# Review of the application of fuzzy inference systems in river flow forecasting

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## ABSTRACT

This paper provides a general overview about the use of fuzzy inference systems in the important field of river flow forecasting. It discusses the overall operation of the main two types of fuzzy inference systems, namely Mamdani and Takagi–Sugeno–Kang fuzzy inference systems, and the critical issues related to their application. A literature review of existing studies dealing with the use of fuzzy inference systems in river flow forecasting models is presented, followed by some recommendations for future research areas. This review shows that fuzzy inference systems can be used as effective tools for river flow forecasting, even though their application is rather limited in comparison to the popularity of neural networks models. In addition to this, it was found that there are several unresolved issues requiring further attention before more clear guidelines for the application of fuzzy inference systems can be given.

**Key words** | fuzzy logic, possibility theory, river flow forecasting, uncertainty analysis

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## INTRODUCTION

Models belonging to the black-box type, or systems-based, attempt to describe the relationship between the input and output variables of a real system without explicit consideration of the internal physical processes that lead to this transformation. Applications of black-box-type models in the field of river flow forecasting have been available for decades. Most traditionally, these models have imposed a rigid functional structure on the input–output transformation. For example, several black-box-type rainfall–runoff models relying on an *a priori* definition of the functional relationship between rainfall (and other input variables) and discharge have been proposed in the literature (Todini & Wallis 1977; Nash & Barsi 1983; Kachroo & Natale 1992).

More flexible modelling techniques, which attempt to capture the input–output relationship of a system by extracting the patterns shown in the input–output data in an adaptive manner, without prior specification of a functional structure, have also been applied. Among these modelling techniques, artificial neural networks (ANN) are prominent for having been successfully applied in a number of river flow forecasting studies, especially during the last decade. In this context, the multilayer feedforward neural network is the most widely used network structure (e.g. Halff *et al.* 1993; Hsu *et al.* 1995; Shamseldin 1997; Rajurkar *et al.* 2004; Vos & Rientjes 2005). Although the possibility of developing models that have a flexible formal structure is certainly attractive, these models still have the enormous drawback of not being transparent, in the sense that their functioning is rather obscure and not easily interpretable. Recent research has attempted to establish a relation between different neural network model components and specific physical processes inside the catchment (e.g. Wilby *et al.* 2003; Jain *et al.* 2004), but these results are not conclusive, as only a few model structures and case studies have been considered.

Fuzzy inference systems (FIS), also referred to as fuzzy models, are nonlinear black-box models that describe the relation between the inputs and the output of a real system using a set of fuzzy IF–THEN rules. The internal

operation of FIS requires the application of the inference rules of fuzzy logic, which can be defined as the "basis to what might be called approximate reasoning, that is, a mode of reasoning in which the truth values and the rules of inference are fuzzy rather than precise" (Zadeh 1975). FIS provide flexible solutions, because their model structure and inference mechanisms can be adapted to the modelling problem.

An important advantage of FIS over traditional black-box modelling approaches is their ability to infer the behaviour of complex systems purely from data, without prior specification of a functional structure. A further advantage of FIS over other data-driven modelling techniques, such as ANN, is that the functioning of FIS is more transparent. Because the structure of fuzzy rules can be extracted from the knowledge available about the real system, they can provide a formal representation with a more readily physical interpretation. In fact, subjective knowledge provided by experts can be incorporated into the model in a natural manner by translation into fuzzy IF–THEN rules, which can then be consecutively calibrated to fit the data. In the sense that their internal operation can be relatively easily understood by humans, FIS are sometimes considered "gray-box" rather than black-box type models (Lindskog 1997).

In spite of these merits, the use of FIS in river flow forecasting is not as widespread as that of other data-driven modelling techniques such as ANN. One reason that probably explains the reluctance of hydrologists towards FIS is that a fuzzy-rule-based representation is inadequate to provide an accurate description of the input–output relationship in cases where too many input variables are required and/or the input–output relation is too complex. In fact, one of the main weaknesses of FIS is what is sometimes called the "curse of dimensionality" (Kosko 1997), which refers to the fact that the number of fuzzy rules that is necessary to model the input–output relation increases exponentially with the number of inputs, eventually resulting in a non-parsimonious model which can be very difficult to calibrate. Reducing the number of rules, however, generally decreases the approximation capabilities of the FIS. Evidently, this problem becomes more serious as the complexity of the input–output relationship increases. Another issue affecting the applicability of FIS

is the lack of clear guidelines for calibrating the model parameters in such a manner that their major advantage, namely the model interpretability, is maintained.

## FUZZY INFERENCE SYSTEMS

### General structure of the fuzzy rules

The structure of a fuzzy rule can be formally expressed as

$$\text{IF} (X_1 \text{ is } A_{1,m}) \text{AND} (X_2 \text{ is } A_{2,m}) \text{AND} \ldots \text{AND}$$
$$(X_K \text{ is } A_{K,m}) \text{ THEN } Y \text{ is} \ldots \tag{1}$$

where $X = (X_1, X_2, \ldots, X_K)$ is the input vector and $Y$ is the output variable of the model. The rule antecedents (i.e. the IF part of the rules) are defined by the terms $A_{k,m}$, representing fuzzy sets (Zadeh 1965) used to partition the input space into a number of overlapping regions, such that the feasible range of each input variable $X_k$ is divided into a series of fuzzily defined and overlapping intervals $A_{k,m}$. Each fuzzy set $A_{k,m}$ is defined by a fuzzy membership function $\mu_{k,m}$, which assigns a membership value $\mu_{k,m}(x_k)$ to every possible input $X_k = x_k$. The membership values $\mu_{k,m}(x_k)$ typically range between 0 and 1, where 0 indicates absolute non-membership and 1 indicates full membership of $X_k = x_k$ in the fuzzy set $A_{k,m}$. The structure of the rule consequents (i.e. the THEN part of the rules) depends on the type of FIS under consideration. There are two main types, namely Mamdani (Mamdani 1974) and Takagi–Sugeno–Kang (Takagi & Sugeno 1985; Sugeno & Kang 1988)-type FIS.

### Types of FIS

The rule consequents of a Mamdani-type FIS can be formally represented by the following equation:

$$Y \text{ is } B_m \tag{2}$$

where $B_m$ represents a fuzzy set in the output space. The $m$th rule of a Mamdani-type FIS expresses a tendency of input vectors $x = (x_1, x_2, \ldots, x_K)$ in the region described by the fuzzy sets $A_{1,m}, A_{2,m}, \ldots, A_{k,m}$, to be associated with outputs $y$ in the region defined by the fuzzy set $B_m$. For a given input $X = x = (x_1, x_2, \ldots, x_K)$, the inference

mechanisms of fuzzy logic (Zadeh 1975) are used to obtain the fuzzy output of each rule and to derive the overall (crisp) output $Y = y$ of the FIS. The characteristics of the input–output relationship of the FIS depends on the fuzzy logic operators chosen (see, e.g., Yager & Filev 1994; Bárdossy & Duckstein 1995; Piegat 2001). The question of which model operators are more suitable in the context of river flow forecasting remains an unresolved issue.

A schematic diagram illustrating the functioning of a multiple-input single-output Mamdani-type FIS is shown in Figure 1. First, the degree of fulfilment (DOF) of each rule, measuring the degree to which $X = x = (x_1, x_2, \ldots, x_K)$ belongs to the region described by the fuzzy sets $A_{1,m}$, $A_{2,m}, \ldots, A_{k,m}$, is evaluated. This information is then presented to the consequent fuzzy sets $B_m$ in order to obtain the response of each rule. The individual rule responses are combined to derive the fuzzy system's response $B$, which is finally converted into the model's output $Y = y$ in a process called defuzzification.

In the case of Takagi–Sugeno–Kang (TSK)-type FIS, the rule consequents are explicit functions of the input variables, as expressed by the following equation:

$$Y = f_m(X_1, X_2, \ldots, X_K) \qquad (3)$$

As pointed out by Takagi & Sugeno (1985), each fuzzy rule of a TSK-type FIS can be interpreted as a local model of the real system under consideration, valid for the vaguely defined region of the input space that is described by the antecedent fuzzy sets $A_{1,m}, A_{2,m}, \ldots, A_{k,m}$. The functions $f_m$

are commonly first-order polynomials such as

$$f_m(X_1, X_2, \ldots, X_K) = b_{0,m} + b_{1,m}X_1 + b_{2,m}X_2, \ldots, b_{K,m}X_K \qquad (4)$$

which implies that the local models are linear functions of the input variables $X_k$.

The overall operation of a multiple-input single-output TSK-type FIS is shown in Figure 2. The model's output, for a given input $X = x = (x_1, x_2, \ldots, x_K)$, is calculated as a combination of the individual rule responses $f_m(X_1, X_2, \ldots, X_K)$. The inference mechanisms of fuzzy logic (Zadeh 1975) are used for determining the relative contribution of each rule, obtained by evaluating the compatibility of the input vector with the rule antecedent (i.e. the degree of fulfilment (DOF) of the rule), generating an overall input–output relationship that may be highly nonlinear, even if the functions $f_m$ are first-order polynomials.

## Rule development issues

The continuity of the model response as a function of the input variables is a desirable feature if the input–output relationship of the real system being mapped does not exhibit jumps. This property can be achieved by choosing continuous fuzzy logic operators and continuous membership function types (see, e.g., Bárdossy & Duckstein 1995; Piegat 2001) and ensuring that at least one rule is activated for every possible input vector. This latter condition, sometimes referred to as completeness in the specialized literature (Bárdossy & Duckstein 1995; Cordon et al. 2001), requires that the sub-regions of the input space defined by the antecedent fuzzy sets must overlap.
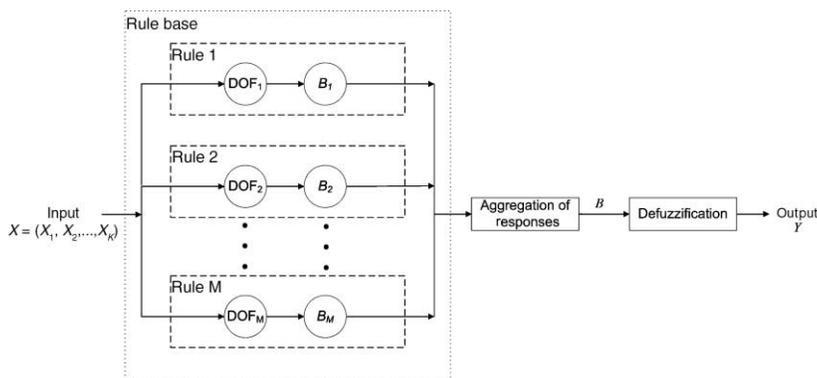


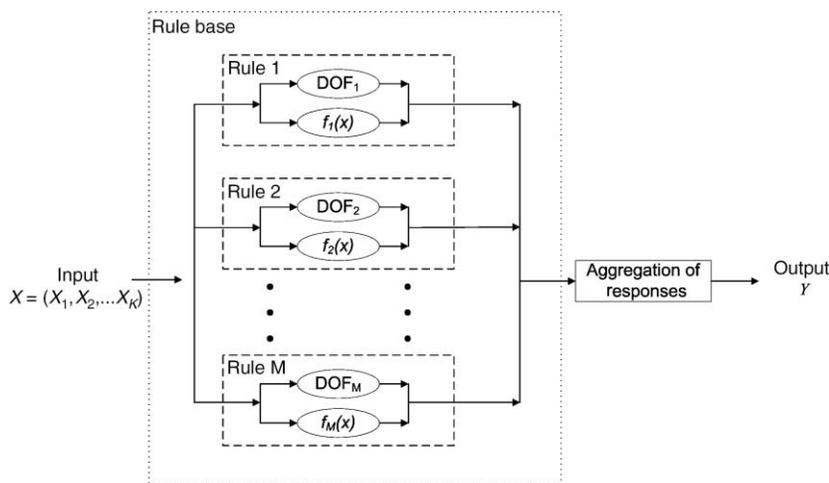**Figure 1** | Functioning of a multiple-input single-output Mamdani-type FIS.

**Figure 2** | Functioning of a multiple-input single-output Takagi–Sugeno–Kang-type FIS.

The number of fuzzy rules which is necessary to approximate the input–output relation of the real system depends on the complexity of the real system, increasing with both the number of input variables and with the inherent complexity of the input–output relationship itself (Kosko 1997). A larger number of fuzzy rules can provide a finer partition of the input space, which would generally lead to a better approximation of a nonlinear real system. However, an excessive increase in the number of fuzzy rules may result in a non-parsimonious model that may be difficult to calibrate and risks overfitting the calibration data (Piegat 2001). Thus, there is a need to strike a balance between model complexity and model performance. Unfortunately, there are no clear guidelines with respect to the selection of the appropriate number of fuzzy rules. This number is often estimated by trial and error, or through the use of clustering algorithms.

### Calibration of FIS

The application of FIS for simulating the behaviour of a real system (e.g. catchment) requires the calibration of the model, understood as the process of adjusting the values of the model parameters so that its response is as close to the observations as possible. Although the definition of the fuzzy logic operators may also be adapted during the calibration process (see, e.g., Cordon et al. 2001), these are usually predefined by the modeller prior to calibration of the model parameters. The parameters of a Mamdani-type

FIS include the parameters defining the antecedent and consequent membership functions. Similarly, the parameters found in a TSK-type FIS consist of those of the antecedent membership functions and the parameters of the consequent functions $f_m$. Accordingly, in both cases the total number of parameters involved in the FIS is directly proportional to the number of fuzzy rules.

In principle, the process of estimation of the parameters of an FIS is essentially a nonlinear optimization problem. There are several studies concerned with the development of *ad hoc* calibration strategies, both for Mamdani-type (e.g. Delgado et al. 1998; Wang et al. 1999; Schnitman & Yoneyama 2000; Ali & Zhang 2001) and TSK-type FIS (e.g. Park et al. 1997; Kim et al. 1998; Chen & Wang 1999; Cordon et al. 2001). Extensive descriptions of methods with a well-established tradition (e.g. recursive least-squares, backpropagation, clustering, etc.) can be found in the work of Mendel (2001), Piegat (2001) and Ross (2004), among others, whereas the application of more recent methods based on genetic algorithms are discussed in the work of Cordon et al. (2001), for example. A breed of hybrid model known as "neuro-fuzzy" has also been used in a number of river flow forecasting studies (e.g. Gautam & Holz 2001; Bazartseren et al. 2003; Nayak et al. 2005; Chang & Chang 2006). This modelling approach is based on (1) finding an equivalent neural network-like structure for the FIS and (2) using ANN training methods (e.g. back propagation and conjugate gradient) to calibrate the FIS.

During the calibration of the parameters of an FIS, it is normally useful to implicitly constrain the values of the antecedent parameters by imposing conditions on the degrees of fulfilment of the rules. Firstly, limiting the overlap between the rule antecedents helps in preserving the interpretability of the FIS. For example, separating the rule antecedents of TSK-type FIS allows us to maintain the interpretation of the FIS as a series of local models of the real system. Secondly, the antecedent fuzzy sets must be such that at least one rule is significantly activated for every possible input to the fuzzy system, in order to ensure an appropriate cover of the antecedent input domain. This requirement is related to the issue of model completeness, discussed in the last section. Finally, the existence of fuzzy rules not sufficiently supported by data must be avoided, by preventing the antecedent fuzzy sets from being trapped in a region where they are not significantly activated during the calibration period. In addition to being useful for preserving the interpretability of the rule system, the last two restrictions also help to alleviate undesirable features in the input–output relationship of the fuzzy models, such as local declinations or bumps (see, e.g., Piegat 2001).

### Applicability of Mamdani versus TSK-type FIS

The structure of Mamdani-type fuzzy inference systems is conceptually simpler than that of TSK fuzzy models, because their rules can be more easily understood by humans. In addition to these, Mamdani-type fuzzy models are more suitable for representing expert knowledge that is given in the form of vague descriptions of the real system's behaviour (Cordon et al. 2001; Piegat 2001). For instance, an expert could provide a set of statements describing the operation of a sluice gate that regulates the water level upstream in a channel. One of these statements could be: "if the water level is high, then the sluice gate must be raised"; this statement can be easily translated into a Mamdani-type fuzzy rule such as: "IF (water level is HIGH) THEN (sluice gate position is HIGH)".

However, the derivation of the model output of a Mamdani-type FIS requires very computationally intensive calculations. As a result, the tuning the model parameters of these models by means of automatic calibration techniques may become a numerically complex problem (see, e.g., Yager & Filev 1994; Piegat 2001; Mendel 2001), not easily resolved with a personal computer. Deriving the output of a TSK-type FIS is simpler and less computationally demanding than in the case of a Mamdani-type model. Accordingly, the parameters of a TSK-type FIS are more easily calibrated using computer algorithms (see, e.g., Piegat 2001; Mendel 2001).

Finally, TSK-type FIS have the advantage of allowing the representation of a complex real system by means of a series of local models, each analytically describing the behaviour of the real system in a particular region of the input space. This kind of representation is certainly useful in the case of real systems that are known to modify their apparent behaviour in different operational conditions (Yager & Filev 1994; Piegat 2001).

## EXISTING APPLICATIONS OF FIS IN RIVER FLOW FORECASTING

First proposed by Mamdani (1974) as a modelling tool for controlling a dynamic plant, FIS have been subsequently applied in a variety of fields (see, e.g., Zimmermann 2001; Ross 2004). A review of existing applications of fuzzy logic in various research areas within the broader field of earth sciences can be found in the work by Demicco & Klir (2004), for example. A number of studies have used FIS as a solution to complex hydrological modelling problems, such as the interdependence between global circulation and precipitation (e.g. Galambosi et al. 1998; Pongracz et al. 2001), reconstruction of missing precipitation events (e.g. Abebe et al. 2000), groundwater flow and solute transport in the unsaturated zone (e.g. Bárdossy et al. 1995; Bárdossy 1996; Dou et al. 1999) and infiltration (e.g. Zeigler et al. 1996). More recently, FIS have also begun to be employed in the context of river flow forecasting.

### Flow forecasting combination

One of the earliest applications of fuzzy inference systems in this area has been the development of flow forecasting combination methods. It has been observed that the accuracy of flow forecasts can be improved by combining the discharge estimates from several individual models working in parallel

rather than relying on a single individual model, which may be unable to simulate the behaviour of the catchment in all operating conditions (Shamseldin et al. 1997).

Previous studies devoted to this topic have shown that FIS are an effective tool for producing such forecast combinations. For example, See & Openshaw (2000) used Mamdani-type FIS for combining the outputs of a series of river level forecasting models, finding that this combination method yielded better results than more traditional model combination strategies, namely an unweighted average and a Bayesian approach. Similarly, Xiong et al. (2001) used TSK-type FIS for combining the outputs of five rainfall–runoff models, comparing the performance of this combination method with that of a simple average, a weighted average and an ANN model. Overall, the FIS-based combination was as accurate as the weighted average and the ANN model.

## Autoregressive models for time series forecasting

Using a time series modelling approach, some authors have applied FIS to construct time series forecasting models with a purely autoregressive input structure. These models are intended to find a relationship between past flow measurements and future discharges, differing from more traditional autoregressive models (see, e.g., Salas et al. 1980) in that this relationship is not *a priori* restricted to being linear.

To start with, it has been observed that FIS-based time series forecasting models can outperform ARIMA models (Nayak et al. 2004; Lin & Chen 2005). Nayak et al. (2004) also found that a TSK-type FIS-based flow forecasting model can have a similar performance to that of an ANN using the same input information. In any case, the main drawback of a time series approach to hydrological forecasting is that, although autoregressive models are able to provide good forecasts in some situations, they are not useful in cases with a weaker relationship between past and future time series values.

## Flow routing models

Flow routing models are yet another interesting application of FIS in river flow forecasting. Several types of flow routing models based on this modelling technique have been proposed in the literature, differing mainly in their input structure. Some studies have used lagged upstream water levels or flows as the only inputs to a TSK-type FIS (e.g. Chang et al. 2001; Bazartseren et al. 2003; Jacquin & Shamseldin 2004). This kind of model is intended to approximate the propagation of the flood wave along the river, without considering the influence of other external forcings. Evidently, this strategy has the disadvantage that the model is not applicable in cases where the contribution of rainfall over the catchment cannot be neglected (e.g. large humid catchments). Other researchers have incorporated rainfall amounts, in addition to lagged upstream measurements and recent water level or flow information at the catchment outlet, as input variables to either Mamdani-type (e.g. See & Openshaw 2000) or TSK-type FIS-based flow routing models (e.g. Nayak et al. 2005). Because these latter models have an autoregressive input component given by recent flow measurements at the catchment outlet, they operate in updating rather than in simulation mode. All the above-mentioned flow routing models have been quite successful at producing water level/flow forecasts in rivers, but they would probably not be very useful in estuaries, where astronomical tides largely influence the water levels. Following this line of work, Chang & Chen (2003) constructed a fuzzy inference system that successfully modelled changes in the water level in an estuary, using an input structure consisting of lagged upstream water levels, recent water level measurements at the estuary cross section and astronomical variables (lunar month, lunar day and time) as input variables.

In general, flow routing models based on FIS have shown good forecasting abilities. However, there are also cases where the forecasts obtained with this modelling technique are not really superior to those of far simpler methods, such as ARMA models (e.g. See & Openshaw 2000) or multiple linear regressions (e.g. Jacquin & Shamseldin 2004). These findings only confirm the need to evaluate the quality of any model against real data, as "no such 'perfect' model exists that will perform well on all catchments, at all times, and in all circumstances" (O'Connor 2005).

## Rainfall–runoff modelling

Finally, the application of fuzzy inference systems in the development of rainfall–runoff models has also been

investigated in the past. An early example of this line of work is the study by Hundecha *et al.* (2001), who developed a conceptual type rainfall–runoff model using Mamdani-type fuzzy models. This model was constructed by replacing each processing module of an existing rainfall–runoff model, namely the HBV model (Bergström 1995), by a fuzzy inference system. Each one of these fuzzy models was intended to approximate the internal functioning of the corresponding conceptual processing element, without imposing a rigid functional structure to its input–output relationship. Quite disappointingly, the model developed did not show a good performance, obtaining discharge estimates that were not superior to that of the original HBV model. A possible explanation for the poor performance of the fuzzy HBV model is likely to be the chosen calibration procedure.

Other researchers have applied first-order TSK (e.g. Gautam & Holz 2001; Boukhris *et al.* 2001; Vernieuwe *et al.* 2005) and Mamdani-type FIS (e.g. Chang & Chen 2001; Chang *et al.* 2002), for developing black-box rainfall–runoff models with an autoregressive component, that is, including recent discharge measurements among their input variables. Given that previous discharge measurements are used as input information, these models are suitable for operation in updating mode. Even though this line of work has produced promising results in terms of model fitness, it is not clear whether the good agreement between the model discharge estimates and the discharge observations is mainly due to the contribution of the autoregressive component of the model, rather than to an appropriate description of the rainfall–runoff relationship. Studies dealing with the application of FIS for the development of black-box-type rainfall–runoff models that are suitable for operation in simulation mode suggest that fuzzy models are a suitable alternative for modelling the rainfall–runoff relation using a systems-based approach (e.g. Jacquin & Shamseldin 2006, 2008; Sen & Altunkaynak 2006).

## RECOMMENDATIONS FOR FUTURE RESEARCH

There are two main research areas on which future studies should focus, in order to give FIS a wider acceptability as effective modelling tools for river flow forecasting. It is possible that modelling guidelines that are specific to each kind of application need to be developed. First, the process of building the rule base has to be further analysed, as the issue of rule explosion is a major problem affecting the applicability of FIS. Even though some work has been done in this direction, it is advisable that future research provides clearer guidelines with respect to the selection of input variables, selection of the appropriate number of rules, rule construction and removal of unnecessary rules. Future research work also should aim at developing guidelines for model calibration that allow keeping of the interpretation of the fuzzy rules with a hydrological basis. For example, it was mentioned that constraining the antecedent parameters by limiting the overlap between the rules, ensuring an appropriate cover of the antecedent input domain and avoiding the existence of fuzzy rules not sufficiently supported by data may improve the interpretability of the rule system. However, it is not clear to what extent these constraints may affect the model performance.

## CONCLUSIONS

FIS are black-box-type models that allow approximating a nonlinear input–output relationship (see, e.g., Piegat 2001; Zimmermann 2001). The main advantage of this modelling technique over other data-driven methods is that their model structure is more easily interpretable by humans. There are two kinds of FIS that are usually found in the literature, namely Mamdani and TSK types. While Mamdani-type FIS have a more human-friendly formal structure, TSK-type FIS are less computationally demanding and easier to calibrate.

There are several previous studies concerned with the application of FIS, both of Mamdani and TSK type, in hydrological modelling. The use of fuzzy inference systems for the development of river flow forecasting models is currently being intensively investigated. These applications include flow forecasting combination methods (e.g. See & Openshaw 2000; Xiong *et al.* 2001), flow routing models (e.g. Chang *et al.* 2001; Bazartseren *et al.* 2003) and rainfall–runoff modelling (e.g. Hundecha *et al.* 2001; Chang *et al.* 2002). In general, these studies have shown

that FIS are a suitable alternative for river flow forecasting problems. Nevertheless, it was also seen that FIS are far from being infallible and model evaluation against field data remains necessary, as is also the case with any other modelling technique.

Finally, gauging by the number of publications, the use of FIS in river flow forecasting is not widespread in comparison with ANN. This may be an indication of a reluctance from hydrologists to accept the use of fuzzy models, partly because there are no clear guidelines for building the rule base and calibrating the model.

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