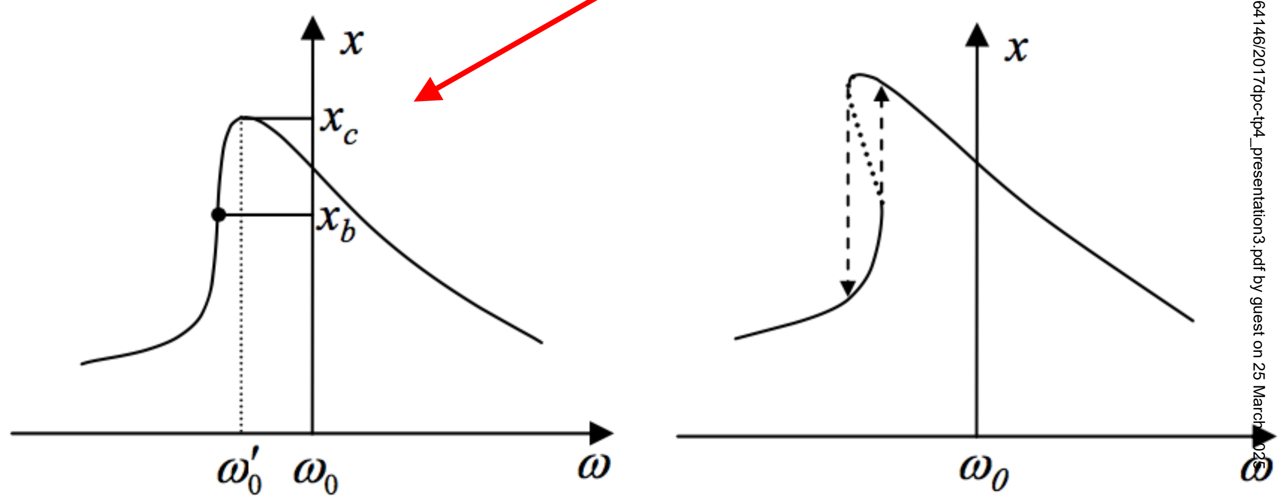
ω

NONLINEAR RESONANCE EFFECTS SET IN

Nonlinearity Measure

$$k_{NL} = \frac{3}{8}k_2 - \frac{5}{12}k_1^2$$



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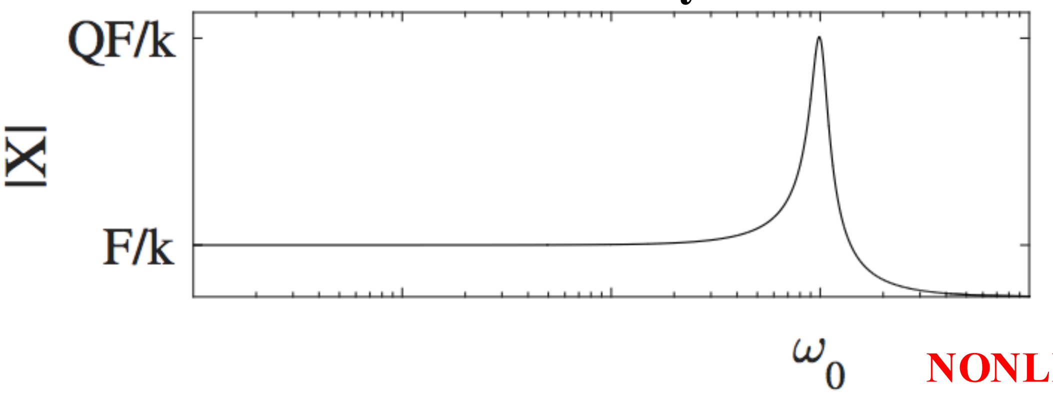

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NONLINEAR RESONANCE EFFECTS SET IN

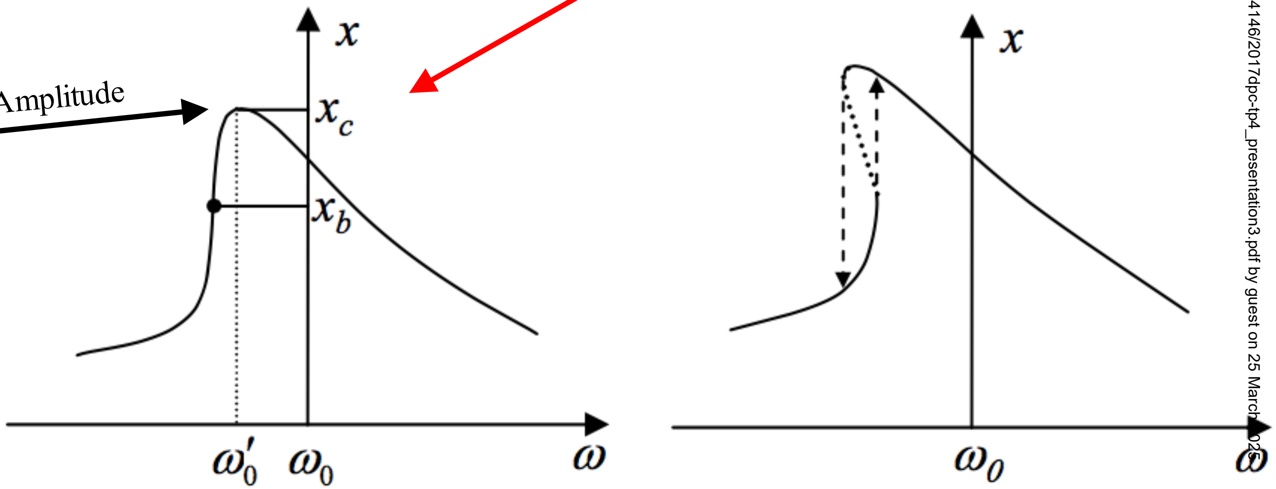
Nonlinearity Measure

ω

$$k_{NL} = \frac{3}{8}k_2 - \frac{5}{12}k_1^2$$

$$x_c = \frac{2}{\sqrt{3\sqrt{3}Q|k_{NL}|}}$$

Greatest Amplitude



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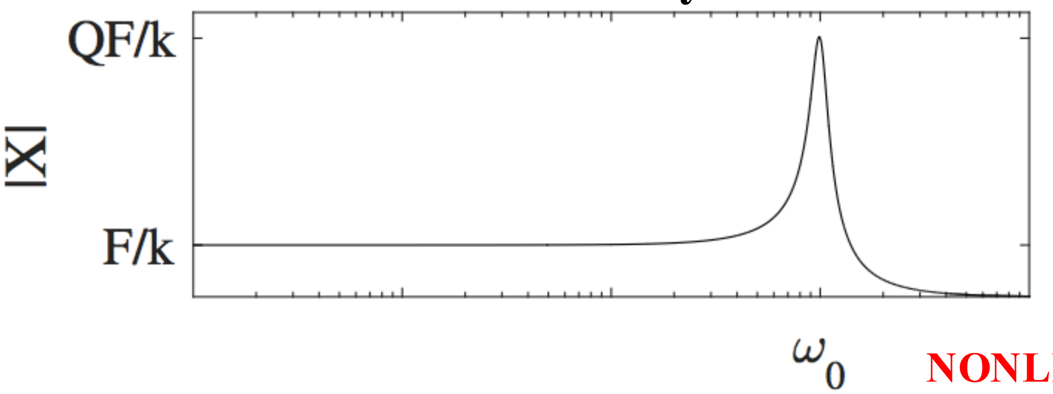

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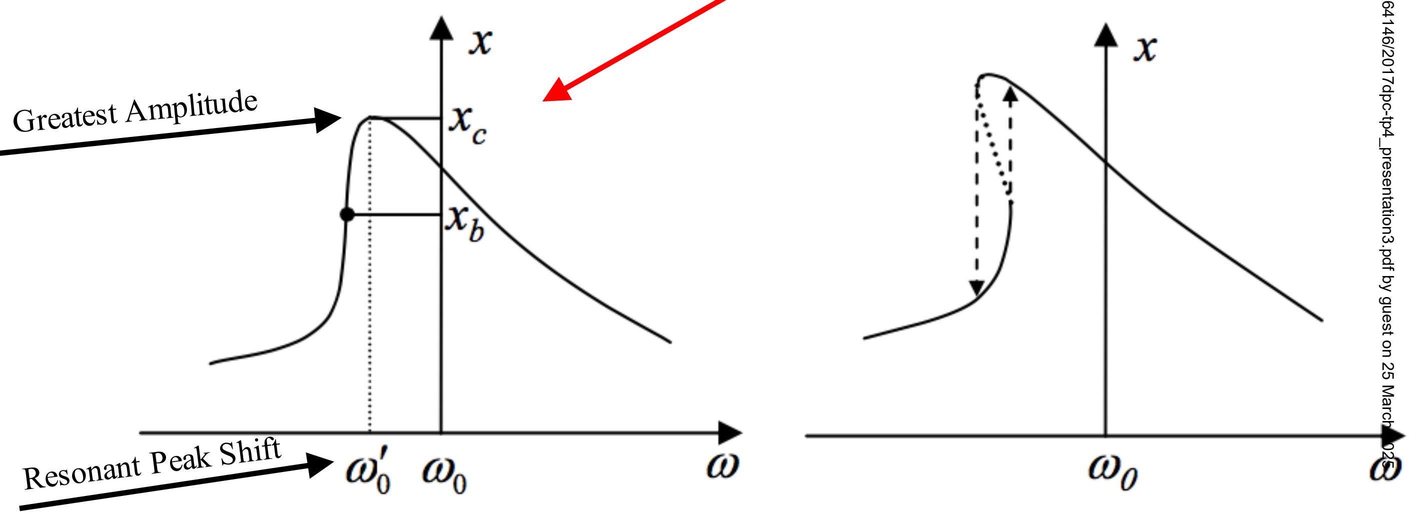

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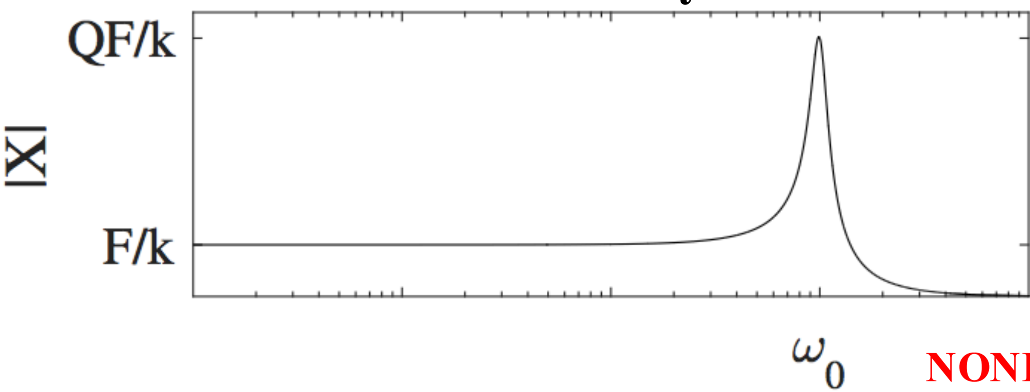

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ω

NONLINEAR RESONANCE EFFECTS SET IN

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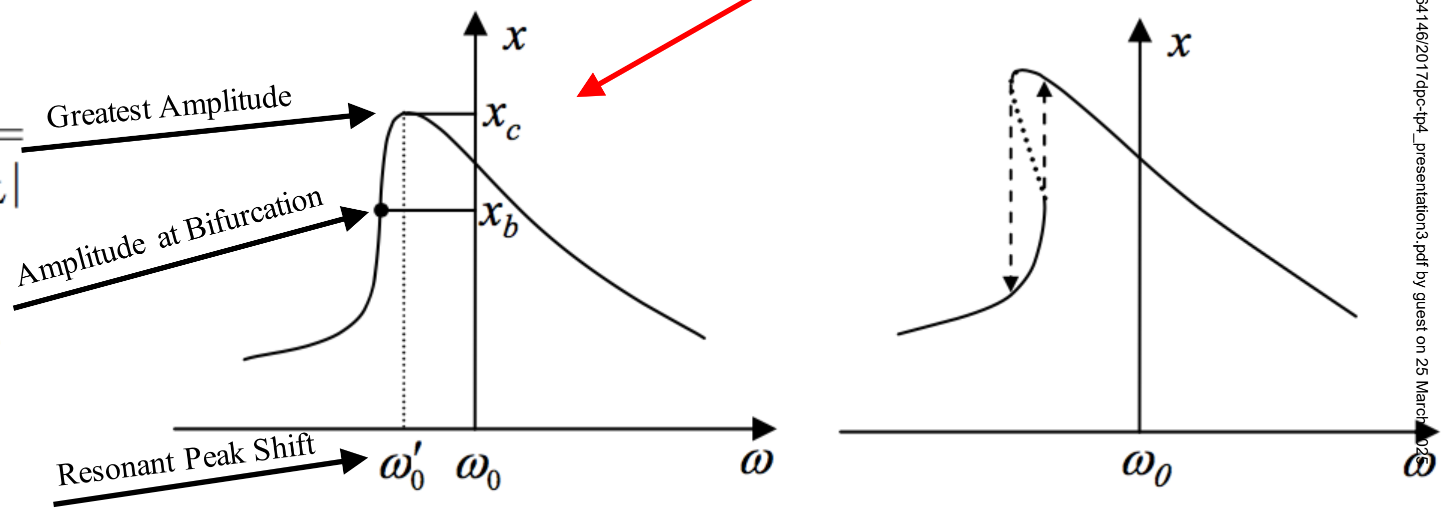
$$k_{NL} = \frac{3}{8}k_2 - \frac{5}{12}k_1^2$$

$$x_c = \frac{2}{\sqrt{3\sqrt{3}Q|k_{NL}|}}$$

$$x_b = \frac{1}{\sqrt{\sqrt{3}Q|k_{NL}|}}$$



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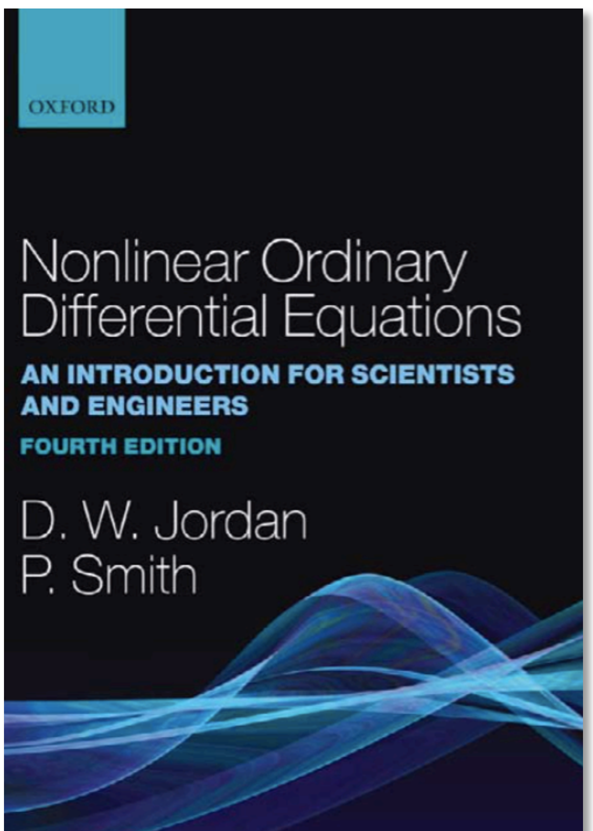
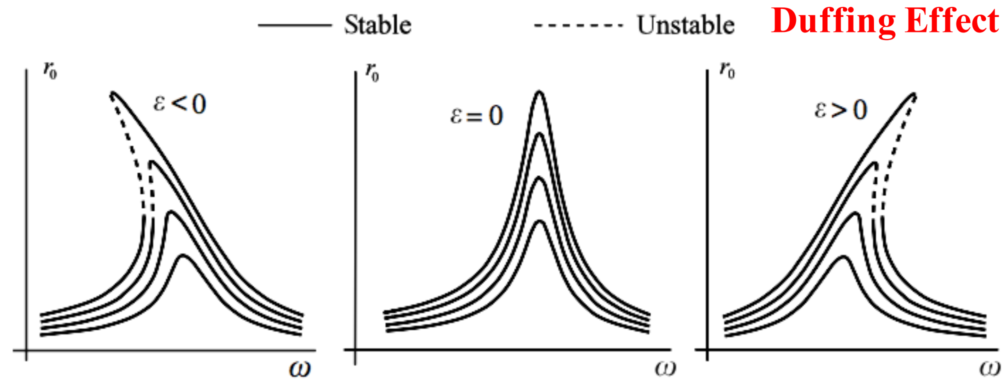
SIMILARITY TO DUFFING EQUATION

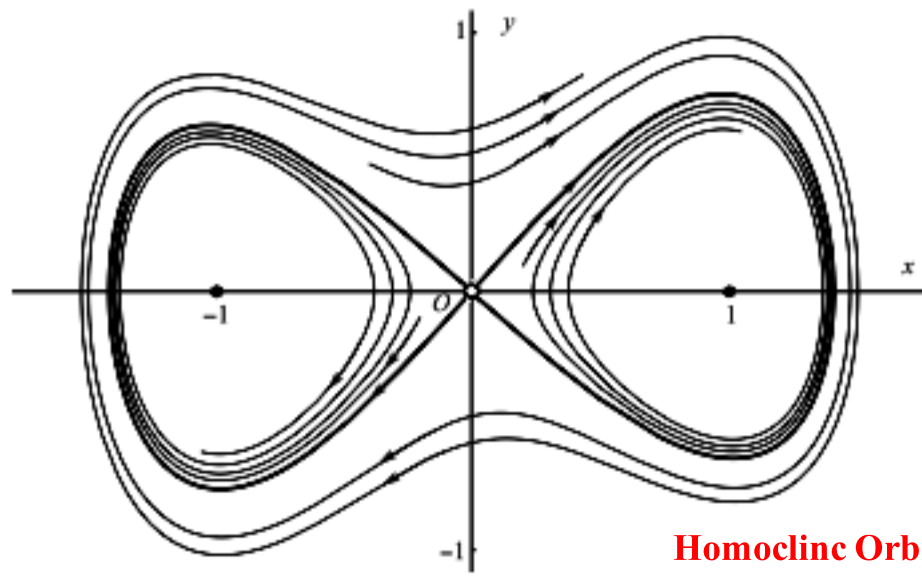
Higher order spring terms

$$\ddot{x} + k\dot{x} + cx + \overbrace{\varepsilon g(x)} = F\cos\omega t \qquad \ddot{x} + k\dot{x} + \overbrace{cx + \varepsilon x^3} = F\cos\omega t$$



Duffing Effect





Homoclinc Orbits

1.D. W. Jordan and P. Smith, *Nonlinear ordinary differential equations: an introduction to dynamical systems*. Oxford University Press, USA, 1999, vol. 2.

OUTLINE

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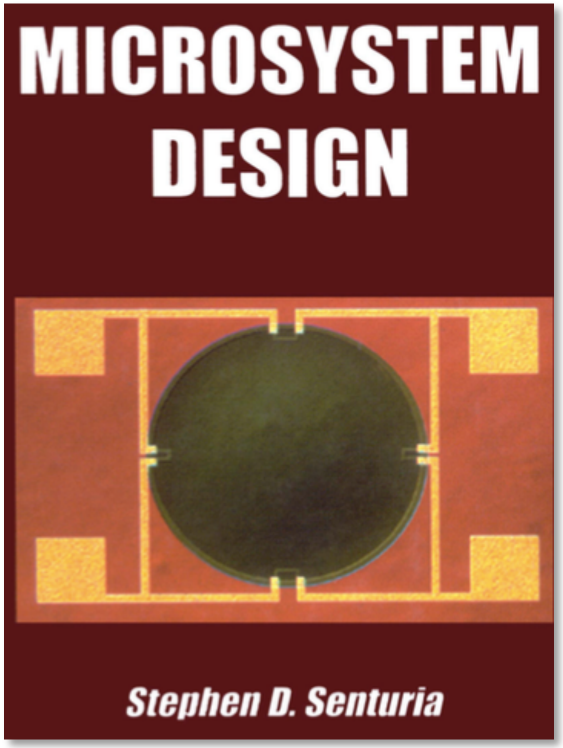
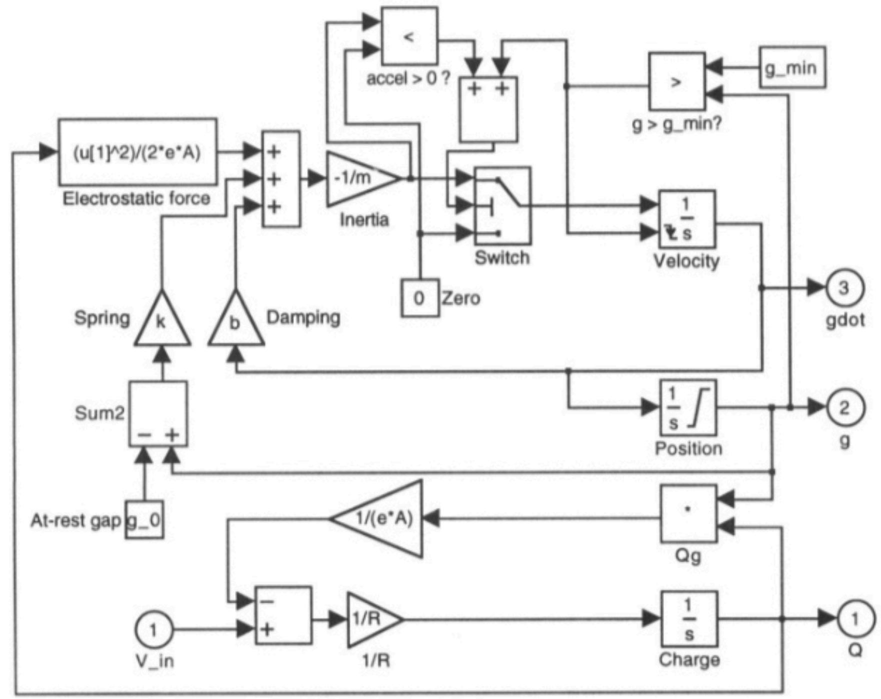
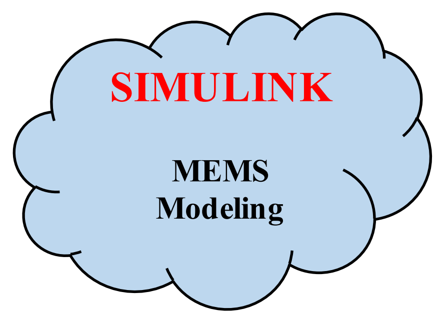
NONLINEAR MODELING



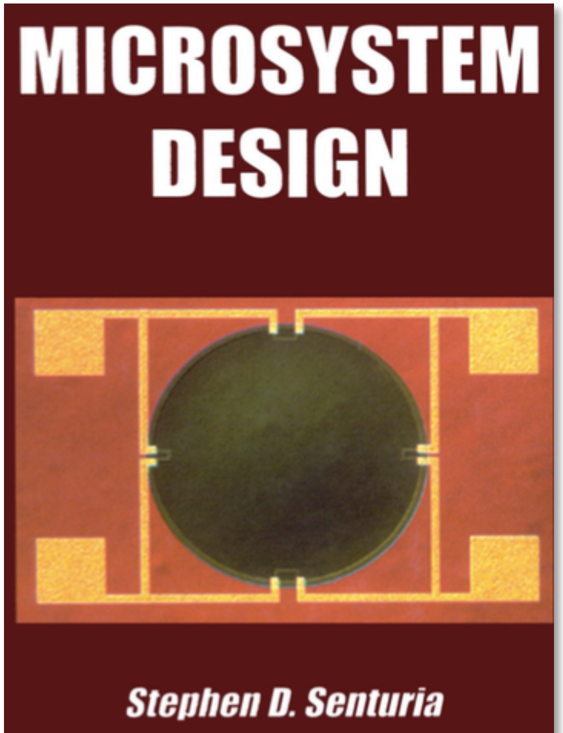


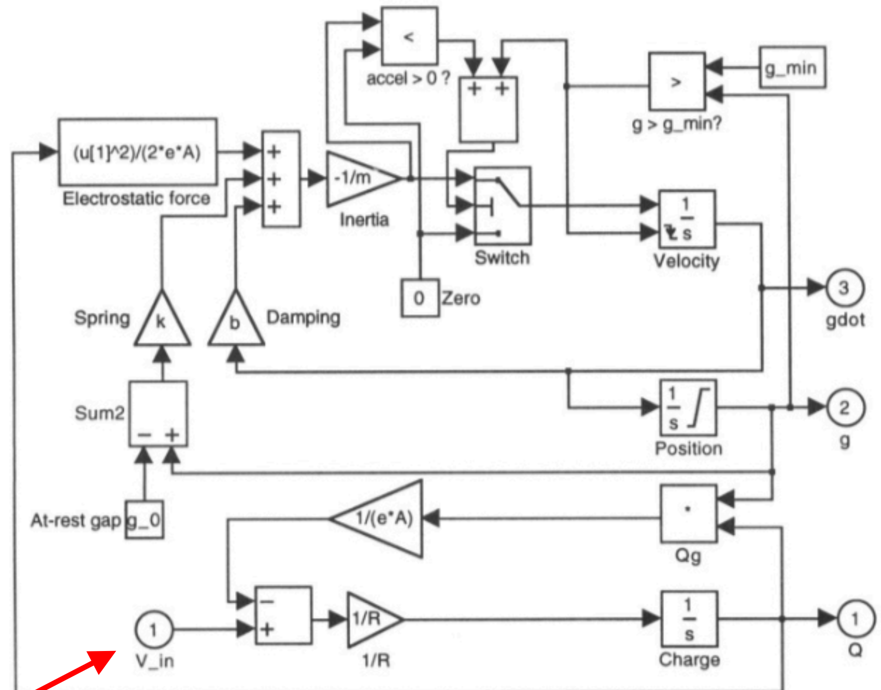
Parallel Plate Actuator with Nonlinear Pull-In SIMULINK Model



S.D.Senturia, *Microsystemdesign*. Springer Science & Business Media, 2007.

NONLINEAR MODELING





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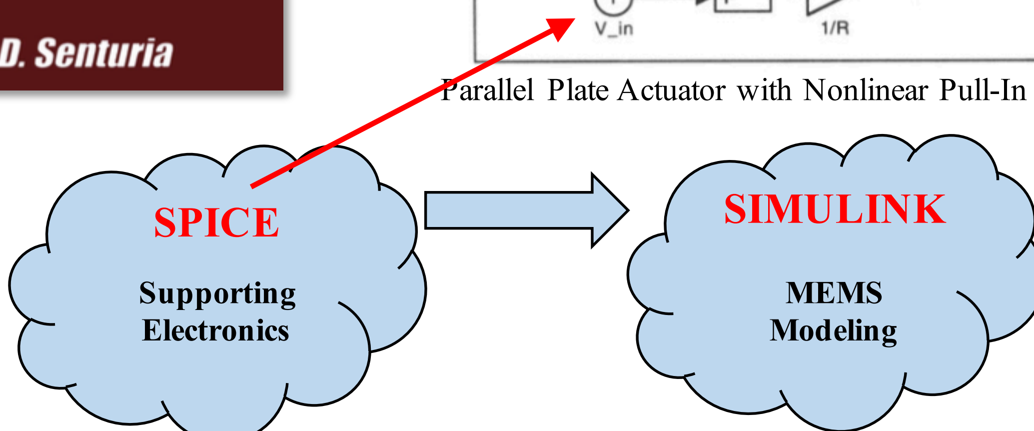

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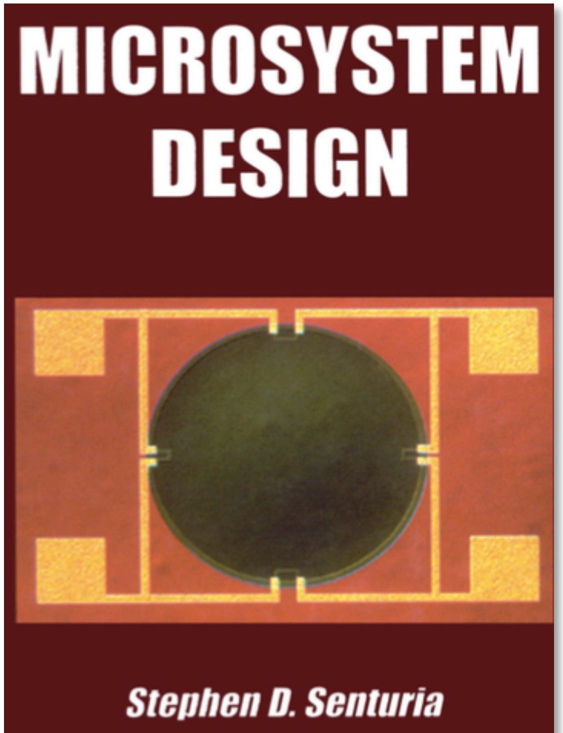
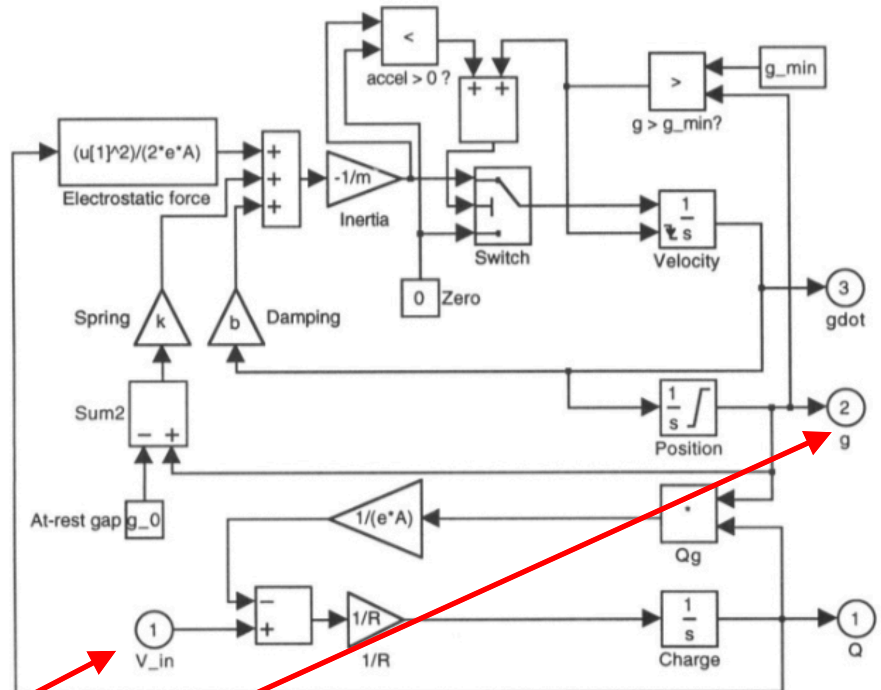
NONLINEAR MODELING





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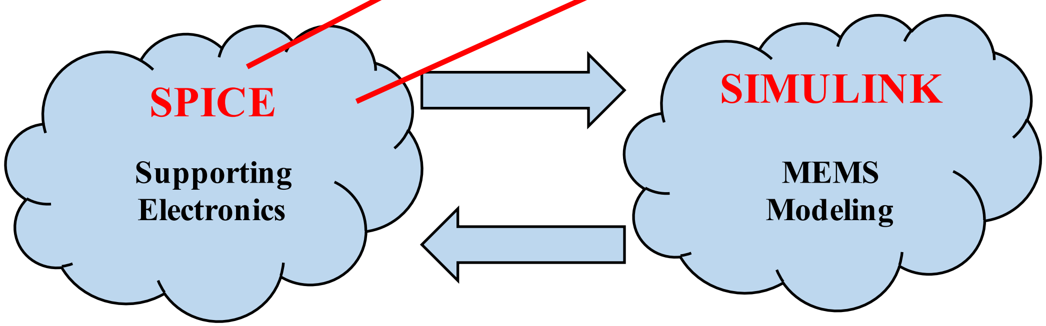

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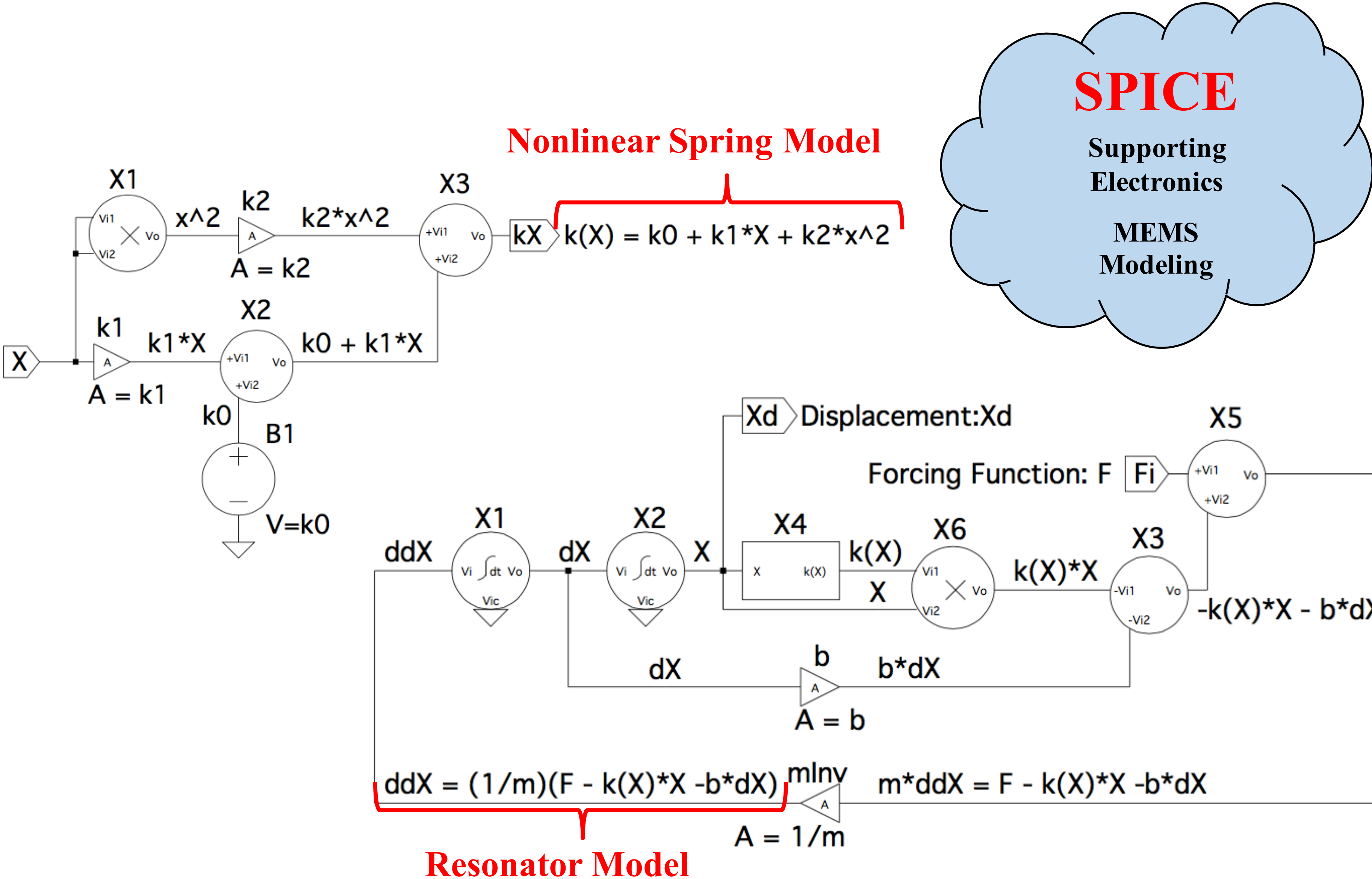
NONLINEAR BEAM MODELING IN SPICE



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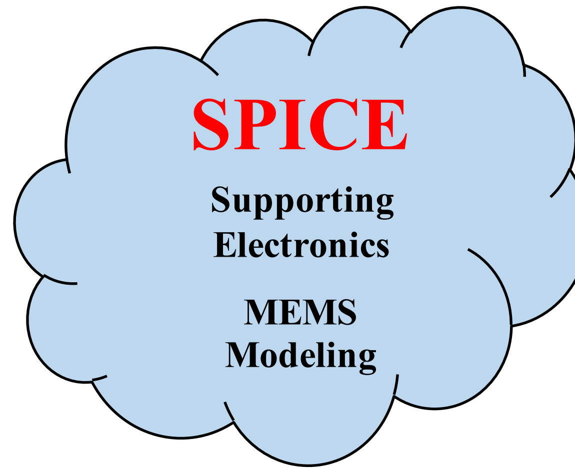

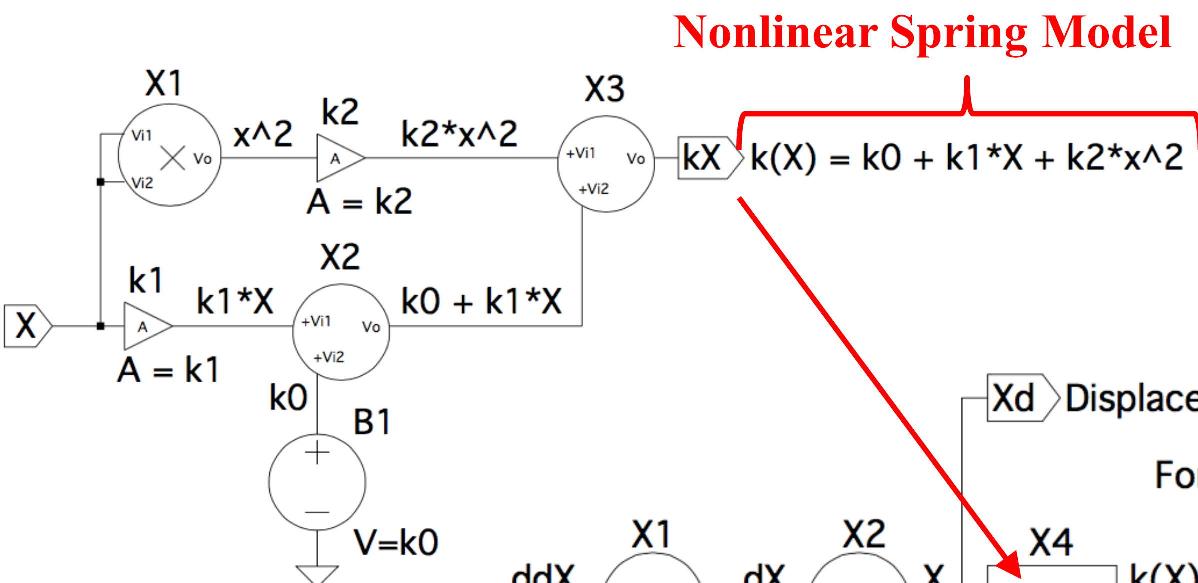

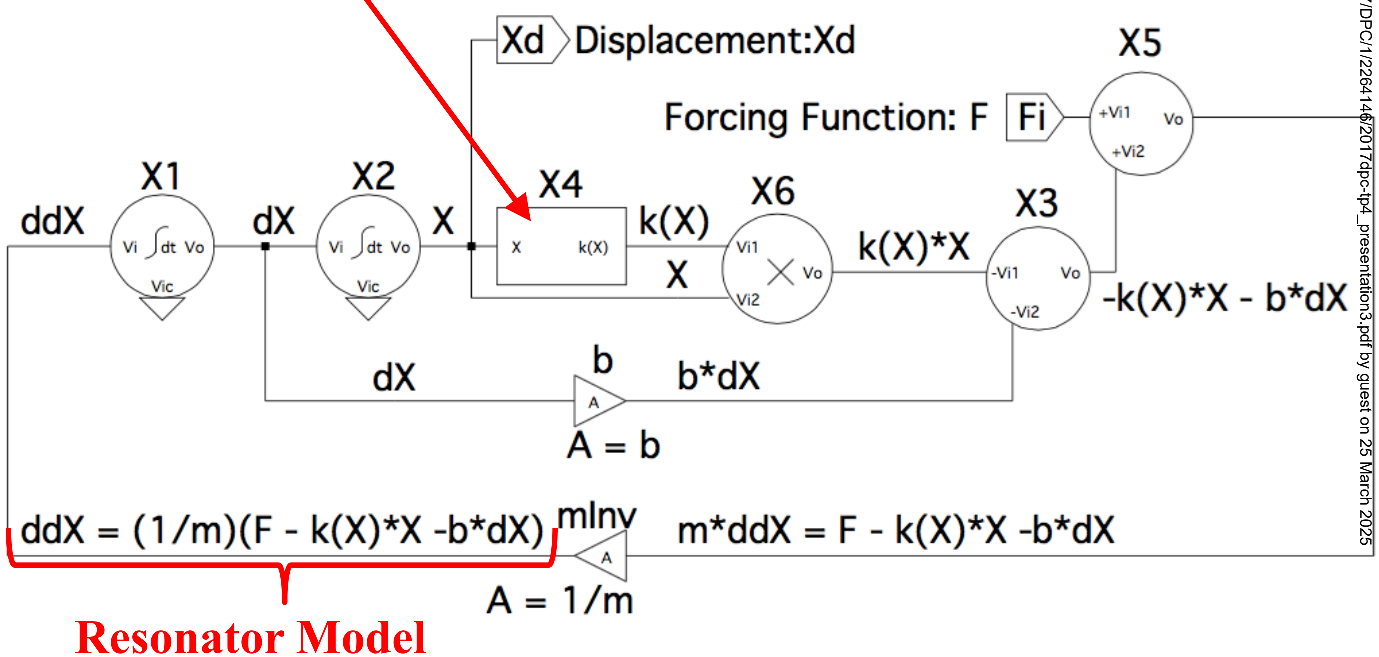

- **-Uses hierarchy**
- **-Uses SPICE primitives**
  - -Integrators
  - -Multipliers
  - -Gain
  - -Etc…
- **-Single software package**

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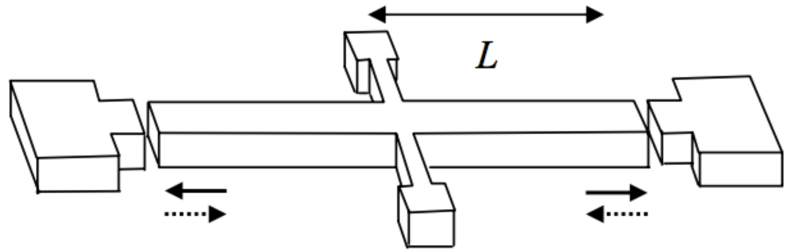
SIMULATION RESULTS



BAW resonator with a length of $200\mu m$

TABLE I: MEMS Parameters

Parameter	Symbol		Units
Linear spring constant	k_0	50m	Nm
	k_1	25	10^3
	k_2	12	10^8
Nonlinearity Measure	k_{NL}	256	
Linear Resonance	ω_0	1.54	kHz
Mass	m	21n	g
Quality factor	Q	1	10^5
Damping	γ	3.33	10^{-5}
Critical vibration amplitude	x_c	44	μm
Energy at x_c	E_c	5	nJ
Power at x_c	P_c	516	pW

V.Kaajakari,T.Mattila,A.Lipsanen,andA.Oja,"Nonlinearmechanical effects in silicon longitudinal mode beam resonators," ***Sensors and Actuators A: Physical*****, vol. 120, no. 1, pp. 64–70, 2005.**

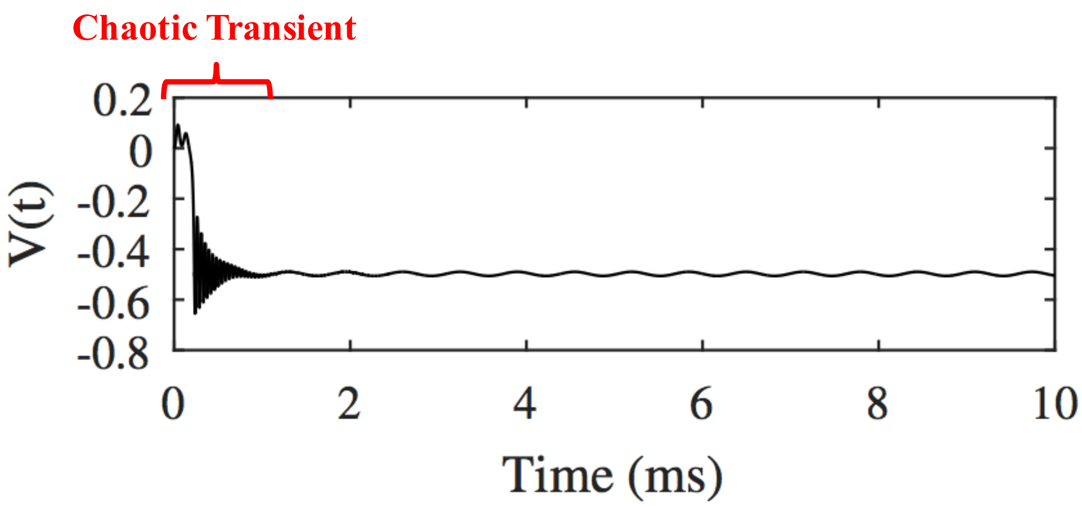
SIMULATION RESULTS

SMALL FORCING AMPLITUDE

$$A_F = 100mV$$

$$F(t) = A_F \cos(\omega_F t)$$

$$f_F = f_0 = \frac{\omega_0}{2\pi} \rightarrow 1.54kHz$$



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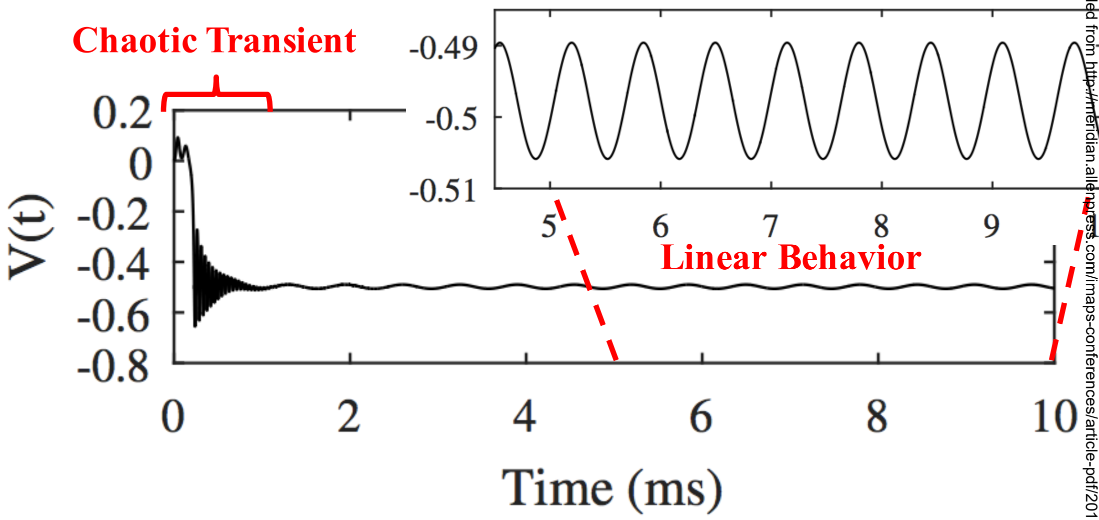

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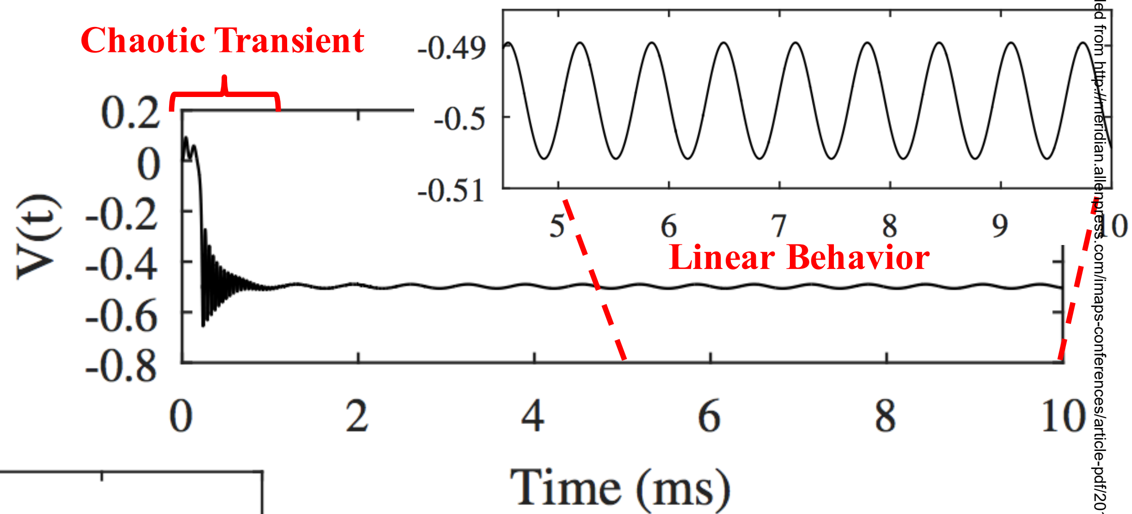

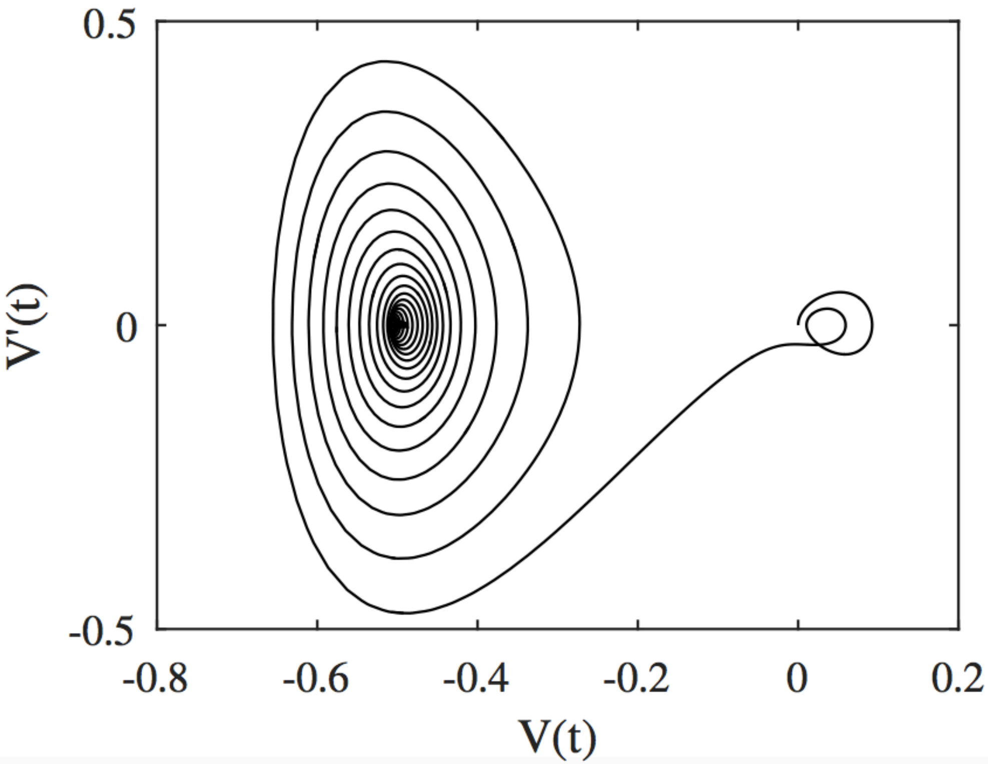

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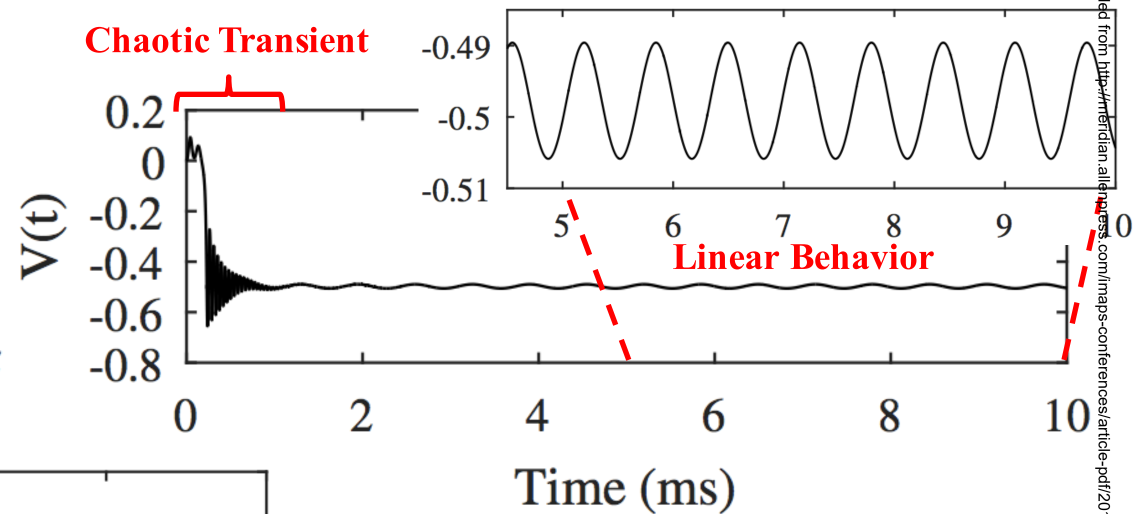

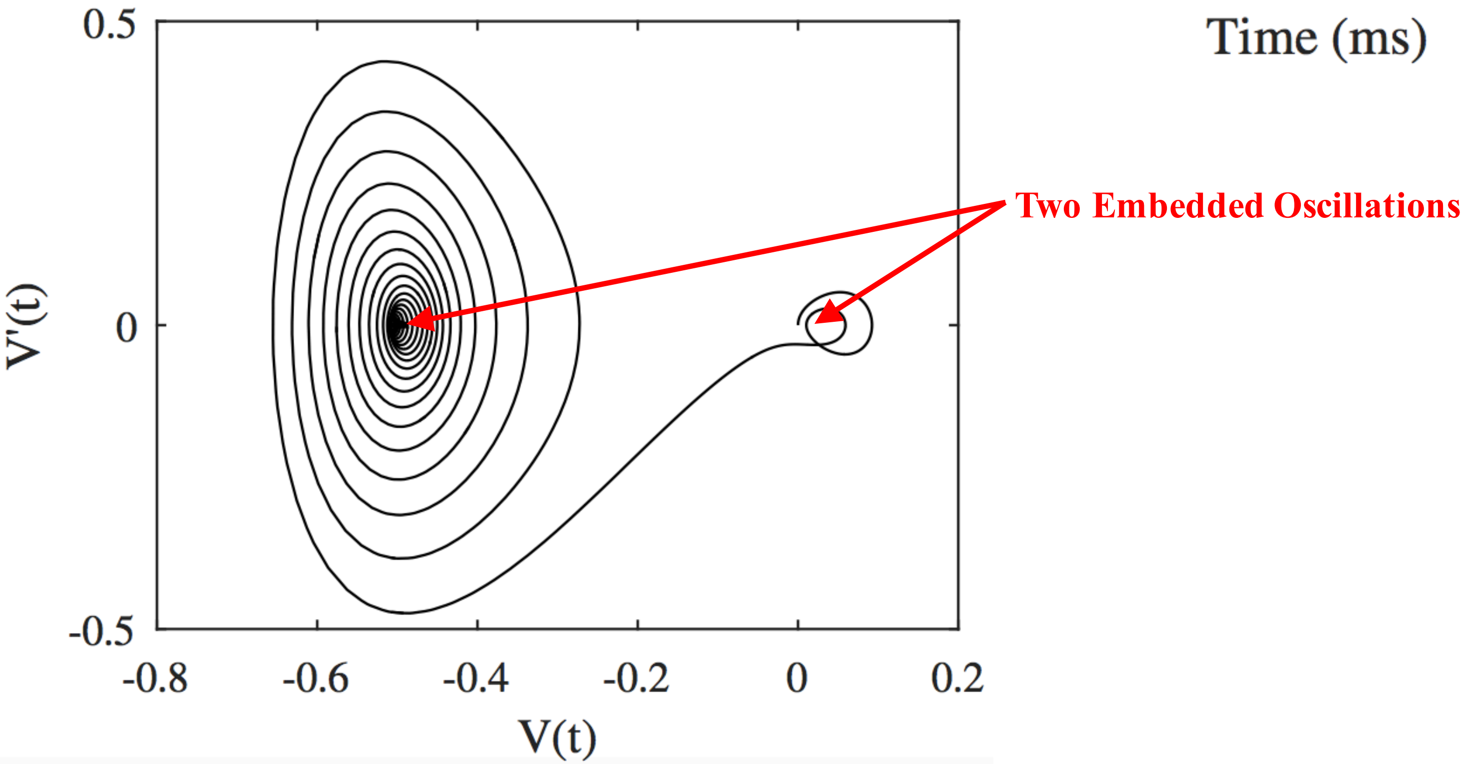

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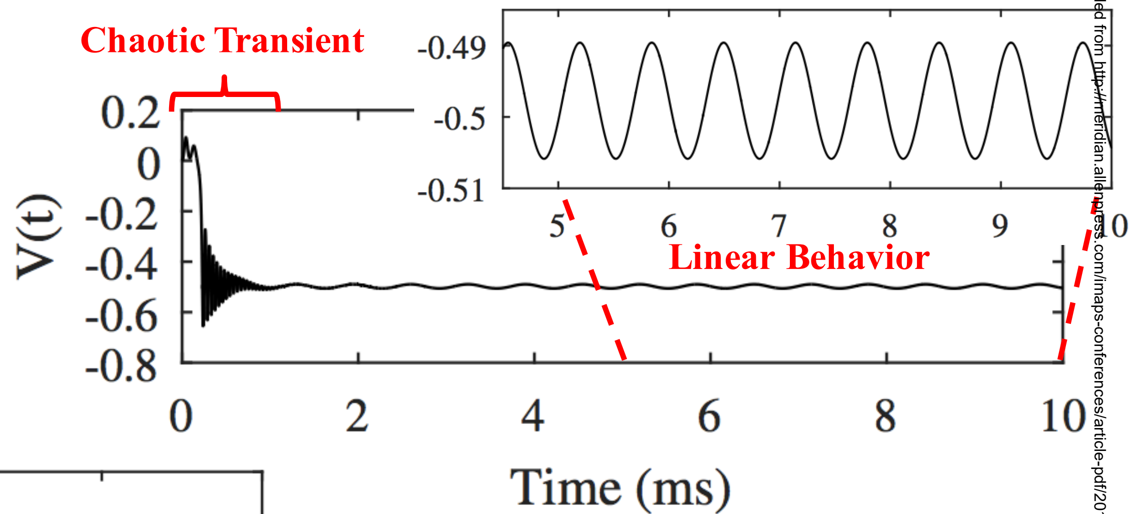

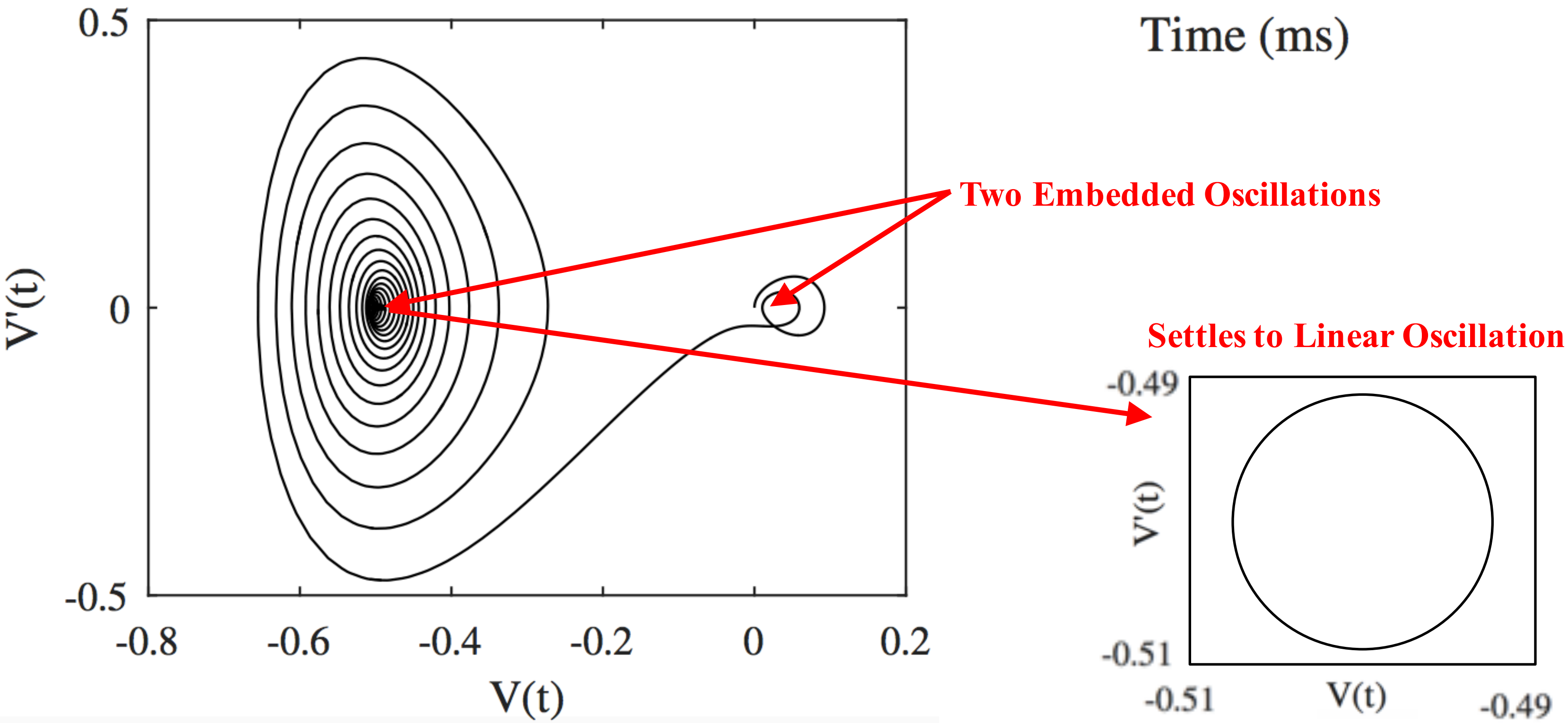

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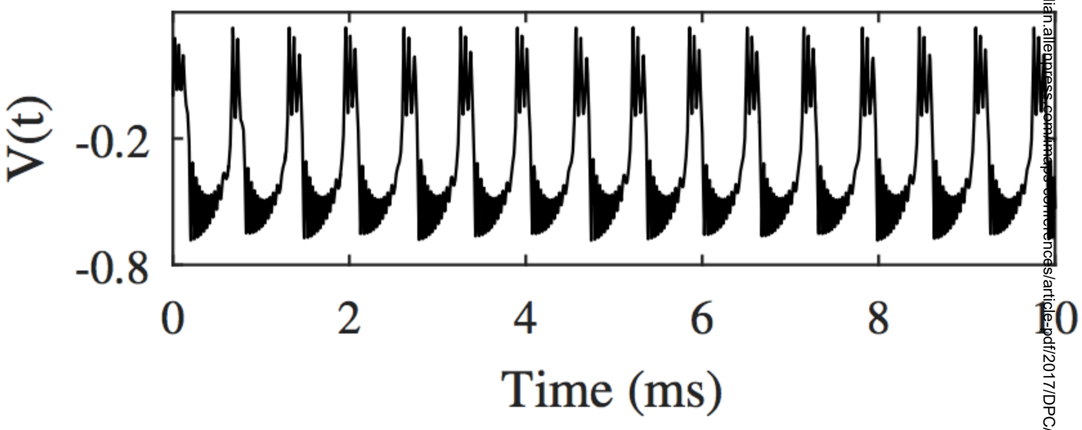

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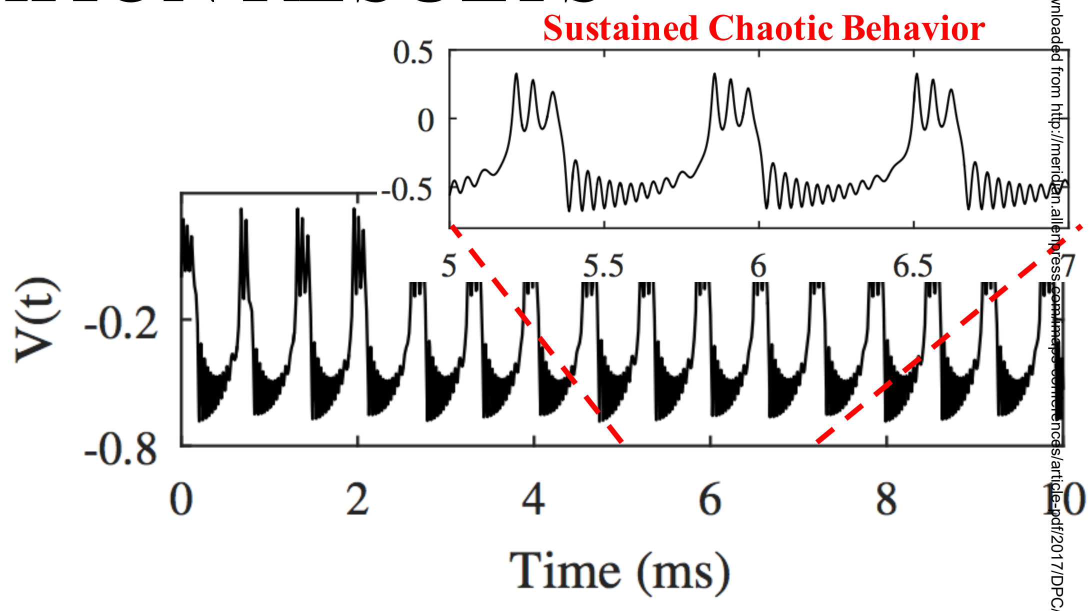

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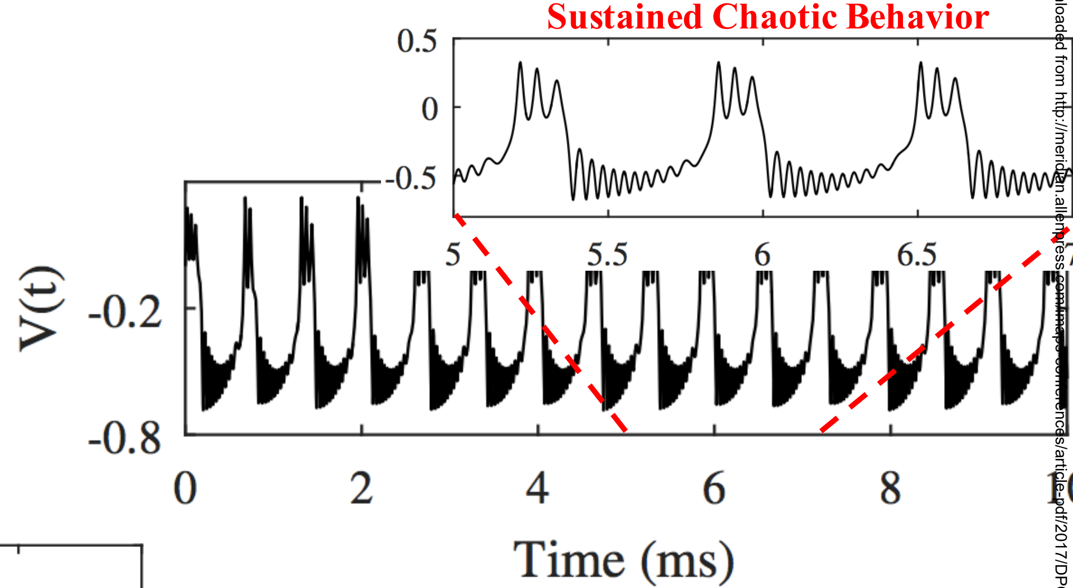

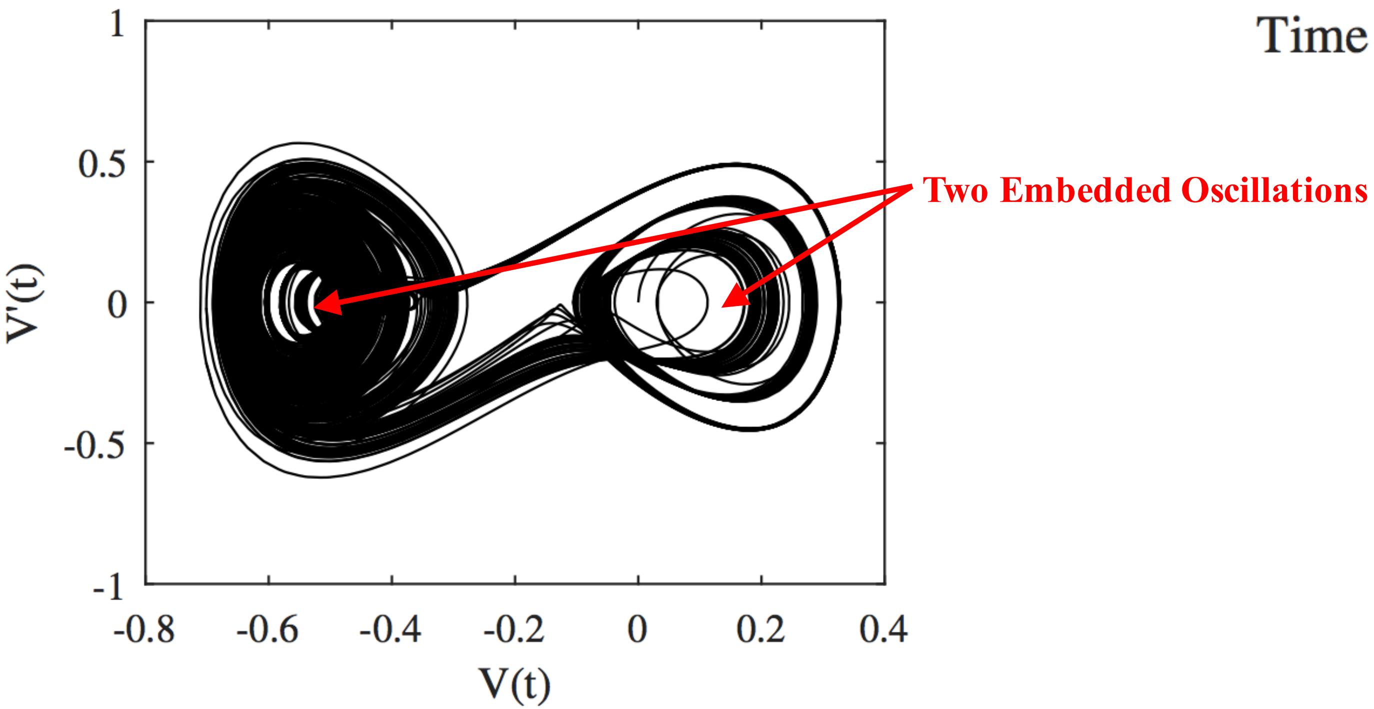

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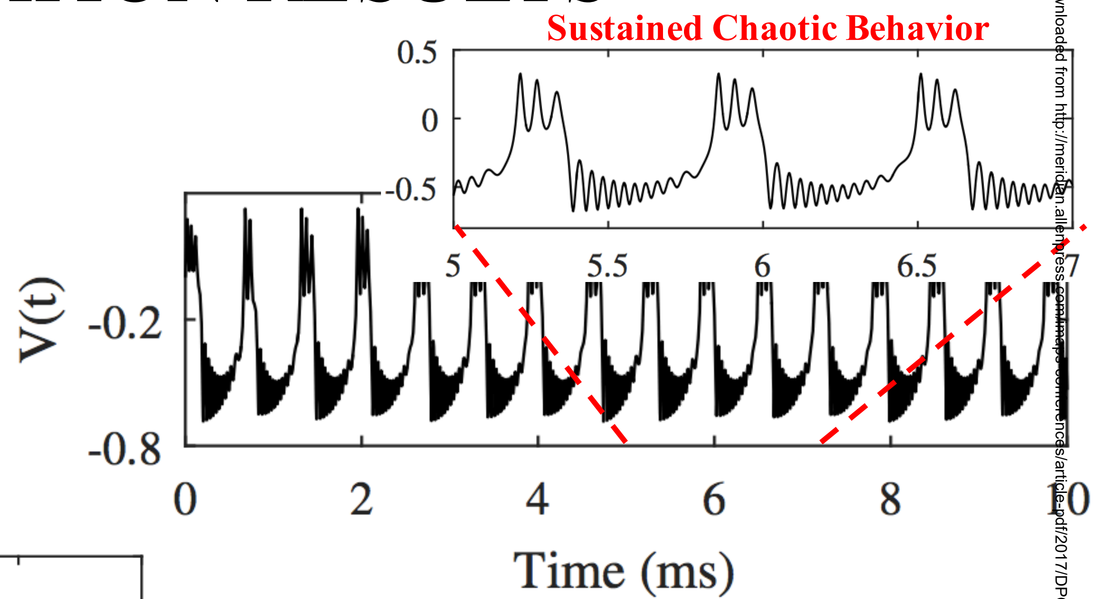
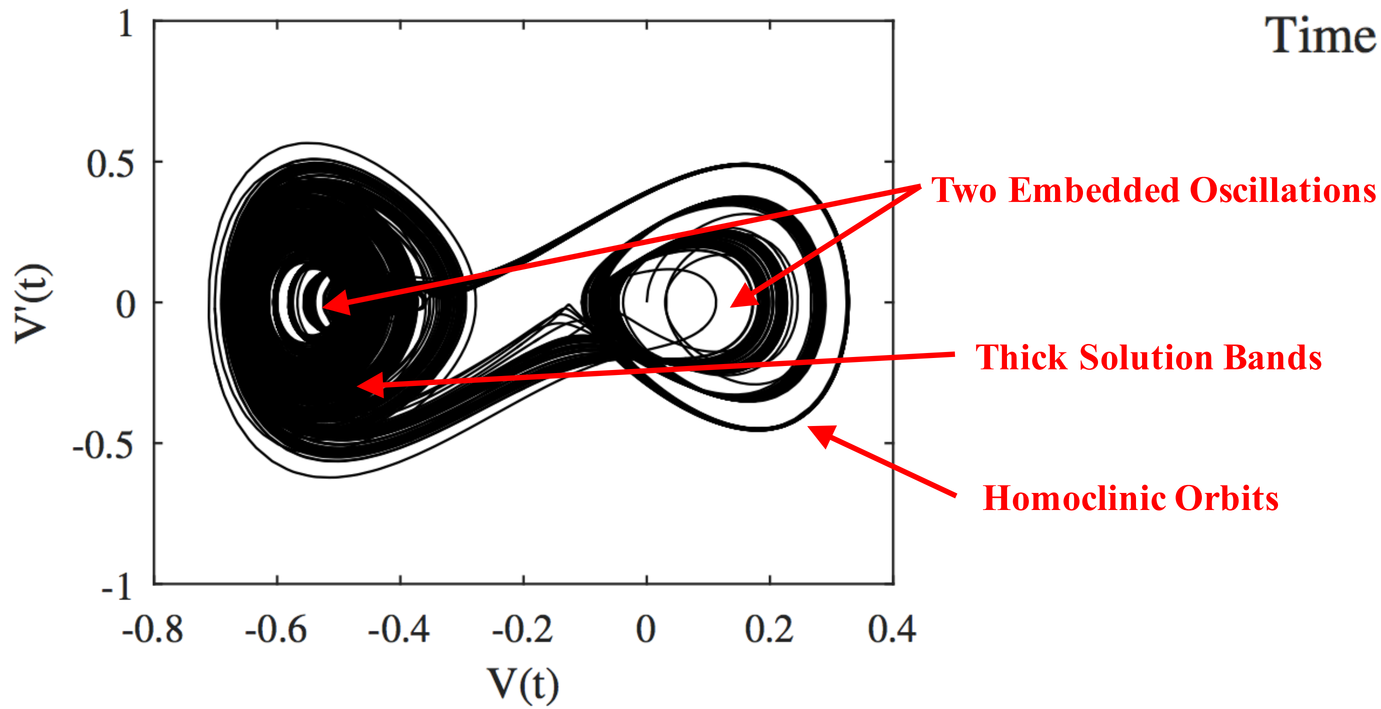
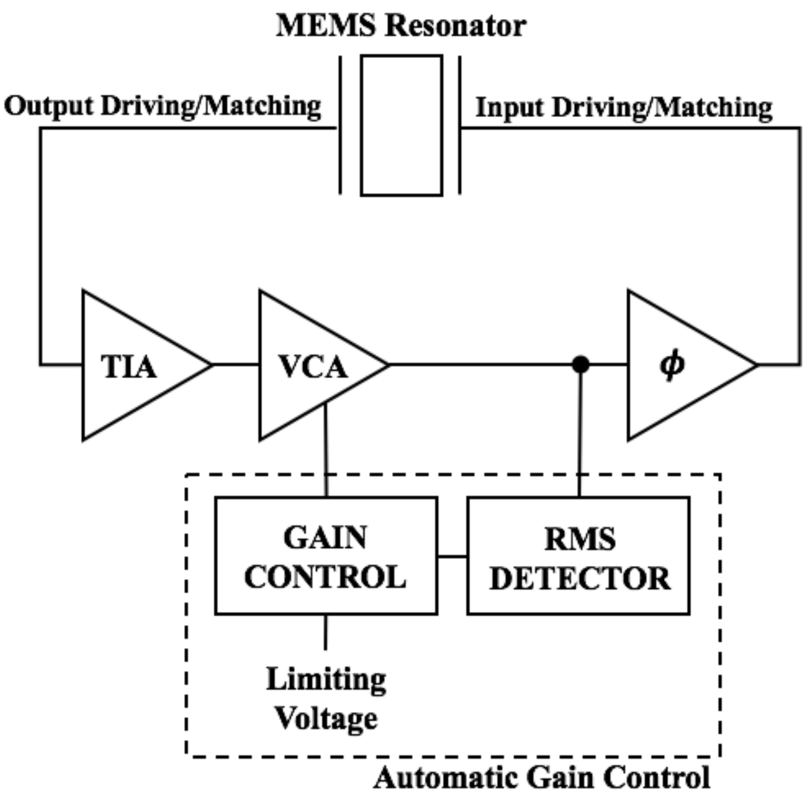




BIFURCATION & SUPPORTING ELECTRONICS



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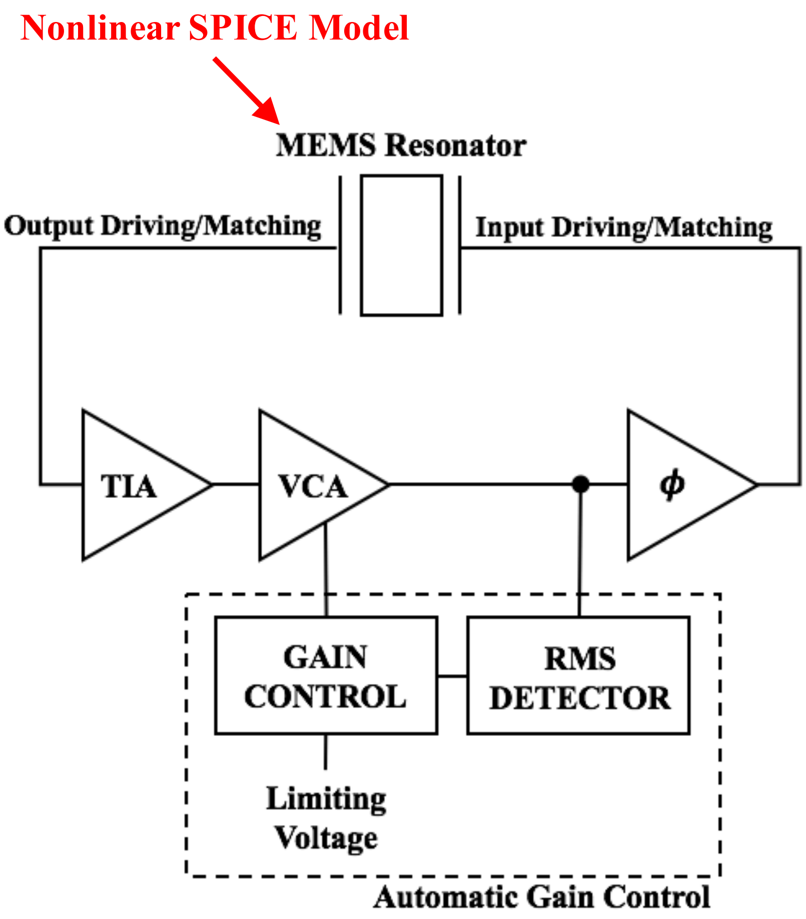
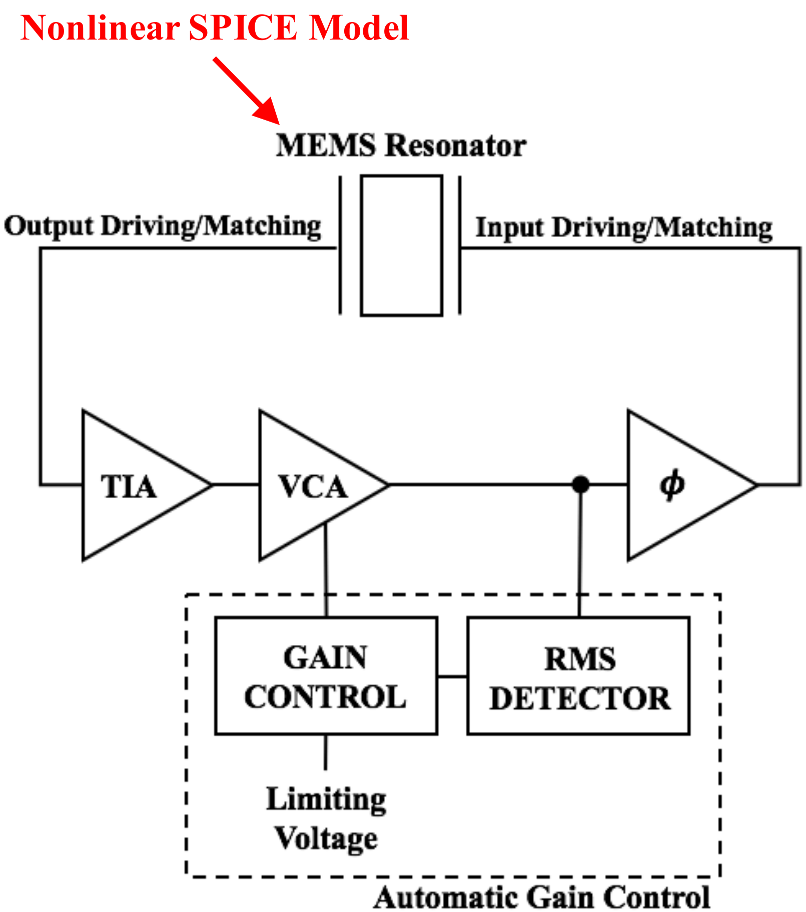
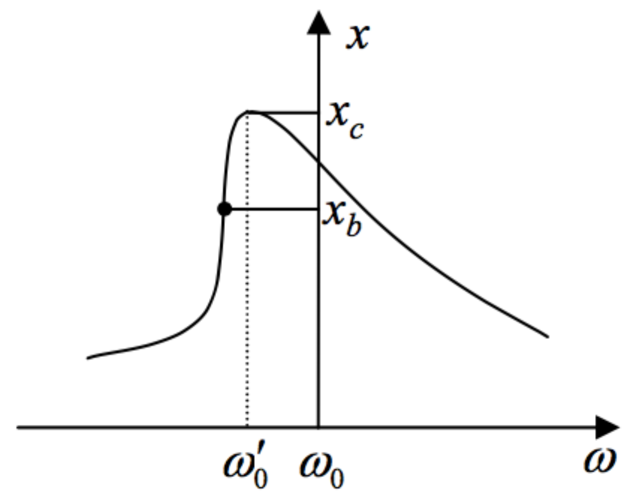


BIFURCATION & SUPPORTING ELECTRONICS

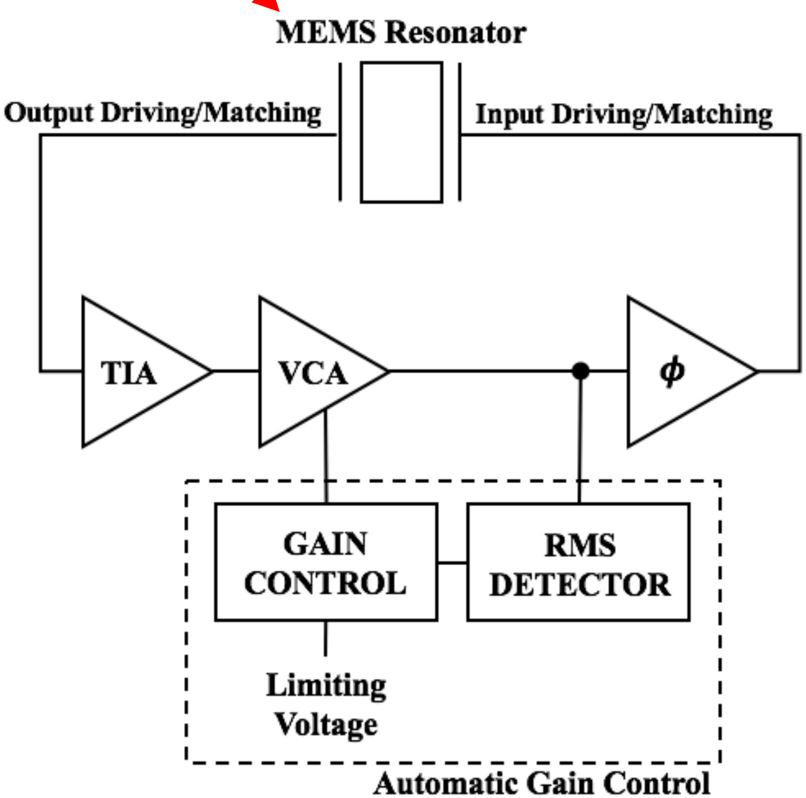




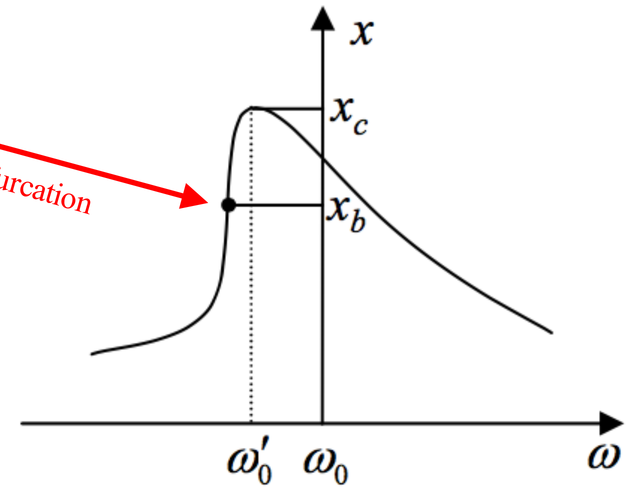
BIFURCATION & SUPPORTING ELECTRONICS

$$x_b = \frac{1}{\sqrt{\sqrt{3}Q|k_{NL}|}}$$

Nonlinear SPICE Model



Amplitude at Bifurcation



BIFURCATION & SUPPORTING ELECTRONICS

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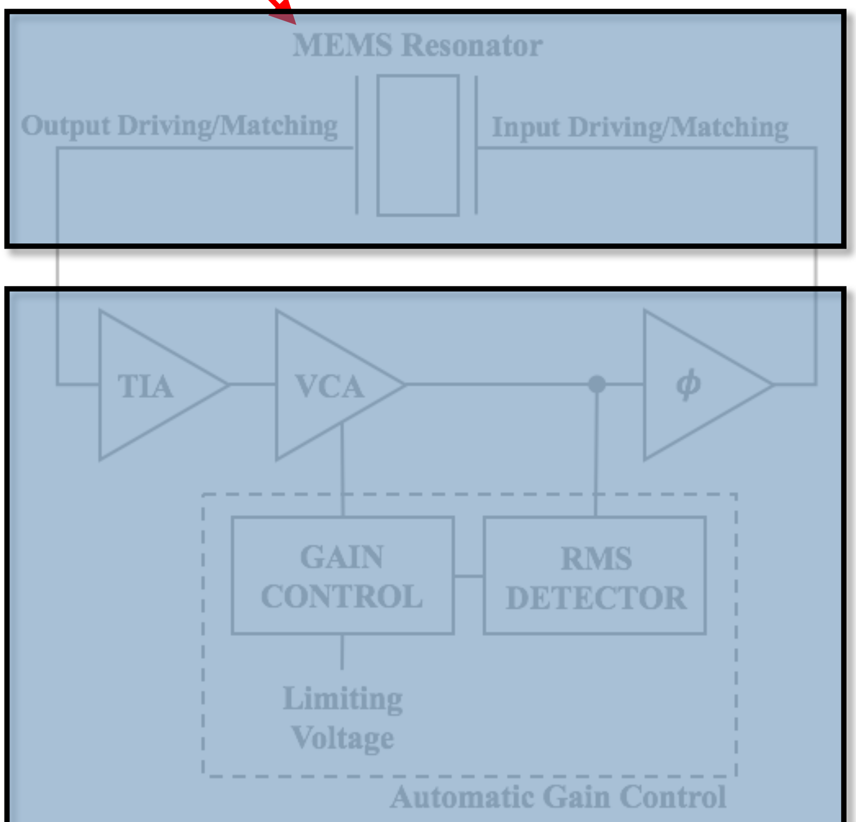
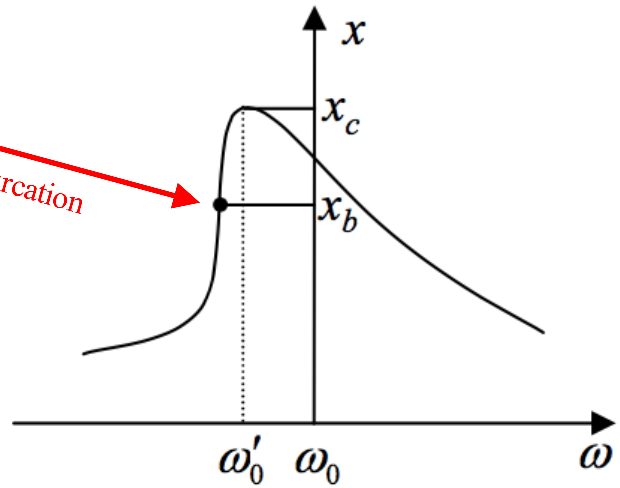


Amplitude at Bifurcation



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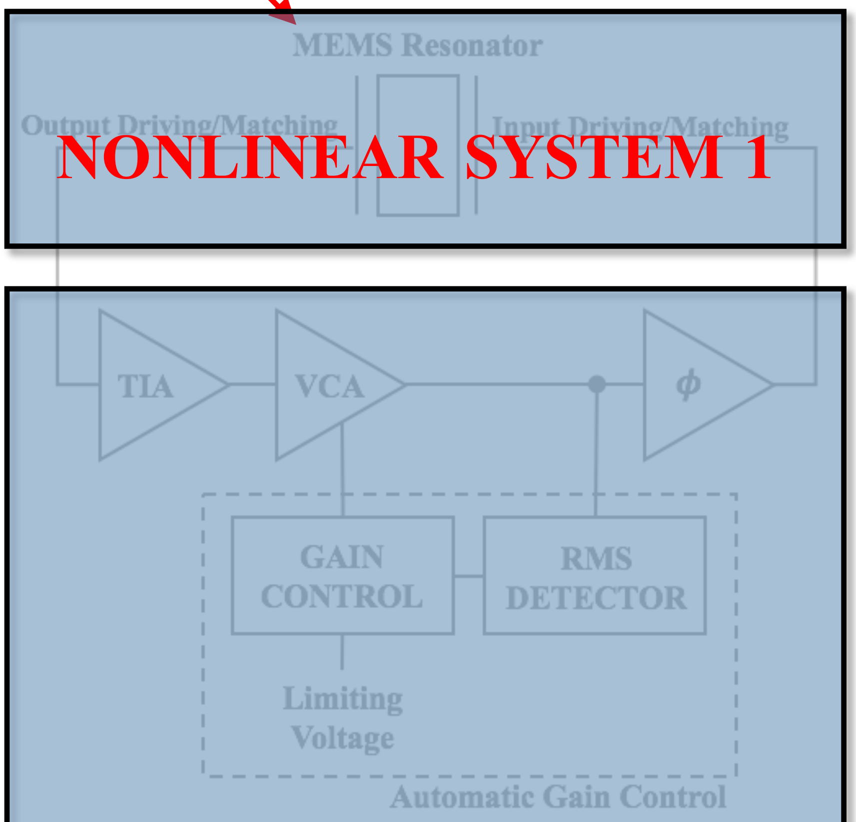

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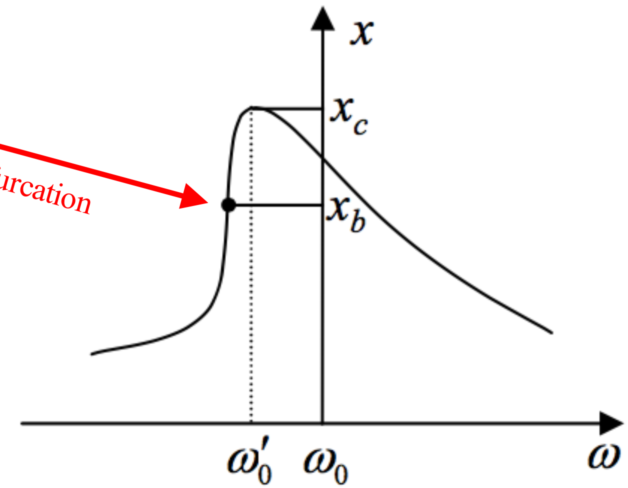

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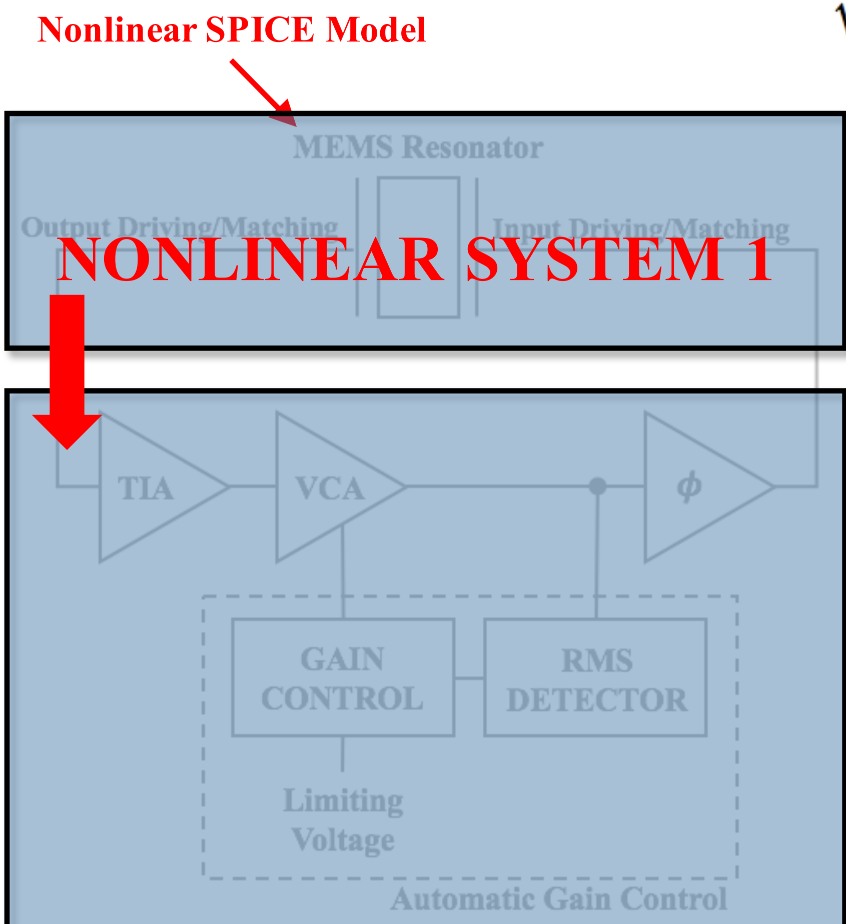

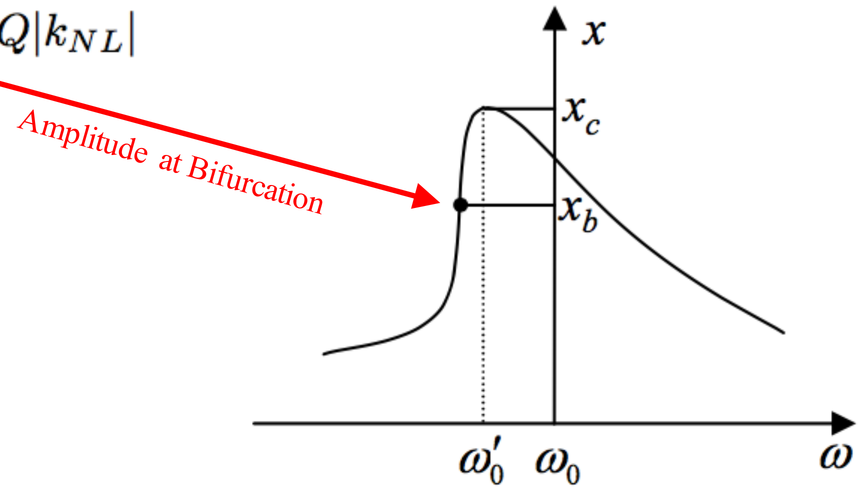

BIFURCATION & SUPPORTING ELECTRONICS

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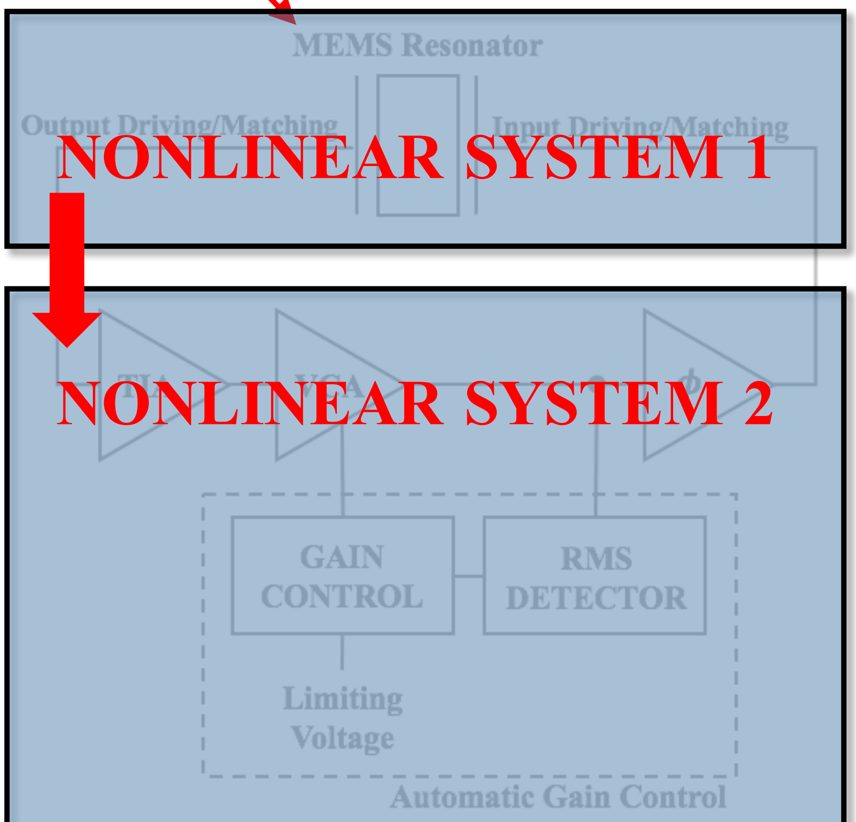
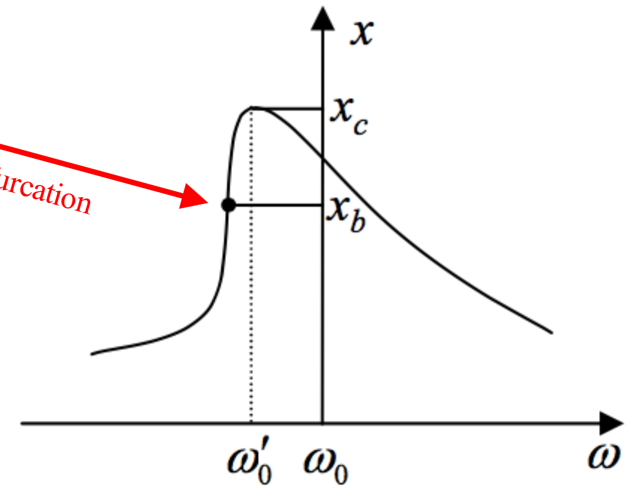
Nonlinear SPICE Model



Amplitude at Bifurcation



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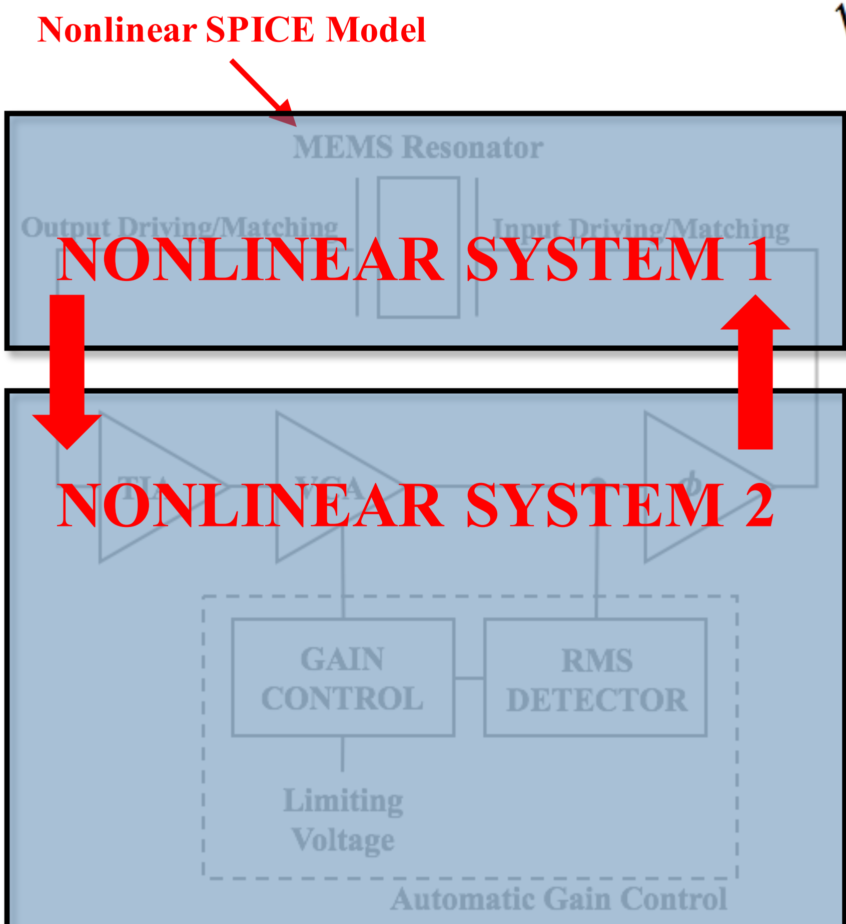

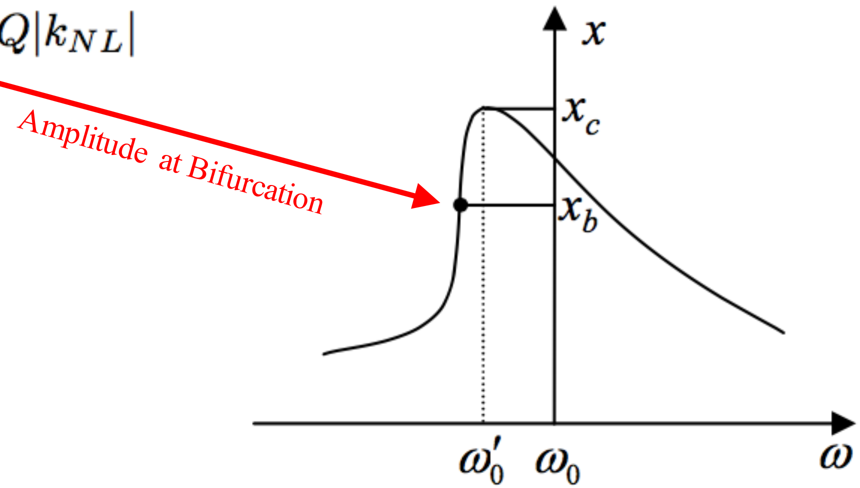

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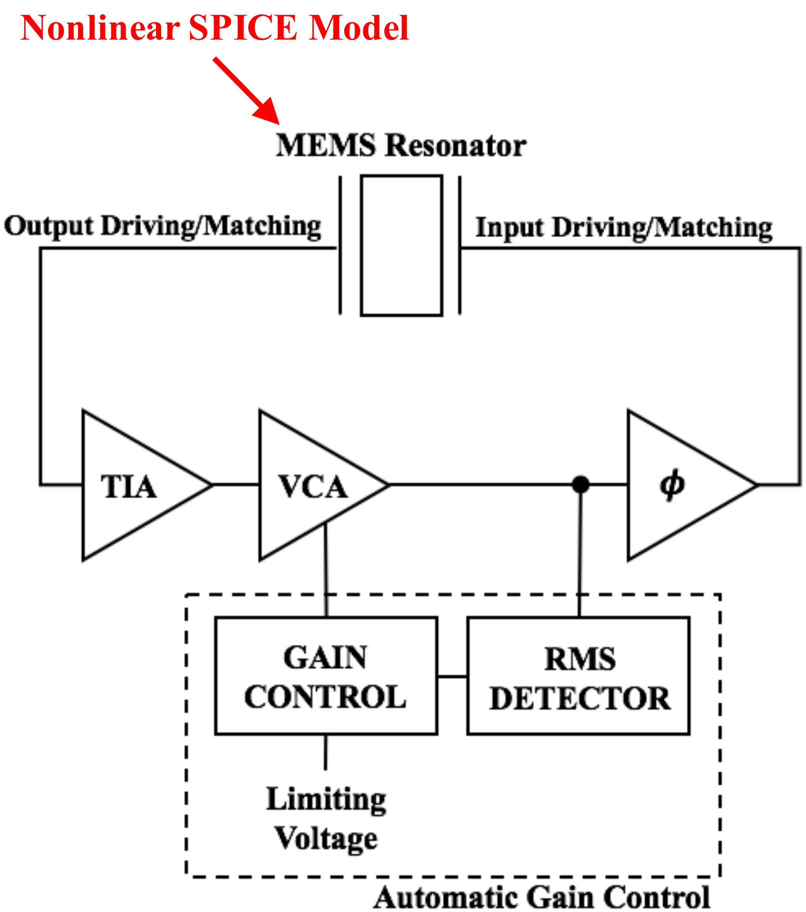
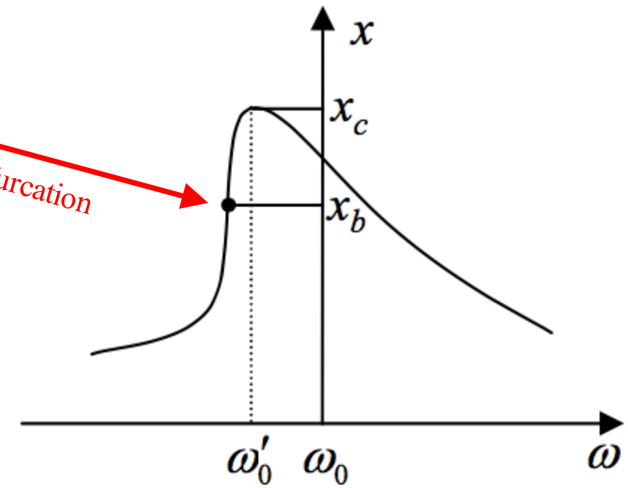
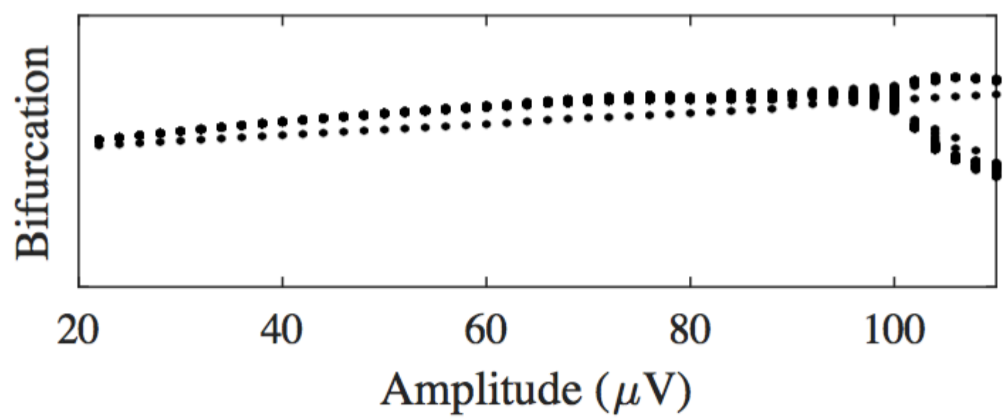


Amplitude at Bifurcation





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BIFURCATION & SUPPORTING ELECTRONICS

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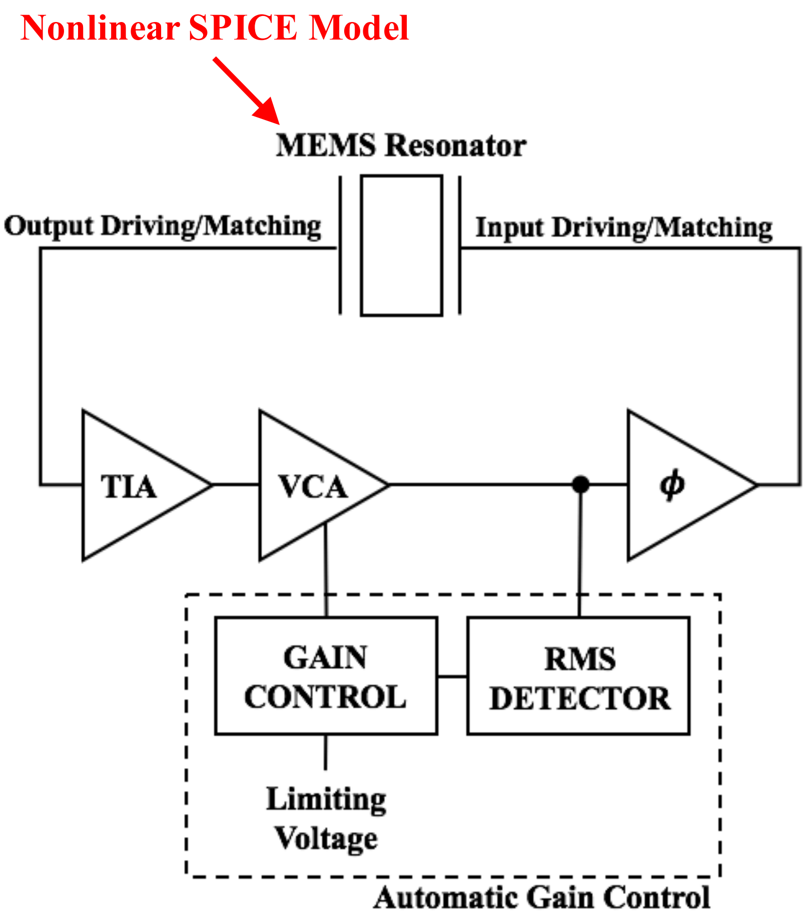

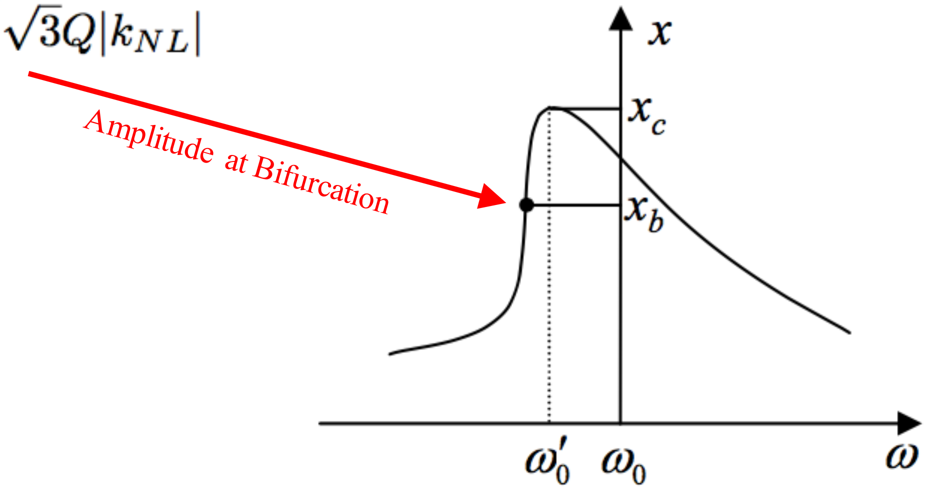

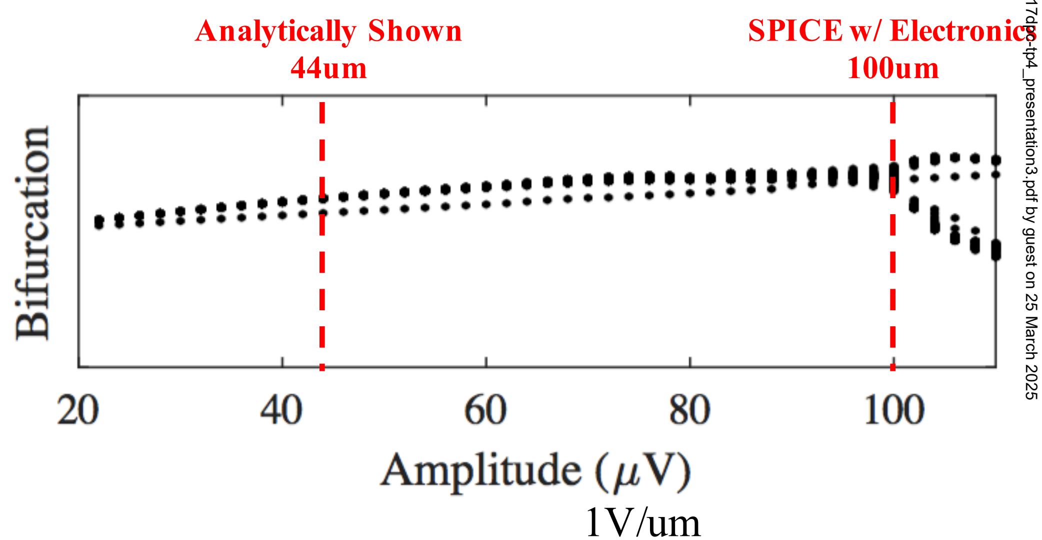

CONCLUSION

- **Nonlinear effects of MEMS beam were modeled**
  - Nonlinear spring constant
  - Due to geometry
  - Due to material properties
- **Exclusively in SPICE**
  - Chaos due to large amplitude
  - Chaotic transients
  - Changes in bifurcation due to supporting electronics

FUTURE WORK

- **Simulate other sources of nonlinearity**
- **Temperature simulations**
- **Expand to other electromechanical systems**
  - Macro
  - Micro
- **Physical Device Modeling**
  - Large amplitude characteristics
  - Model turn-on and chaotic transients
  - Identify bifurcation points
    - Analytically
    - Changes due to supporting electronics

THANK YOU

QUESTIONS?

Direct questions to aubreybeal@gmail.com