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MAPPING RUNOFF BY THE GRID SQUARE TECHNIQUE

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Runoff is a difficult variable to map because the information from river gauging stations only indicates average runoff from the catchment area above the gauging point. A method of mapping runoff by grid-based extrapolation of two components of the water balance equation (evapotranspiration and precipitation) and application of a runoff model in each grid square, is suggested as a means of overcoming many of the problems related to runoff mapping.

Although a knowledge of the spatial distribution of runoff is essential to plans for estimating and developing the water resources of an area, this variable is particularly difficult to map. The main problems result from the fact that discharge measurements at a gauging station are an integral measure of runoff over the catchment area and are not point estimates. Consequently it is difficult to construct an isoline map of runoff around the available measurement sites.

The problems of mapping runoff vary according to the scale considered. For example, on the macro scale, catchment areas become small in comparison with the total area to be mapped, and can, therefore, be considered as point estimates. In these circumstances, isolines can be easily interpolated around the point data, and in this way Woodruff & Hewlett (1970) were able to present a hydrologic response pattern for the Eastern United States. Another method for mapping runoff at this scale, which is particularly suitable for use in areas of limited stream flow information, is to use a water balance approach.

This can be achieved by a comparison of the rainfall and evapotranspiration distributions for the area (Garnier 1960) or by using analytical methods to predict runoff from water balance information. Liebscher (1971) used the latter approach to produce a runoff map of West Germany on the basis of precipitation and air temperature data.

On the micro scale, and specifically within individual catchments, it is

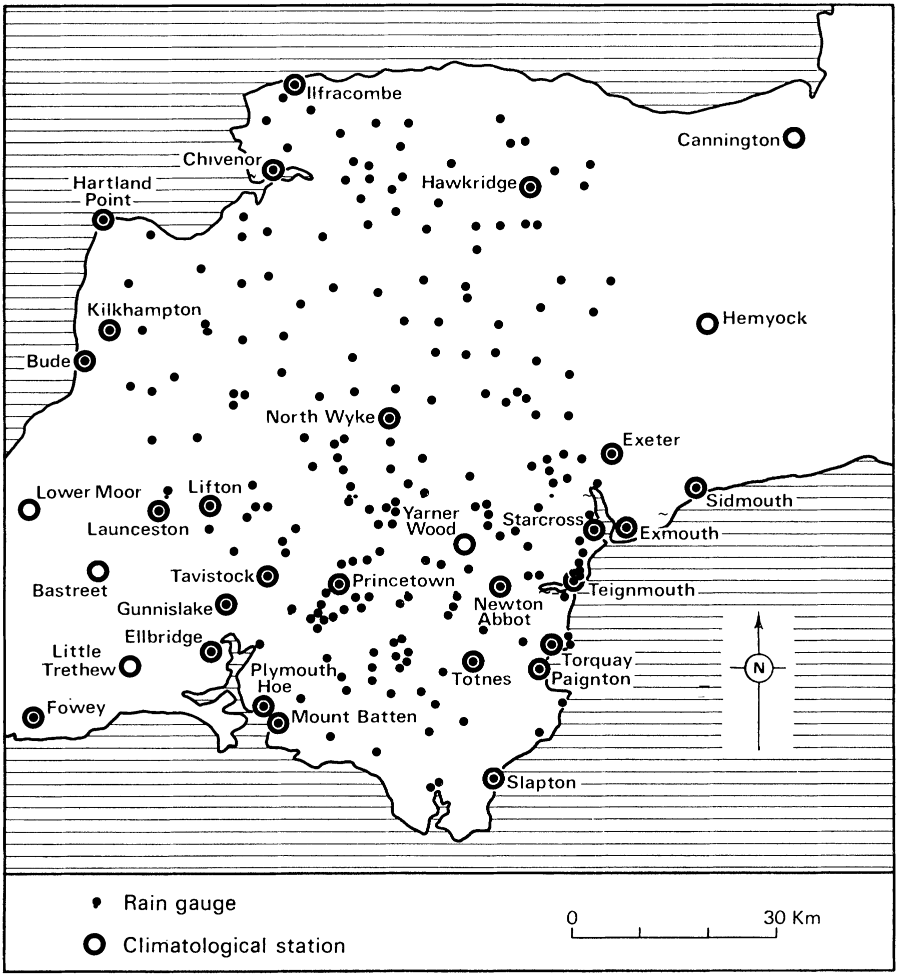


Fig. 1.

Network of climatological stations and rain gauges from which data were obtained for this study.

possible to use specially prepared field information and laboratory analysis. For example, Goczan (1972) suggests the comparison of detailed slope maps and maps of the hydrological characteristics of the soil as a means of extrapolating and mapping runoff information.

However, the present paper is concerned with the meso scale, or a scale at which assumptions of point discharge are invalid but detailed field work is impossible. Attempts at mapping runoff at this scale have involved either the interpolation of generalised isolines on the basis of stream flow records or the application of a water balance approach. McMahon (1969) mapped runoff for the Hunter Valley, constructing isolines subjectively "in the same way that isohyets are interpreted" on the basis of catchment rainfall and stream flow data. Pentland (1968) adopted a similar method to map average runoff in the Great Lakes Basin. He calculated average unit runoff for each gauging station, plotted the results at the centroid of the catchment area, and then constructed isolines around the results. Nevertheless, for a more detailed result, a water balance approach seems to be the most suitable technique. The Devon River Authority (Hall 1967) have used this approach by mapping precipitation and losses on an average annual basis, and then subtracting the results to obtain a runoff map. The subjectivity of this type of method was overcome by Solomon et al. (1968), who extrapolated precipitation and evapotranspiration over a grid. They were then able to apply the water balance equation within each grid square to evaluate and map the distribution of mean annual runoff in Newfoundland.

A method of utilising grid-based extrapolation to map meteorological variables, particularly evapotranspiration, has been described elsewhere (Foyster 1973). The grid base was chosen in order to facilitate computer analysis, storage and retrieval of information, and to allow rapid compilation and mapping of the results. The present paper investigates an extension of this work to runoff mapping by application of the water balance equation to data from each grid square.

MAPPING EVAPOTRANSPIRATION

A part of Southwest England was used as a study area for developing the mapping technique (see Fig. 1). A 65 × 65, 2 km intersection grid was superimposed over the area, and using information from 1:63,360 Ordnance Survey topographic maps, estimates of mean slope, aspect, altitude, location and distance to the sea were calculated and recorded for each grid square. This physiographic information was then used as a basis for extrapolating the mean monthly values (1960 to 1969 inclusive) of precipitation and all the constituent variables of Penman's (1962) potential evapotranspiration equation. In this way, mean monthly estimates of precipitation and potential evapotranspiration were calculated for each grid square. It was then possible to apply a water budgeting technique, described by Penman (1950) and discussed in more detail by Grindley (1967). The method calculates changes in soil moisture content on the basis of variations in effective precipitation, and then adjusts potential values to actual evapotranspiration using assumed root constants. This budgeting technique can be extended to calculate the amount of water available for runoff in any particular month after losses by evapotranspiration and replenishment of any soil moisture deficit have been satisfied. Average annual and monthly maps of both potential and actual evapotranspiration can be constructed by accumulating the monthly values for each grid square, plotting the results and constructing isolines. A more detailed account of this procedure for mapping evapotranspiration is given by Foyster (1973).

MAPPING MEAN ANNUAL RUNOFF

The water budgeting technique keeps account of consumptive use of soil water, replenishment of soil moisture deficits and residual water available for runoff. In this way, mean monthly values of potential runoff, or surplus water available for runoff, were accumulated on a grid basis, and these values provided a means of mapping runoff.

The main problem associated with transforming potential runoff information into an actual runoff map, was that of redistributing the potential values in time to take account of delays in movement of water across the area. In other words, although a particular grid square may have a water surplus in a given month, this does not mean that the surplus will actually contribute to river discharge during that period. As a result, a mean monthly potential runoff map does not constitute a map of actual runoff during that month. It will only indicate areas with a water surplus; this surplus might contribute to the recharge of soil moisture deficits in surrounding areas or might pass into temporary storage, rather than directly forming river flow.

The problems of the effects of variable storage complicated the production of monthly runoff maps. However, by assuming that storage was similar at the beginning and end of an average calendar year, it was possible to accumulate the monthly potential runoff values to produce an average annual runoff map (Fig. 2). The storage assumption seems valid in the context of the

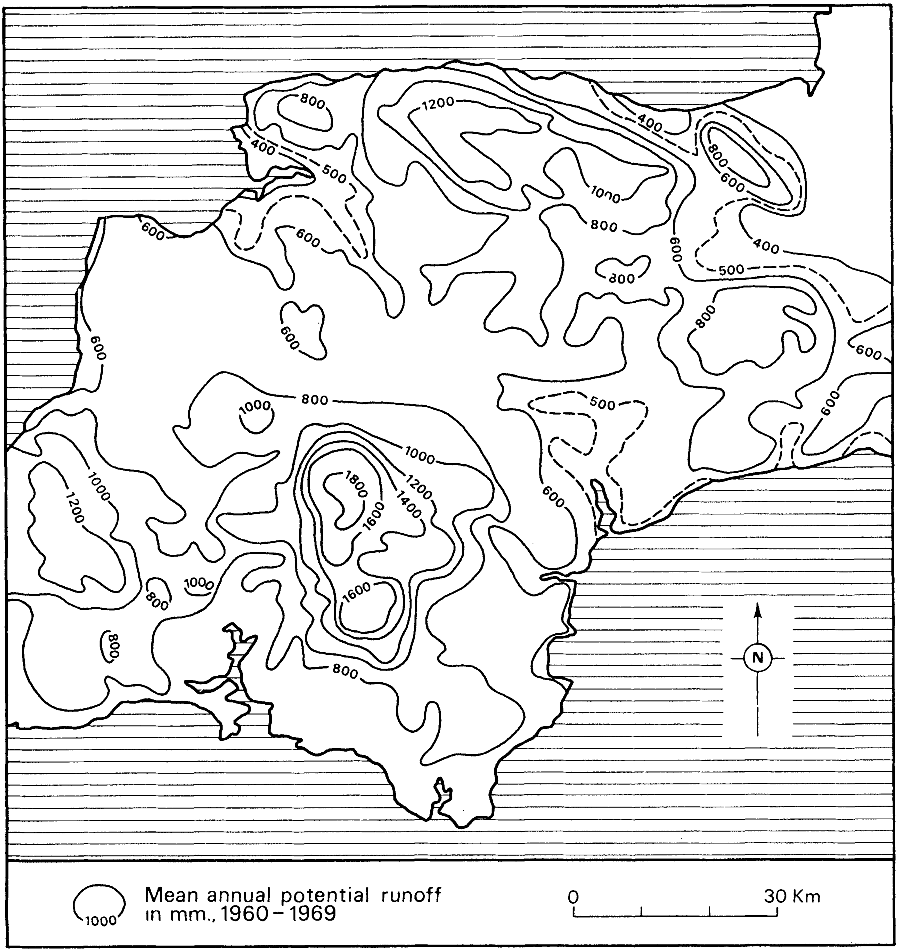


Fig. 2.

Mean annual (potential) runoff in a part of Southwest England, 1960–1969.

study area, because of the limited ground water contribution to river flow and because soil moisture can generally be assumed to return to field capacity each winter.

The accuracy of the runoff map was tested using discharge records for five gauging stations (Fig. 3); Thorverton on the Exe, Preston on the Teign, Dotton on the Otter, Newbridge on the Torridge and Austin's Bridge on the Dart. The catchment areas of these stations were defined in terms of the grid

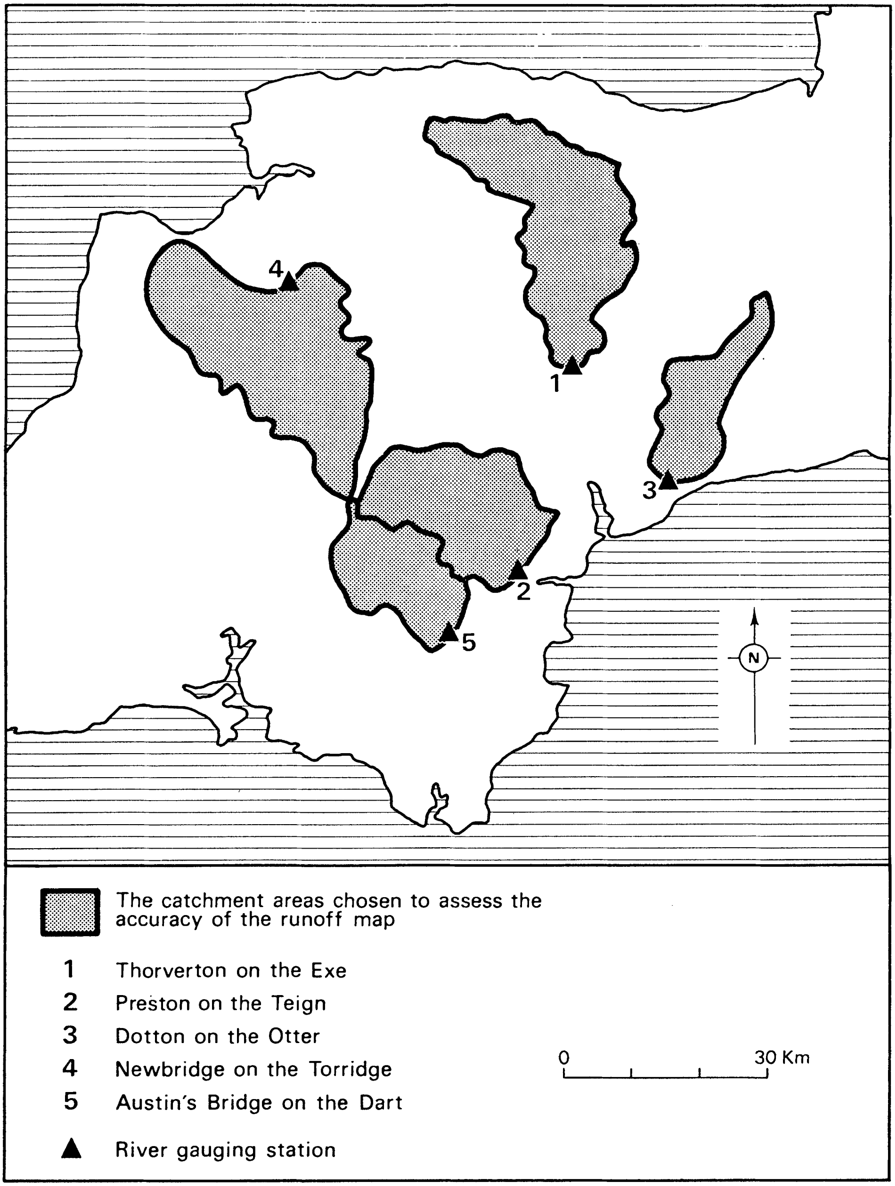


Fig. 3.

Gauging stations and catchment areas used to assess the accuracy of Fig. 2.

Table 1.
The values of runoff derived from the grid square technique and from gauging station measurements, for the same catchment areas, for the period 1960–1969.

Catchment	2km Grid Result	Measured Discharge
The Exe at Thorverton	15.92	15.99
The Teign at Preston	11.71	10.06
The Otter at Dotton	3.75	3.34
The Torridge at Newbridge	15.78	14.87
The Dart at Austins Bridge	10.85	11.69

The values are average discharge in m^3/s for the period 1960–1969 inclusive.

network, and grid square annual potential runoff estimates were then accumulated within each catchment and converted into a value representing average annual discharge at the gauging station. Table 1 compares the observed and calculated average discharges for the five catchments. The similarity between these figures suggests that the map is a worthwhile approximation to the true runoff pattern.

THE EXE WATER BALANCE

In an attempt to investigate changes in the runoff pattern during an average year and also the influence of grid scale on map accuracy, a detailed study of the Exe was undertaken. The grid square technique was repeated for this catchment using a 1 km intersection grid base. Although sufficient precipitation data were available for the Exe basin above the Thorverton gauging station, there was only one climatological station. As a result, regional relationships were used to estimate evapotranspiration, but precipitation values were calculated from local information. Fig. 4 shows the results obtained from using the two grid scales on the Exe catchment. In spite of the increase in isoline crenulations, the finer grid mesh gave virtually the same estimate of average annual discharge as the coarser mesh; 15.70 m^3/s and 15.92 m^3/s respectively, in comparison with the observed value of 15.99 m^3/s. Any difference between these values must largely be attributed to difficulties in accurately defining the watershed in terms of a square grid.

Although the map of mean annual runoff (Fig. 3) seemed to be accurate, a method was needed to give some indication of monthly fluctuations. The generation of a synthetic mean annual hydrograph on the basis of the total monthly potential runoff figures, by employing a runoff model, provided a means of looking at total monthly fluctuations in the Exe catchment.

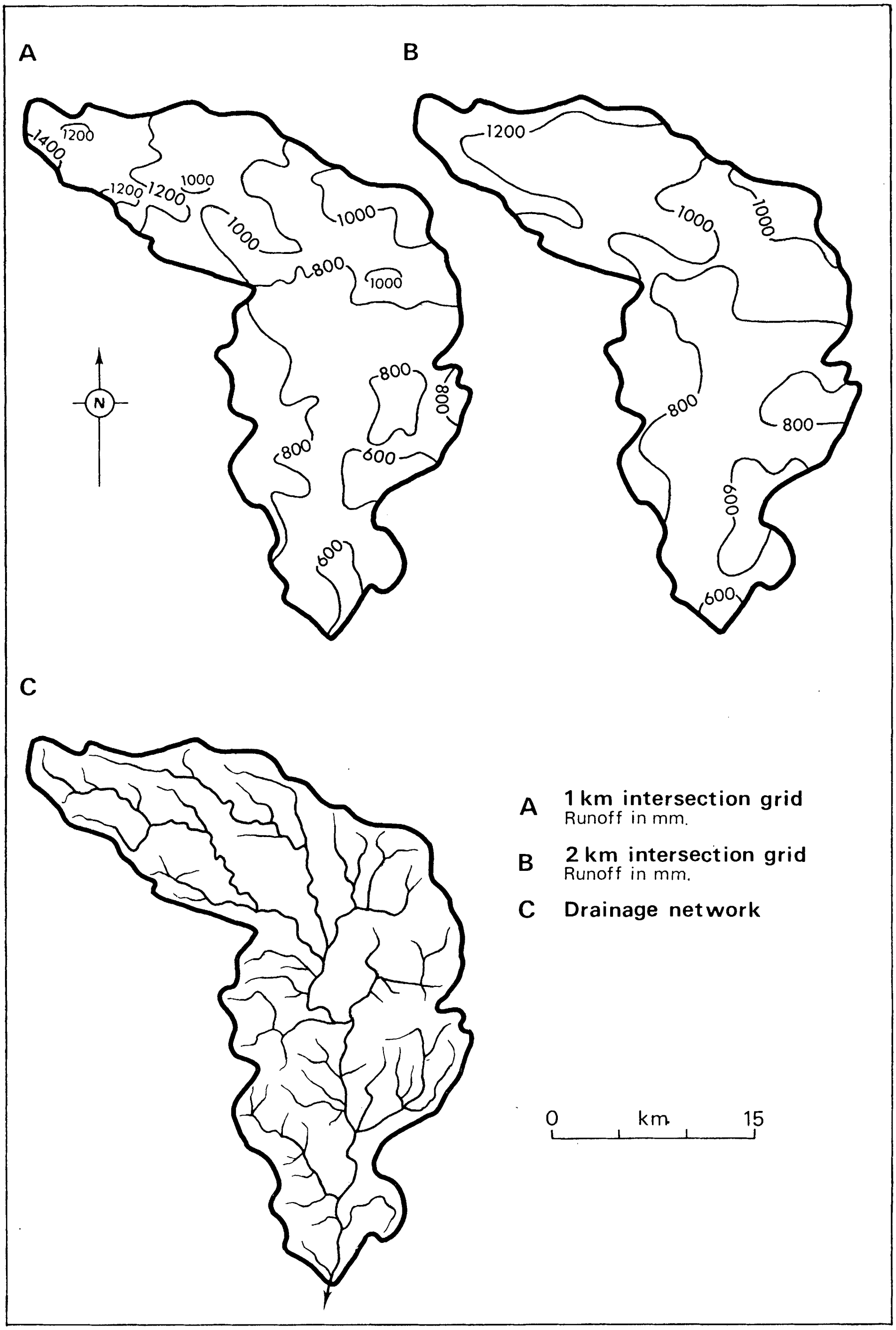
There is a wide range of models available for simulating discharge from a catchment, ranging from the many versions of the complex Stanford Watershed Model (e.g. Crawford & Linsley 1963, 1966) to the simple Thornthwaite (1948) climatic water balance approach. Although the former model could have been used in the present context, its application would have been very expensive and time consuming. However, experimental work with the much simpler climatic water balance technique described by Ward (1972), seemed to give very promising results, and showed that the method could be easily combined with the soil moisture budgeting technique already used in the present work, to calculate potential runoff figures. The model assumes that a fixed percentage of the water available for runoff in any time period (in the present context, 1 month) will provide discharge from the catchment, the remainder being held over to the following month, when the same percentage of the water in total temporary storage (i.e. the remainder of previous month's potential runoff plus that from the present month) will provide actual runoff. In this way, the discharge of potential runoff is delayed in its removal from the catchment. The model has the advantage of being simple to apply, but it does not take into account the effects of variable storage on outflow, assuming that a fixed, rather than a variable, proportion of the total surplus water stored will be discharged, regardless of the area of the catchment contributing to that temporary storage. The percentage value applied is dependent upon the response characteristics of the catchment and the time lags being studied, but a value of 41 % was found to be suitable for the Exe, using the method described in the appendix.

In spite of the inherent problems of the model, the results for the Exe catchment (Fig. 5) show that a fairly good approximation to the true hydrograph can be obtained from the mean monthly potential runoff figures. The model also provides a means of constructing monthly runoff maps.

The fine drainage network in the Exe catchment (Fig. 4) allows every 1 km grid square to either contain or be very close to part of the permanent drainage network. This suggests that much of the delay between the occurrence of potential runoff and the discharge of the water from the catchment must

Fig. 4.

Mean annual (potential) runoff maps, for the Exe catchment above the Thorverton gauging station, based on estimates for (A) a 1 km and (B) a 2 km intersection grid. (C) The drainage network for discharging the water surplus.


A
B
C
1400
1200
1000
800
600
N
A 1 km intersection grid
Runoff in mm.
B 2 km intersection grid
Runoff in mm.
C Drainage network
0 km 15

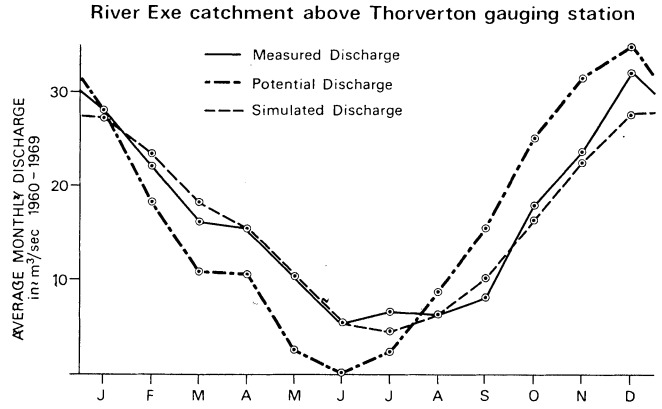


Fig. 5.

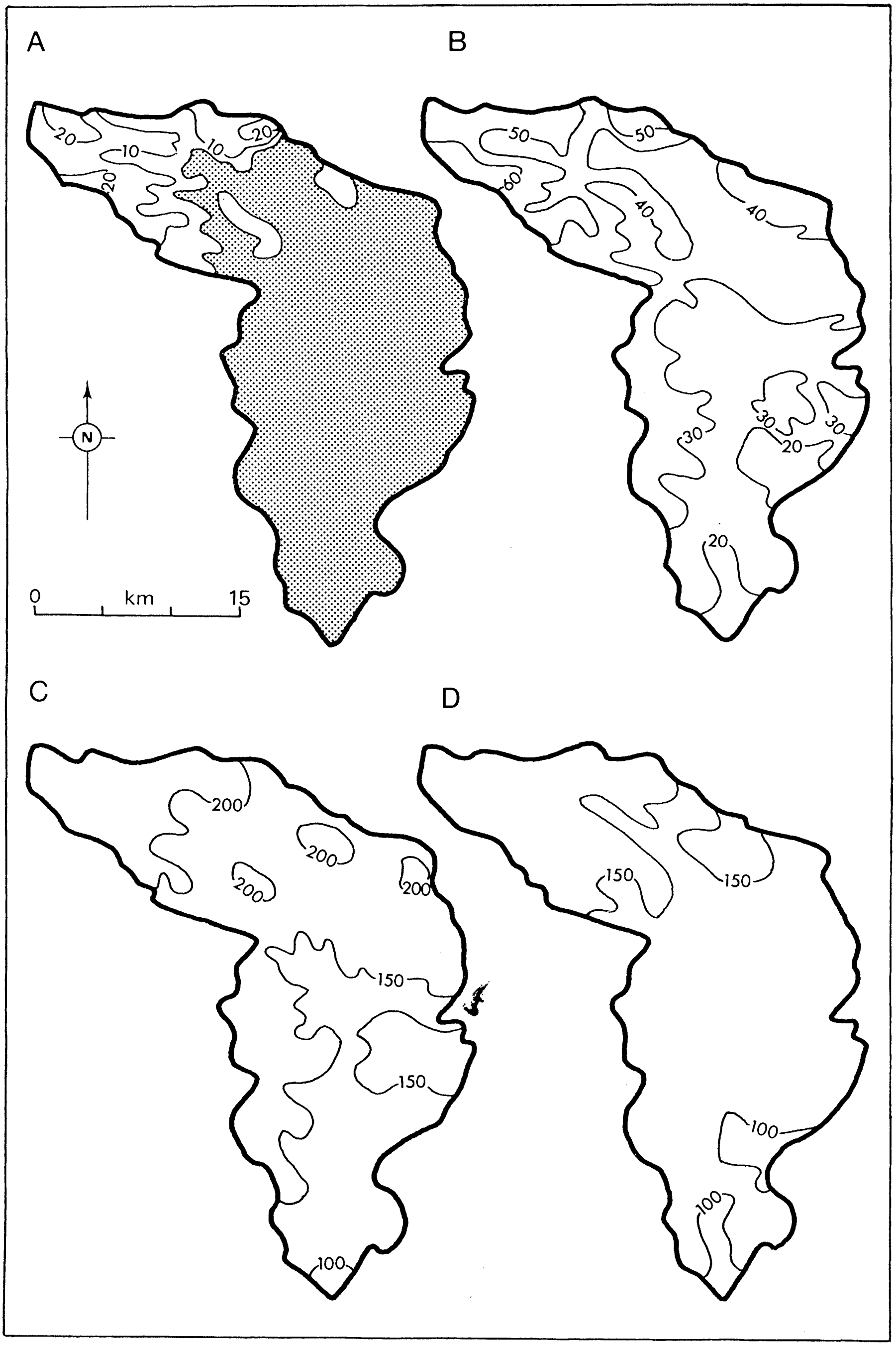
Mean monthly measured discharge for the Exe, 1960–1969 compared with mean monthly potential discharge and simulated discharge using the Thornthwaite climatic water balance model.

occur very near the source of the surplus water, because once the water reaches a permanent drainage channel it will quickly flow out of the catchment. Since the climatic water balance model provides a means of simulating mean monthly discharge from the catchment, and since the major cause of differences between potential and actual runoff is related to temporary storage of surplus water near its source, the Thornthwaite model and derived storage constant were applied to potential runoff data for individual grid squares to produce maps of actual runoff.

The mean monthly runoff estimates for a particular square were a measure of the amount of surplus water that would reach the drainage network from that square in a given month. Application of the same storage constant to every part of the catchment must inevitably give generalised results, but the runoff maps produced give a more realistic picture than the potential runoff maps for the same months. Fig. 5 compares maps of potential and actual runoff in millimetres for both June and December. This method of mapping runoff is very simple and it seems to give worthwhile results for the Exe catchment. However, its universal suitability has yet to be demonstrated.

Fig. 6.

Mean monthly runoff maps, 1960–1969, in mm, for the Exe catchment above Thorverton gauging station. (A) June potential runoff (shaded area has no water surplus). (B) June actual runoff. (C) December potential runoff. (D) December actual runoff.


A
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C
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200
200
200
150
150
100
D
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100

CONCLUSIONS AND POSSIBILITIES FOR FUTURE RESEARCH

Application of the grid square technique to automated, objective mapping of hydrological variables, particularly evapotranspiration and runoff, has been described. The accuracy of the results are difficult to assess, but by investigation of individual catchments, the method seems to produce a map of mean annual runoff for part of Southwest England, 1960–1969, which gives good approximations to observed average discharge in five sample catchments. A synthetic hydrograph for the Exe was also successfully generated by applying Thornthwaite's climatic water balance technique to mean monthly potential runoff figures and this has provided a means of mapping monthly catchment runoff. However, the study area is not complicated by marked ground water storage, and so the next stage in the development of this technique must, inevitably, concentrate on the storage term in the water balance equation.

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APPENDIX

Storage is split into long term soil moisture storage within the catchment and temporary storage which is surplus water in the process of being evacuated from the catchment. Water is input each month by precipitation and is lost by evapotranspiration. The difference between these two terms is a measure of the amount by which precipitation fails to meet or exceeds the water needs of the vegetation. If precipitation falls short of the evapotranspiration rate in any month then the long term storage is depleted. Once a deficit has been created, precipitation excess in any future months must first make up this deficit, before any water can pass into temporary storage. However, once water has passed into temporary storage, it is available to feed the river until the supply is exhausted.

According to Thornthwaite's model, a fixed percentage of the total surplus water available in temporary storage will actually reach the gauging station within that month. Thornthwaite suggested a figure for discharge as 60 % of the total water held in temporary storage. However, the appropriate percentage (x) must inevitably depend upon the rate of catchment response and the time periods being studied. Once the amount of water contributing to temporary storage has been calculated for each month, it is a relatively simple process to use these data in conjunction with the observed discharge data, to calculate a relevant value for x. Let initial temporary storage be S_o, then by using the value of potential discharge, PQ_n, for month n, and Q_n, the observed discharge for month n it is possible to calculate x in the following manner:–

Month 1

$$\begin{aligned} Q_1 &= x\,(PQ_1 + S_o) \\ S_1 &= (1-x)\,(PQ_1 + S_o) \end{aligned}$$

i.e.

$$Q_1 + S_1 = PQ_1 + S_o$$

Storage at the end of month 1, plus discharge during month 1 must, by continuity, be equal to the sum of storage at the beginning of the month and the effective precipitation during the month.

Similarly for Month 2

$$\begin{aligned} Q_2 &= x\,(PQ_2 + S_1) \\ S_2 &= (1-x)\,(PQ_2 + S_1) \\ &= (1-x)\,PQ_2 + (1-x)^2\,PQ_1 + (1-x)^2\,S_o \\ Q_2 &= x\,(PQ_2 + (1-x)\,PQ_1 + (1-x)\,S_o) \end{aligned}$$

Similarly, for Month n

$$\begin{aligned} S_n &= (1-x)\,PQ_n + (1-x)^2\,PQ_{n-1} + \ldots\ldots \\ &\quad + (1-x)^n\,PQ_1 + (1-x)^n\,S_o \\ Q_n &= x\,(PQ_n + S_{n-1}) \\ &= x\,(PQ_n + (1-x)\,PQ_{n-1} + (1-x)^2\,PQ_{n-2} + \ldots\ldots \\ &\quad + (1-x)^{n-1}\,PQ_1 + (1-x)^{n-1}\,S_o) \end{aligned}$$

Hence

$$\begin{aligned} \frac{Q_{n+1}}{Q_n} &= \frac{x\,(PQ_{n+1} + (1-x)PQ_n + \ldots\ldots + (1-x)^nPQ_1 + (1-x)^n\,S_o)}{x\,(\qquad PQ_n + \ldots\ldots + (1-x)^{n-1}PQ_1 + (1-x)^{n-1}S_o)} \\ &= \frac{x\,PQ_{n+1}}{Q_n} + (1-x) \end{aligned}$$

$$x = \frac{\dfrac{Q_{n+1}}{Q_n} - 1}{\dfrac{PQ_{n+1}}{Q_n} - 1}$$

Thus, x can be estimated from the observed runoff for two consecutive months, and the effective precipitation for the second month. By using the information for every pair of consecutive months to estimate x, twelve possible values of x were obtained, which were averaged to give the best estimate of x.

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