\[ \text{Var} \{ \phi \} = \sigma^2 \sum_{i=1}^{4} \left( \frac{\partial \phi}{\partial x_i} \right)^2 + \sigma^2 \sum_{i=1}^{5} \left( \frac{\partial \phi}{\partial \theta} \right)^2 + R^2 \]  

(13)

The coefficients \( \frac{\partial \phi}{\partial x_i} \) and \( \frac{\partial \phi}{\partial \theta} \) can be evaluated numerically. For example,

\[ \frac{\partial \phi}{\partial x_i} \approx \frac{\phi(\theta_1, \mu_1, \mu_2 \ldots \mu_5) - \phi(\theta_1, \mu_1, \mu_2 \ldots \mu_5)}{h} \]

(14)

where \( h \) is a sufficiently small number.

Fig. 8 shows the three-sigma mobility band calculated for a given mechanism with tolerances of \( \pm 0.005 \) in. and clearances of 0.001 to 0.002 in. The interpretation of this band is that the probability that a manufactured mechanism will have an error outside the band at a given \( \theta \) is small (about 0.003 for approximate normality). The stars represent the values obtained independently by sampling, and give an indication of the accuracy of the process. It is this value (three-sigma value), evaluated at particular intervals of the input angle, that is referred to in earlier papers by the authors [5] as the "sensitivity factor."

If this band were combined with or "warped" around the structural error curve, the total variation of the output from the desired output could be seen. It is this information which is valuable in the optimization phase of the realistically synthesized mechanism. Figs. 9, 10, and 11 show the results of four-bar mechanisms synthesized for generating \( y = x^2, y = \sin x, y = \log x \). The dimensions of the mechanisms can be seen in Table 1.

### Table 1: Four-bar mechanisms

<table>
<thead>
<tr>
<th>Function</th>
<th>( X^2 )</th>
<th>( \sin X )</th>
<th>( \log_10(X) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \theta ) (deg)</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>( \Delta \phi ) (deg)</td>
<td>147.002</td>
<td>116.211</td>
<td>8.602</td>
</tr>
<tr>
<td>( \phi_1 ) (deg)</td>
<td>233.353</td>
<td>72.85</td>
<td>100.681</td>
</tr>
<tr>
<td>( B_1 )</td>
<td>2.70687</td>
<td>2.06980</td>
<td>1.34063</td>
</tr>
<tr>
<td>( B_3 )</td>
<td>3.50957</td>
<td>2.42146</td>
<td>2.47267</td>
</tr>
<tr>
<td>( B_4 )</td>
<td>0.36506</td>
<td>0.74013</td>
<td>2.32348</td>
</tr>
<tr>
<td>Max. error (zero tol. and cl)</td>
<td>0.075 deg</td>
<td>0.238 deg</td>
<td>0.655 deg</td>
</tr>
<tr>
<td>Fig</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

### Conclusions

The consideration of variances or sensitivity of the output due to tolerances and clearances in an optimization technique has the effect of scanning the error criterion surface with an area or spot of varying size, rather than scanning with an infinitesimal point. Therefore, the design procedure may not only avoid unfavorable sharp peaks on the error criterion surface but it should also result in the attainment of more designs. It may also be possible to obtain a more favorable design (from the standpoint of link-length ratios and transmission angles), and a more accurate design than the one synthesized without allowing for eventual mechanical errors.

### Acknowledgments

The authors wish to express their appreciation to Professor L. Cote for his advice in the application of the statistical techniques. This work was undertaken in part with the aid of a National Aeronautics and Space Agency Traineeship during 1965 and 1967. Computing time was provided by the Purdue University President’s Computer Reserve Fund.

### References


### DISCUSSION

D. A. Kugath

I suspect the increase in width is related to dead center positions. One also wonders how one interprets the mobility bands when the linkage is preloaded. Does Fig. 8 give us a hint?

To summarize, I think the paper’s methods are clear enough that anyone who has access to a computer could investigate on their own the effect of tolerances and clearances. However, for those not so fortunate, I think there is room for more interpretive discussion.

D. C. Tao

"Effect of Tolerance and Clearance in Linkage Design," by R. E. Garrett and Allen S. Hall, Jr., describes the application of sophisticated mathematical techniques for optimum result in the design of linkage mechanisms. Since the practical aspect of linkage mechanism design is taken into consideration, such techniques would be extremely valuable to industry people as well as to academicians. I should like to see more papers of this caliber in the ASME TRANSACTIONS.

The techniques described in this paper will result in optimum mechanism design; however, in actual production (due to varied manufacturing differentials), no two parts are exactly alike; thus, in critical applications, a finer adjustment is needed for consistent and exact results. This fine adjustment of the parameters of the linkage, e.g., link length and initial phase angle of certain members, is essential.

Although there have been some articles published in this area, the synthesis of linkage mechanisms with fine adjustments needs to be investigated in greater depth.

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### Authors’ Closure

The authors wish to thank Dr. Kugath and Dr. Tao for their remarks. With regard to Dr. Kugath’s concern over preloading, we should like to point out that there was no intention of considering the linkage preloading problem here.