shown in Figs. 3 and 4 compare fairly well with the theoretical profile computed by Tollmien. Equations (11) for the profile cannot be correct near the centerline since the velocity profile has a peak there. Alternative assumptions to equations (10) for the Reynolds stresses would have lead to another differential equation. The changes in theoretical velocity profile are probably not detectable in the data presented, thus it is argued that the data do not confirm the theoretical assumptions. The important consequence of the experimental profile measurements is that a nearly self-similar form is achieved at $\eta \approx 1.15$ although analysis suggests that it occurs at much larger values.

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DISCUSSION

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Professor Chanaud has presented some very interesting quantitative data on flow beyond the edge of a rotating disk. It was realized many years ago that flow at the edge of a disk involved large scale entrainment because measurements of disk friction showed a large parasitic drag when compared with Cochran's analysis [2], leading some experimenters toward use of paper-thin disks. The effect of disk thickness on drag was progressive and large. Another indicator of the strong edge flow was discovered by Cobb and Saunders [17];[3] they found that heat transfer coefficients at the edge of an isolated disk were disproportionately large, necessitating use of an edge guard heater.

Rotating disks are rarely isolated in practice, so that most studies of the wake-jet region beyond the disk have concentrated on flows with stationary planes or other rotating disks nearby. An example of such work from Professor Kreith's laboratory was published in 1960 [18], and in a discussion on this paper the writer mentioned briefly some studies of edge flow from isolated rotating disks, but did not give many details.

Experiments were performed by the writer with disks ranging in size from 1.5 to 10 in. in radius, and speeds of 100 to 2700 rpm. Flow visualization with Titanium tetrachloride, and by other means, showed that vortices form at the edges of rotating disks throughout this range, and with disk thicknesses from about 0.014 to 1.10 in. The vortices appeared to be the order of four times as long (in the direction of flow) as they were wide. This confirms Professor Chanaud's conclusion about the nature of the flow at the edge. The "damping" of the space-correlation which he shows in Fig. 7 is a consequence of slight irregularities in the flow which are quite natural with formation of wake vortex structures, and is seen also in wake autocorrelations [19]. The entrainment associated with these vortices was found to be very large; for example, with a disk of radius 4.2 in. rotating at 1810 rpm the entrainment which had occurred by $r = 4.9$ in. had augmented the boundary layer flow at the disk edge by about 15 times. Perhaps Professor Chanaud could reinterpret his data to indicate rate of entrainment as a function of radius. The region of high entrainment appeared to extend radially outwards as much as 30 percent of the disk radius.

The writer also measured yaw angles, and can confirm that the flow angle in the center plane may reverse its decreasing trend in the wakelike region, as shown by Professor Chanaud in Fig. 6, provided that the disk is thin enough. With thicker disks, it was observed that the yaw angles did not have this sigmoid-section.

With this phenomenology in mind, it is easy to see that analysis is very difficult. In the region close to the disk edge itself, the flow is definitely periodic, and a mean-turbulent motion approximation is likely to be as unsatisfactory there as it is in other periodic wake flows; it is much more realistic to make an unsteady analysis for the region, e.g., Gerrard [20]. At disk Reynolds numbers sufficiently low to give laminar separation we may even find Mrs. Bloor's [21] instability waves in the separated shear layers, with their known sensitivity to acoustic disturbance and effect on vortex shedding [22, 23]. From this region of definite periodicity one moves radially outward to a region where the periodicity is less dominant, and this region should perhaps be considered as the starting point for evaluation of a mean-turbulent analysis such as sketched by Professor Chanaud. Even in this region we may find a tendency to a whirl instability; there is a negative radial gradient of the mean centrifugal stress. In the face of all these difficulties, it is interesting that the theory compares with the data as well as it does.

Let us hope that further experimental work on this problem will expand the range of disk Reynolds numbers and thickness ratios.

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