DISCUSSION

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The analysis and conclusions presented for the magnetohydrodynamic slider bearing are valid for the specific case considered and electromagnetic pressurization is indeed possible. However the analytical approach is needlessly complicated by the use of superposition and the specific restrictions on the general solution have not been explicitly discussed.

The superposition of the solutions for channel and Couette flow does not add to the physical or mathematical understanding because it tends to obscure the approximations and assumptions which are implicit in the model that is used to describe the semi-infinite slider bearing. The equation of motion (equation (1) of the paper) describes the linearized one-dimensional viscous flow between stationary or moving parallel plates. The *exact* solution of this equation, treating dP/dx as an unknown function of only x and using the boundary conditions given in equation (2*b*), is identical to that which is given in equation (4*c*). Now the approximation can be made that this solution can be used to represent the velocity profile for the case where the plates are no longer parallel (but skewed only by a small perturbation). Then the flow rate and pressure distribution can be found as in equations (5) to (9) where the film thickness is a function of x and the boundary conditions on pressure are those given in equations (8*a*) and (8*b*). It is evident that, for a constant film thickness, the pressure is uniform and the solution for Couette flow is obtained. These remarks also apply to the MHD solution.

The requirement that the tangential magnetic field, H_x, be zero at the outer surfaces of the plates specifies that the total current over the cross-section of the fluid and the plates is zero which implies open circuit conditions. This restriction on the general solution, equation (19*a*), was not mentioned explicitly and it should be pointed out that it is possible to locate stationary electrodes in the x-y plane on the z axis and derive a general set of boundary conditions on H_x as a function of the electrical loading at the electrodes. This approach would provide a complete description of the MHD slider bearing and approximate solutions for nonconducting plates and large values of M could still be obtained.

The discussion presented in Appendix C on the contribution of the diagonal terms of the Maxwell stress tensor to a normal stress at a wetted surface or interface is open to some question. It can be shown (and is discussed in reference [9], p. 97) that *any arbitrary* term δ_{ij} which has zero tensor divergence can be added to the stress tensor. Then one could conclude that the normal stress at the interface would be $(T_{yy} + \delta_{yy})$ and the pressure P' would become

$$P' = P - T_{yy} - \delta_{yy}$$

The point is that only the tensor divergence of the Maxwell stress tensor has any physical meaning. A complete formulation of the electromagnetic body force on the fluid (neglecting only the electrostriction forces and forces due to an inhomogeneous permitivity) is

$$\vec{f}_e = \rho_e \vec{E} + \vec{J} \times \vec{B} - \frac{1}{2} H^2 \nabla \mu_m + \nabla \left[\frac{1}{2} \rho H^2 \frac{\partial \mu_m}{\partial \rho} \right]$$

where ρ_e is the electric charge density, μ_m is the magnetic permeability of the medium and the units are RMKS. Then using this form of the electromagnetic body force it can be shown that a pressure discontinuity at the interface can only occur if there is a discontinuity in μ_m at the interface or if magnetostriction effects are present (reference [9], p. 162, which discusses the electric field analog). However, since this paper considers a nonmagnetic fluid and nonmagnetic plates, the pressure should be continuous across the interface of the wetted surface.

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Author's Closure

The author wishes to express his appreciation to Professors Elco and Hughes for their comments which are helpful in clarifying certain points in the paper.

Concerning the comment on the superposition of solutions used, it is the author's contention that a recognition of the equivalence of a superposition of Couette and channel solutions to the straightforward integration of the equation of motion does aid in physical insight into the problem. This is a subjective viewpoint, however, and depends on one's choice of alternative interpretations possible. The important point technically is that because of the linearity of the equation of motion, a superposition interpretation is a valid one.

The comment that the electromagnetic boundary conditions of Equation (15) (discussed in Appendix A) correspond to the open circuit case is correct. This open circuit condition was implied by assuming the induced magnetic field vanishes outside the channel walls, although not stated explicitly. That the solution obtained corresponds to the open circuit case can be seen as follows.

The current density from the Maxwell equation is

$$\frac{4\pi}{c} J_z = -\frac{dH_x}{dY}$$

The total current per unit channel length, including current in the channel walls as well as the fluid, is

$$I = \int_{-h_s}^{0} J_z dy + \int_0^w J_z dy + \int_0^{w+h_m} J_z dy$$

or

$$I = -\frac{c}{4\pi} \{H_{xw})_{y=0} - H_{xw})_{y=-h_s} + H_{xf})_{y=w} - H_{xf})_{y=0} + H_{xw})_{x=w+h_m} - H_{xw})_{y=w}\}$$

where subscript w refers to the wall and f refers to the fluid. Since the field is continuous across the interfaces between wall and fluid, we have

$$H_{xw})_{y=0} = H_{xf})_{y=0}$$
$$H_{xw})_{y=w} = H_{xf})_{y=w}$$

and

$$I = -\frac{c}{4\pi} \{H_{xw})_{y=w+h_m} - H_{xw})_{y=-h_s}\}$$

The total current is thus the difference between the values of induced field at the outside boundaries of the two walls. Taking the field to be zero at the outer surfaces of the plates then gives $I = 0$ which corresponds to an open circuit condition.

The discussers' comment on the role of the Maxwell stress tensor is not too clear, and additional clarification can be obtained as follows. It can be shown that, if $\bar{f}$ is the electromagnetic body force and T_{ij} is the Maxwell stress tensor, then

$$f_i = \frac{\partial T_{ij}}{\partial x_j}$$

The complete Maxwell stress tensor (excluding electrostriction and magnetostriction terms) can be written

$$T_{ij} = \frac{K}{4\pi}\left(E_iE_j - \frac{1}{2}\delta_{ij}E_eE_e\right) + \frac{\eta}{4\pi}\left(H_iH_j - \frac{1}{2}\delta_{ij}H_eH_e\right)$$

where K is the dielectric constant, η is the magnetic permeability, and δ_{ij} is the Kronecker delta.

Writing the equation of motion in the X_2-direction (corresponding to y of the paper) gives

$$f_2 - \frac{\partial P}{\partial x_2} = 0 = \frac{\partial T_{2j}}{\partial x_j} - \frac{\partial P}{\partial x_2}$$

The term $\frac{\partial T_{2j}}{\partial x_j}$ reduces to $\frac{\partial T_{22}}{\partial x_2}$ since $\frac{\partial}{\partial x_1} = 0 = \frac{\partial}{\partial x_3}$. For the problem considered in the paper,

$$T_{22} = -\frac{1}{2}KE_3^2 - \frac{1}{2}\eta(H_1^2 - H_2^2)$$

and since E_3 and H_2 are uniform across the interface

$$\frac{\partial T_{22}}{\partial x_2} = -\frac{1}{2}E_3^2\frac{\partial K}{\partial x_2} - \frac{1}{2}\eta\frac{\partial H_1^2}{\partial x_2} - \frac{H_1^2}{2}\frac{\partial \eta}{\partial x_2} + \frac{H_2^2}{2}\frac{\partial \eta}{\partial x_2}$$

Integrating the equation of motion in the solid region and fluid region independently gives

$$(P - T_{22})_f = C_f$$

$$(P - T_{22})_w = C_w$$

where C_f and C_w are constants referred to fluid and wall regions, respectively. The condition of equilibrium at the interface requires that $C_f = C_w$ and thus

$$P_w - P_f = T_{22})_w - T_{22})_f$$

Thus, if K and η are different for fluid and plate, a pressure discontinuity may occur.