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Stochastic Simulation of Evaporation Data for Australia

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Stochastic models are developed to simulate annual, monthly and daily evaporation data for Australia. For annual and monthly synthesis, the evaporation values are generated independently and dependently of rainfall; on the other hand, generation of daily evaporation is based on the knowledge of occurrence of daily rainfall. Data from nine Australian stations are used to illustrate applications of the models.

Introduction

One major use of evaporation data in conjunction with rainfall data is in computer simulation modelling of rainfall/runoff processes. Hydrologic models like those of Boughton (1966), Crawford and Linsley (1966), or the Australian Representative Basins Model (Chapman 1968), require evaporation data along with rainfall as input. Crop growth models like those of Ritchie (1981) and Saxton et al. (1982) require, in addition to rainfall, net radiation or evaporation as a measure of energy input. In irrigation simulation studies, both rainfall and evaporation are also required.

A special characteristic that must be preserved in stochastic modelling with more than one variable is the cross correlation between variables. This study deals essentially with developing stochastic generation procedures for evaporation at yearly, monthly and daily intervals, in both a multi-variable system and as an independent data set. In the multi-variable system, rainfall is considered to be the second variable. The methods developed are applied to nine Australian locations.

Literature Review

Using annual rainfall and evaporation, Hoy (1977) demonstrated the existence of an inverse relationship between annual totals of pan evaporation and rainfall over most of the interior and north of Australia. He also observed that the correlation was most pronounced in the interior of the continent and decreased towards the coastline, where for most sites it was of little practical significance.

Singh (1979) found that the following models were satisfactory in correlating monthly evaporation ($E_{i,j}$) with flow ($Q_{i,j}$) or rainfall ($R_{i,j}$)

$$E_{i,j} = a_i + b_i \log Q_{i,j} + SE_i \, t_{i,j} \quad (1)$$

$$E_{i,j} = a_i + b_i R_{i,j} + SE_i \, t_{i,j} \quad (2)$$

i = months 1,2,7,8 9,10,11 and 12 (for (1) and (2))

$$E_{i,j} = a_i + SE_i \, t_{i,j} \qquad i = \text{months } 3,4,5 \text{ and } 6 \quad (3)$$

in which

SE – standard error or estimate obtained by regression of monthly evaporation on monthly flow or rainfall,

t – random normal deviate, and

i,j – month and year respectively.

Even though this procedure takes into account the correlation between evaporation and rainfall, the serial correlation between evaporation values is ignored. Also, the skewness of monthly evaporation is not taken into account. If monthly rainfall is normally distributed, then the generated evaporation will be also normally distributed. If monthly rainfall is skewed, then the generated evaporation will have an implied skew which is a function of cross correlation and the skewness of rainfall. In addition, when the monthly values are aggregated, the annual parameters, in general, will not be preserved.

Jones et al. (1972) hypothesized that daily evaporation could be obtained from the time of the year, and the occurrence of rainfall on both the day in question and the preceding day. Evaporation was assumed to be normally distributed and the parameters were estimated for each week of a year from 17 years of data. Polynomial functions were fitted to the parameters to predict evaporation parameters. Daily evaporation was simulated by Monte Carlo type sampling from a normal distribution with evaporation parameters chosen according to the state of the present and preceding days. The main drawback with this procedure is that the skewness and autocorrelation of daily evaporation values are ignored.

Nicks and Harp (1980) generated daily temperature and solar radiation data using a lag one Markov model dependent on the state of present and preceding days. A normal distribution was assumed for both the temperature and solar radiation. This model could be used to generate daily evaporation with necessary modifications to account for the skewness.

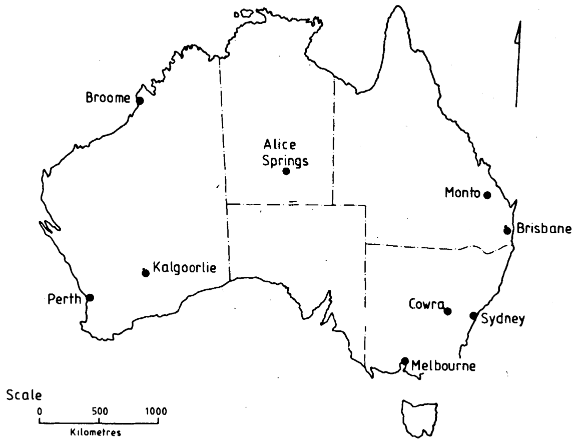

Fig. 1. Location of evaporation stations used in application of models.

Evaporation Data

Evaporation data were obtained from the Bureau of Meteorology, Melbourne, for nine stations listed in Table 1 (Fig. 1). These data correspond to a Class A pan fitted with a bird screen which affects the natural mechanism of evaporation from the pan. The effect varies not only from place to place, but also with seasons. Van Dijk (1982) observed that the reduction in monthly evaporation totals due to the

Table 1 – Evaporation data as measured in Class A pan with bird screen

		Length of Data		Annual Parameters					
Station	AWRC Ref No	Annual & monthly analysis	Daily analysis	Mean (mm)	Coeff. of variation	Coeff. of skewness	Lag one autocor-relation	Maxi-mum+	Mini-mum+
Melbourne	86071	11	10	1397	0.068	1.216*	0.471*	1.161	0.898
Sydney	66037	9	8	1820	0.064	0.668	0.258	1.122	0.913
Monto	39104	12	9	1617	0.059	0.590	0.096	1.116	0.917
Cowra	63023	9	8	1427	0.147	−0.484	0.131	1.169	0.766
Brisbane	40214	8	7	1431	0.032	1.180	−0.326	1.06	0.971
Broome	03003	14	13	2838	0.070	0.196	−0.495*	1.125	0.885
Perth	09034	14	13	1813	0.064	0.659	0.311*	1.133	0.909
Alice Springs	15590	16	–	2944	0.155	−0.151	0.621*	1.220	0.749
Kalgoorlie	12038	14	13	2701	0.082	−0.563	0.430*	1.130	0.838

+ Expressed as a ratio of mean annual evaporation.

* Significantly different from zero at 5 % level.

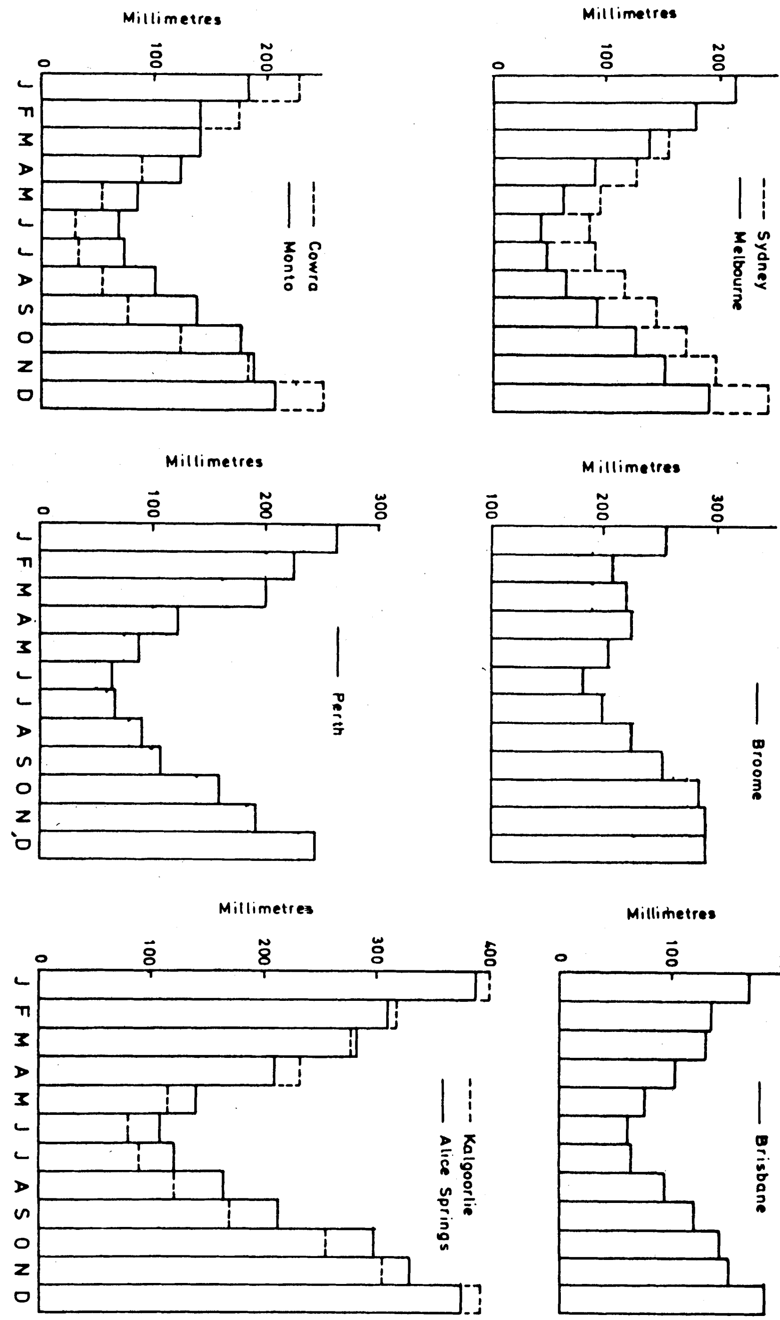

Fig. 2. Mean monthly evaporation from Class A Pan with bird screen.

standard Australian bird screen ranged from 4 to 8 per cent of four test sites. He suggested an adjustment of +7 per cent should be made to monthly evaporation totals measured from screened pans. Even though about 30 years of data is generally considered necessary to adequately estimate population parameters, because of the small variability of evaporation data, 10-15 years is considered sufficient to estimate model parameters.

Mean monthly evaporation at each station is shown in Fig. 2. All the stations show similar seasonal patterns, but with different relative magnitudes.

Annual Evaporation Models

Independent of Rainfall

Because variability, skewness and lag one autocorrelation coefficient of annual evaporation values are low, a first order Markov model with the Wilson-Hilferty transformation (1931) is adequate for data generation. The model equations are set out below. For several stations, a white noise model appears to be sufficient because of their very small skewness and low autocorrelation.

Model 1: First order Markov model

$$E_t = \bar{E} + r_1(E_{t-1} - \bar{E}) + S(1-r_1^2)^{\frac{1}{2}}\varepsilon_t \tag{4}$$

$$\varepsilon_t = \frac{2\left(\left(1 + \frac{g\eta_t}{6} - \frac{g^2}{36}\right)^3 - 1\right)}{g} \tag{5}$$

$$g = \frac{(1 - r_1^3)\gamma}{(1 - r_1^2)^{1.5}} \tag{6}$$

where

- E_t, E_{t-1} – annual evaporation corresponding to years t and t-1,
- $\bar{E}$ – mean annual evaporation,
- S – standard deviation of annual evaporation,
- r_1 – lag one autocorrelation coefficient,
- ε_t – random number with mean zero, variance one and skewness g,
- η_t – normally distributed random number with mean zero and variance one,
- γ – skewness of annual evaporation, and
- g – skewness of random numbers, ε.

For the white noise model, $r_1 = 0$ and ε_t is replaced by η_t in Eq. (2), resulting in

$$E_t = \bar{E} + S\eta_t \tag{7}$$

where E_t, $\bar{E}$, S and η_t are defined above.

Dependent on Rainfall

For situations where evaporation is correlated with rainfall, the latter is first generated by using a first order Markov model or White Noise model. Evaporation is then obtained from generated rainfall, using a linear regression of the form

$$E_t = a + bR_t + SE\eta_t \tag{8}$$

where

R_t – annual rainfall in year t,

a,b,SE – regression coefficients and standard error of estimate obtained by regression of annual evaporation with annual rainfall, and

E_t,η_t – as defined in model 1.

If b is small and not significantly different from zero, then this model degenerates into a white noise model.

The evaporation data generated using Eq. (8) will have an implied skewness and autocorrelation. In terms of the cross correlation (r) between evaporation and rainfall, skewness (γ_R) and lag one autocorrelation coefficient (r_R) of annual rainfall, the implied skewness (γ) and lag one autorcorrelation (r_1) of evaporation can be written as follows

$$r_1 = r^2 r_R \tag{9}$$

$$\gamma = r^3 \gamma_r \tag{10}$$

Because the lag one autocorrelation coefficient of annual rainfall is usually small (generally the maximum value in Australia is about 0.1), r_1 also will be small. But the skewness of annual rainfall may be large and, with high cross correlations, this will result in significant implied skewness.

If one wishes to preserve both the skewness and autocorrelation in the generated evaporation data that were observed in the historical data, the following model should be used.

Model 2: Multisite type model

A multisite type model is of the form

$$[X_t] = [A][X_{t-1}] + [B][\varepsilon_t] \tag{11}$$

where

$[X_t],[X_{t-1}]$ – (2×1) vector of standardized annual rainfall and evaporation at time t, (t-1),

$[\varepsilon_t]$ – (2×1) vector of random deviates with zero mean and unit variance, and

$[A],[B]$ – (2×2) matrices of constant coefficients to preserve the cross correlations in $[X_t]$.

The annual evaporation and rainfall are standardized using

$$X_t^{(i)} = \frac{x_t^{(i)} - \bar{x}_i}{s_i} \tag{12}$$

where

$x_t^{(i)}$ – observed rainfall (i=1) or evaporation (i=2) in time t,
$X_t^{(i)}$ – corresponding standardized value, and
$\bar{x}_i, s_i$ – mean and standard deviation of $x_t^{(i)}$ respectively.

Matrices $[A]$ and $[B]$ are obtained from

$$[A] = [M_1][M_0]^{-1} \tag{13}$$

$$[B][B]^T = [M_0] - [M_1][M_0]^{-1}[M_1]^T \tag{14}$$

where $[M_0]$ and $[M_1]$ are, respectively, the lag zero and lag one cross correlation matrices.

The elements of $[M_0]$ and $[M_1]$ are estimated from

$$r_0^{ij} = \frac{1}{n} \sum_{t=1}^{n} X_t^{(i)} X_t^{(j)} \tag{15}$$

$$r_1^{ij} = \frac{1}{n} \sum_{t=2}^{n} X_t^{(i)} X_{t-1}^{(j)} \tag{16}$$

The solution of $[A]$ from Eq. (13) is straightforward, but no unique solution exists for $[B]$. Since $[B][B]^T$ is a symmetric matrix, for convenience, $[B]$ may be assumed lower triangular. Assuming $[B][B]^T = [C]$, the elements of $[B]$ may be obtained from the following equations (Young and Pisano 1968)

$$b_{11} = (c_{11})^{\frac{1}{2}} \tag{17}$$

$$b_{12} = 0 \tag{18}$$

$$b_{21} = \frac{c_{21}}{b_{11}} \tag{19}$$

$$b_{22} = (c_{22} - b_{21}^2)^{\frac{1}{2}} \tag{20}$$

Once the matrices [A] and [B] have been determined, the standardized rainfall and evaporation may be obtained from

$$X_t^{(1)} \equiv a_{11} X_{t-1}^{(1)} + a_{12} X_{t-1}^{(2)} + b_{11} \varepsilon_t^{(1)} \quad (21)$$

$$X_t^{(2)} = a_{21} X_{t-1}^{(1)} + a_{22} X_{t-1}^{(2)} + b_{21} \varepsilon_t^{(1)} + b_{22} \varepsilon_t^{(2)} \quad (22)$$

Finally, the actual rainfall (i=1) and evaporation (i=2) are obtained from

$$x_t^{(i)} = \bar{x}_i + s_i X_t^{(i)} \quad (23)$$

The above procedure results in normally distributed rainfall and evaporation values, and may be applied to most cases. If the annual evaporation or rainfall or both are skewed, they must be initially normalized and generated in the transformed domain using the above procedure. The actual values are then obtained by using the inverse transformation.

The data can be normalized by using either a logarithmic transformation or Box-Cox transformation (1964). The logarithmic transformation is preferred because the parameters in the log domain can be estimated through moment transformation equations and this ensures that the parameters are theoretically preserved in the actual domain, as well as in the transformed domain.

For a three parameter log-normal distribution,

$$y_t^{(i)} = \log(x_t^{(i)} - a_i) \quad (24)$$

where

x_t^i – actual value,
a_i – location parameter, and
y_t^i – transformed value.

The moment transformation equations are given by

$$\bar{x}_i = a_i + \exp(\bar{y} + 0.5S_i^2) \quad (25)$$

$$s_i^2 = \exp[2(\bar{y}_i + S_i^2)] - \exp[2\bar{y}_i + S_i^2] \quad (26)$$

$$g_i = \frac{\exp(3S_i^2) - 3\exp(S_i^2) + 2}{\{\exp(S_i) - 1\}^{\frac{3}{2}}} \quad (27)$$

$$r_0^{ij} = \frac{\exp(S_i S_j R_0^{ij}) - 1}{\{\exp(S_i^2)-1\}^{\frac{1}{2}}\{\exp(S_j^2)-1\}^{\frac{1}{2}}} \tag{28}$$

$$r_1^{ij} = \frac{\exp(S_i S_j R_1^{ij}) - 1}{\{\exp(S_i^2)-1\}^{\frac{1}{2}}\{\exp(S_j^2)-1\}^{\frac{1}{2}}} \tag{29}$$

where $\bar{x}_i$, s_i, g_i, r_0^{ij} and r_1^{ij} – mean, standard deviation, coefficient of skewness and lag zero and lag one cross correlations respectively in the actual domain, and $\bar{y}_i$, S_i, a_i, R_0^{ij} and R_1^{ij} – mean, standard deviation, location parameter and lag zero and lag one cross correlation coefficients respectively in the log domain corresponding to variable i.

In the case of a two parameter log normal distribution, a_i in Eqs. (24) and (25) is zero, and Eq. (27) is omitted.

The data generation takes place in the log domain, and the generated values in the log domain are obtained from Eqs. (21) and (22) by replacing X by Y. The actual values are then obtained from

$$y_t^{(i)} = \bar{y} + S_i Y_t^{(i)} \tag{30}$$

$$x_t^{(i)} = a_i + \exp(y_t^{(i)}) \tag{31}$$

The above procedure is applied only when both variables (rainfall and evaporation) are skewed. If only one variable (say j) is skewed, then the log transformation is applied only to this variable j, and the parameter in the log domain $\bar{y}_j$, S_j and a_j are calculated from Eqs. (25), (26) and (27). The cross correlations are obtained from (Mejia et al. 1974)

$$R_0^{ij} = \frac{r_0^{ij}\{\exp(S_j^2) - 1\}}{S_j} \tag{32}$$

$$R_1^{ij} = \frac{r_1^{ij}\{\exp(S_j^2) - 1\}}{S_j} \tag{33}$$

where r_0^{ij}, r_1^{ij}, R_0^{ij}, R_1^{ij} and S_j are defined above.

Application of Annual Evaporation Models

Using models 1 and 2, twenty replicates, each of length equal to the historical record, were generated for all the stations. Various parameters were estimated for each replicate and the averaged values were compared with the corresponding

Table 2 – Comparison of historical and generated* annual evaporation parameters

		Mean	Std dev	Coeff of	Lag one	mum$^+$	mum$^+$
Station	Model	(mm)	(mm)	skewness	autocorr	Maxi-	Mini-
	Hist	1397	96	1.21	0.47	1.16	0.90
Melbourne	1	1395	88	0.60	0.23	1.11	0.92
	2	1393	80	0.61	0.19	1.11	0.93
	Hist	1820	116	0.67	0.26	1.12	0.91
Sydney	1	1820	117	0.26	0.10	1.10	0.91
	2	1812	106	0.37	-0.04	1.10	0.93
	Hist	1617	95	0.59	1.10	1.12	0.92
Monto	1	1616	95	0.40	0.00	1.10	0.92
	2	1610	93	0.45	-1.10	1.10	0.91
	Hist	1427	210	-0.48	0.13	1.17	0.77
Cowra	1	1424	219	-0.36	0.04	1.20	0.75
	2	1411	197	0.05	-0.11	1.22	0.81
	Hist	1431	46	1.18	-0.33	1.06	0.97
Brisbane	1	1431	45	0.70	-0.32	1.05	0.96
	2	1431	45	0.74	-0.38	1.06	0.97
	Hist	2838	199	0.20	-0.50	1.03	0.89
Broome	1	2842	205	0.24	-0.42	1.13	0.88
	2	2853	198	0.09	-0.51	1.11	0.88
	Hist	1813	117	0.66	0.31	1.13	0.91
Perth	1	1818	115	0.44	0.16	1.12	0.91
	2	1829	99	0.74	-0.03	1.10	0.92
	Hist	2944	456	-0.15	0.62	1.22	0.75
Alice Springs	1	2965	401	-0.32	0.43	1.23	0.75
	2	2914	436	-0.03	0.45	1.25	0.73
	Hist	2701	223	-0.56	0.43	1.13	0.84
Kalgoorlie	1	2707	207	-0.26	0.25	1.12	0.86
	2	2692	216	-0.05	0.25	1.14	0.87

$^+$ Expressed as a ratio of mean annual evaporation.

* 1 First order Markov model with Wilson-Hilferty transformation.

2 Multisite type model.

Table 3 – Comparison of historical and generated standard deviations, coefficients of skewness and lag one autocorrelation coefficients over full length of generated data

Station	Model	Std dev (mm)	Coeff of skewness	Lag one autocorr
	Hist	96	1.21	0.47
Melbourne	1	95	0.97	0.42
	2	88	1.14	0.36
	Hist	116	0.67	0.26
Sydney	1	117	0.58	0.23
	2	112	0.55	0.19
	Hist	95	0.59	0.10
Monto	1	96	0.59	0.07
	2	97	0.90	0.03
	Hist	210	–0.48	0.13
Cowra	1	208	–0.37	0.14
	2	208	0.08	0.11
	Hist	46	1.18	–0.33
Brisbane	1	46	1.20	–0.31
	2	47	1.04	–0.38
	Hist	199	0.20	–0.50
Broome	1	205	0.24	–0.48
	2	199	0.19	–0.57
	Hist	117	0.66	0.31
Perth	1	122	0.54	0.31
	2	105	0.98	0.08
	Hist	456	–0.15	0.62
Alice Springs	1	459	–0.34	0.62
	2	482	–0.19	0.65
	Hist	456	–0.15	0.62
Kalgoorlie	1	230	–0.57	0.47
	2	227	–0.06	0.42

historical values (Table 2). It was observed from this comparison that the generated standard deviations and skewnesses were considerably smaller than the corresponding historical values. This discrepancy resulted from the small sample sizes of generated data which varied from 8 to 16 years. To illustrate this effect, the generated data were treated as one sample and the various parameters were again estimated and are presented in Table 3 for comparison with historical values.

Table 4 – Comparison of historical and generated cross correlation between annual evaporation and rainfall

Station	Model	Cross correlation
	Hist	–0.12
Melbourne	1	–0.01
	2	–0.15
	Hist	–0.81
Sydney	1	–0.71
	2	–0.81
	Hist	–0.65
Monto	1	–0.58
	2	–0.69
	Hist	–0.93
Cowra	1	–0.89
	2	–0.92
	Hist	–0.27
Brisbane	1	–0.24
	2	–0.32
	Hist	–0.73
Broome	1	–0.56
	2	–0.71
	Hist	–0.09
Perth	1	–0.01
	2	–0.05
	Hist	–0.83
Alice Springs	1	–0.63
	2	–0.86
	Hist	–0.81
Kalgoorlie	1	–0.78
	2	–0.79

Discussion of Annual Results

It can be seen from Table 2 that all the models preserved the means, maxima and minima for all the stations. Standard deviations, coefficients of skewness and lag one autocorrelations appear to be low for some stations. When these were estimated using the whole generated sequence rather than replicates (Table 3), the estimates were close to the historical values, implying that the model performance is satisfactory. Table 4 shows that model 2 preserves the cross correlations between rainfall and evaporation.

Monthly Evaporation Models

Monthly evaporation may be obtained from the generated annual evaporation by the 'Method of fragments' (Svanidze 1980). This method uses the observed pattern of monthly evaporation in disaggregating the annual evaporation into monthly evaporation. Because the evaporation data are short in length (8 to 16 years), available fragments are also limited in number (8 to 16) and therefore this approach to obtain monthly evaporation may not be suitable for some purposes. However, if one has an adequate length of evaporation data, generated monthly evaporation can be obtained by this method from annual evaporation synthesized by one of the models outlined earlier.

Model 3: Method of fragments

The annual evaporation and rainfall are first generated using model 2. The generated annual values are then disaggregated into monthly values by using the observed patterns of concurrent monthly evaporation rainfall.

The observed monthly values are standardized year by year by dividing each monthly value in a year by the corresponding annual value, so that the sum of standardized monthly values in any year equals unity. By doing so, one will have N sets of fragments of correlated monthly evaporation and rainfall from N years of record. The sets of fragments are arranged in the same order as they occurred historically and numbered from 1 to N. The fragments are selected at random by the number computed from INTEGER ($N * U + 1$), where U is a uniformly distributed random number between 0 and 1. The monthly values are obtained by multiplying the annual values by the fragments chosen.

If only monthly evaporation is required, then only annual evaporation is generated, using model 1, and disaggregated into monthly values using the pattern of observed evaporation data.

In the following sections, suitable alternative models to the method of fragments are given for situations where the length of evaporation data is not adequate to use in the method of fragments.

Independent of Rainfall

Model 4: Two-tier model

Annual and monthly evaporation data are generated separately using a first order Markov model (model 1) and Thomas-Fiering monthly model with Wilson-Hilferty transformation. Because variability and skewness of monthly evaporation are low, the Thomas-Fiering monthly model is adequate and is defined as

$$E_{i,j} = r_j E_{i,j-1} + (1-r_j^2)^{\frac{1}{2}} \varepsilon_{i,j} \qquad (34)$$

where

$E_{i,j}, E_{i,j-1}$ – cyclically standardized evaporation for year i and months j and (j-1), respectively,

r_j – serial correlation between the evaporation values for months j and (j-1), and

$\varepsilon_{i,j}$ – random deviate with zero mean and unit variance.

The cyclical standardization is carried out using

$$E_{i,j} = \frac{E^*_{i,j} - \bar{E}_j}{S_j} \tag{35}$$

where

$E^*_{i,j}$ – observed monthly evaporation for year i and month j, and

$\bar{E}_j, S_j$ – mean and standard deviation of monthly evaporation for month j.

The skewness $\varepsilon_{i,j}$ is given by

$$\gamma_\varepsilon(j) = \frac{\gamma_E(j) - r_j^3\,\gamma_E(j-1)}{(1-r_j^2)^{\frac{3}{2}}} \tag{36}$$

where $\gamma_E(j)$ and $\gamma_E(j\text{-}1)$ – skewness of monthly evaporation for months j and (j-1) respectively.

The Wilson-Hilferty transformation is used to introduce skewness in $\varepsilon_{i,j}$.

The generated monthly evaporation values are then adjusted with respect to the generated annual evaporation values using (Harms and Campbell 1967)

$$E'_{i,j} = \frac{E^*_i}{\sum_{j=1}^{12} E^*_{i,j}} \; E^*_{i,j} \tag{37}$$

where

E^*_i – generated annual evaporation for year i,

$E^*_{i,j}$ – monthly evaporation obtained from Thomas-Fiering model for month j and year i, and

$E'_{i,j}$ – adjusted monthly evaporation for month j and year i.

Dependent on Rainfall

Because rainfall records are usually long, monthly rainfall is first generated using the method of fragments. Monthly evaporation is then obtained by linear regression using Eq. (8) by replacing the annual values with the monthly values.

As mentioned earlier, evaporation data generated in this manner will have an implied serial correlation which is a function of the cross correlation and serial correlation of the rainfall. If one desires to preserve the serial correlation in evaporation as well, a multisite type model of the following form should be used.

Model 5: Monthly multisite type model

$$[Y_j] = [A_j][Y_{j-1}] + [B_j][\varepsilon_j] \tag{38}$$

where

- $[Y_j],[Y_{j-1}]$ – (2×1) vectors of cyclically standardized monthly rainfall and evaporation for months j and (j-1),
- $[\varepsilon_j]$ – (2×1 vector of random deviates with zero mean and unit variance, and
- $[A_j],[B_j]$– (2×2) matrices of constant coefficients to preserve the cross correlations.

The elements of $[A_j]$ and $[B_j]$ are obtained from

$$[A_j] = [M_1^j][M_0^{j-1}]^{-1} \tag{39}$$

$$[B_j][B_j]^T = [M_0^j] - [M_1^j][M_0^{j-1}]^{-1}[M_1^j]^T \tag{40}$$

Solution of Eq. (39) is straightforward, while $[B_j]$ can be obtained using Eqs. (17) to (20).

In terms of the elements of $[A_j]$ and $[B_j]$, Eq. (38) may be written as

$$Y_j^{(1)} = a_{11}^j Y_{j-1}^{(1)} + a_{12}^j Y_{j-1}^{(2)} + b_{11}^j \varepsilon_j^{(1)} \tag{41}$$

$$Y_j^{(2)} = a_{21}^j Y_{j-1}^{(1)} + a_{22}^j Y_{j-1}^{(2)} + b_{21}^j \varepsilon_j^{(1)} + b_{22}^i \varepsilon_j^{(2)} \tag{42}$$

where $Y_j^{(1)}$ and $Y_j^{(2)}$ are, respectively, the monthly rainfall and evaporation for month j. But $Y_j^{(1)}$ is known from the method of fragments.

Eliminating $\varepsilon_j^{(1)}$ between Eqs. (41) and (42), evaporation for month j is obtained from

$$Y_j^{(2)} = \frac{b_{21}^j}{b_{11}^j} Y_j^{(1)} + \left(a_{21}^j - \frac{b_{21}^j a_{11}^j}{b_{11}^j}\right) Y_{j-1}^{(1)} + \left(a_{22}^j - \frac{b_{21}^j a_{12}^j}{b_{11}^j}\right)$$

$$Y_{j-1}^{(2)} + b_{22}^j \varepsilon_j^{(2)} \tag{43}$$

It should be noted that j-1 = 12 when j = 1.

Application of Monthly Evaporation Models

Using models 3, 4 and 5, twenty replicates, each of length equal to the historical record were generated for all the stations listed in Table 1. For some months it was not possible to determined the elements of matrix $[B_j]$ in model 5. As a result, for these months the appropriate regression equation (Eq. (8)) was used.

Discussion of Monthly Results

Various monthly and annual parameters were estimated for each replicate and averaged for comparison. Due to lack of space, the results for only one station, Melbourne, are presented in Table 5. Except for the annual standard deviation, coefficient of skewness and autocorrelation for model 5, the parameters are satisfactorily preserved. Table 6 shows that the correlations between evaporation and rainfall are also preserved.

Table 5 – Comparison of historical and generated* monthly and annual evaporation parameters for Melbourne

Month	Model	Mean (mm)	Std dev (mm)	Coeff of skewness	Serial corr	Maxi-mum$^+$	Mini-mum$^+$
January	Hist	212	26	0.59	0.57	2.24	1.48
	4	204	23	0.27	0.48	2.10	1.45
	5	209	23	–0.19	0.02	2.11	1.45
	3	212	28	0.12	0.07	2.22	1.45
February	Hist	179	19	0.24	0.37	1.80	1.31
	4	174	17	0.14	0.26	1.74	1.27
	5	182	18	–0.11	0.09	1.83	1.29
	3	179	16	0.51	0.43	1.80	1.35
March	Hist	138	17	0.07	0.72	1.42	0.92
	4	134	16	0.05	0.66	1.37	0.94
	5	134	23	–0.44	–0.04	1.43	0.80
	3	138	15	0.34	0.67	1.41	1.00
April	Hist	90	14	–0.29	–0.27	0.97	0.54
	4	88	14	–0.22	–0.17	0.94	0.56
	5	87	14	–0.16	–0.18	0.94	0.55
	3	90	14	–0.38	–0.31	0.94	0.56

cont.

Table 5 cont.

May	Hist	62	7	–0.53	0.43	0.61	0.42
	4	61	7	–0.40	0.41	0.61	0.42
	5	62	7	–0.32	0.18	0.62	0.43
	3	61	8	0.08	0.52	0.64	0.43
June	Hist	41	11	2.31	0.07	0.62	0.26
	4	40	10	1.06	0.09	0.51	0.25
	5	41	11	0.20	0.09	0.52	0.21
	3	41	8	1.42	–0.16	0.52	0.27
July	Hist	47	8	1.47	0.17	0.57	0.31
	4	46	9	0.95	0.13	0.55	0.30
	5	47	8	0.01	–0.02	0.52	0.28
	3	46	7	0.83	0.05	0.53	0.32
August	Hist	65	12	0.73	–0.22	0.76	0.44
	4	61	11	0.72	–0.12	0.72	0.40
	5	69	11	–0.29	0.09	0.73	0.43
	3	64	12	0.73	–0.04	0.76	0.43
September	Hist	91	15	1.19	0.28	1.06	0.63
	4	88	14	0.49	0.27	0.98	0.60
	5	94	15	–0.19	0.27	1.00	0.59
	3	90	11	0.62	0.18	0.94	0.66
October	Hist	127	16	0.03	0.39	1.30	0.89
	4	124	15	–0.13	0.36	1.27	0.85
	5	130	15	–0.14	0.25	1.32	0.90
	3	127	16	0.44	0.31	1.35	0.89
November	Hist	153	12	0.91	0.17	1.56	1.19
	4	150	14	0.57	0.26	1.53	1.11
	5	154	12	–0.18	0.29	1.48	1.14
	3	154	15	0.70	0.28	1.56	1.15
December	Hist	192	28	1.66	0.05	2.24	1.39
	4	188	26	0.77	0.14	2.05	1.35
	5	182	82	0.09	0.24	2.78	0.45
	3	191	20	0.69	–0.08	1.95	1.42
Annual	Hist	1397	96	1.22	0.47	1.16	0.90
	4	1357	70	0.27	0.45	1.10	0.90
	5	1392	102	–0.01	–0.11	1.14	0.88
	3	1393	80	0.62	0.19	1.10	0.92

+ Expressed as a ratio of mean monthly or annual evaporation.

*4 Two-tier model.

5 Monthly multisite type model.

3 Method of fragments.

Table 6 – Comparison of the cross correlation between monthly evaporation and rainfall obtained from historical and generated data

Station	Model	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Ann
	Hist	+0.12	+0.31	−0.56	−0.55	−0.51	−0.29	0.00	−0.80	−0.55	−0.83	−0.28	−0.56	−0.12
Melbourne	5	−0.20	+0.07	−0.76	−0.57	−0.44	−0.17	−0.11	−0.73	−0.54	−0.74	−0.28	−0.60	−0.25
	3	0.00	−0.16	−0.37	−0.51	−0.41	−0.35	+0.06	−0.37	−0.54	−0.60	−0.26	−0.47	−0.15
	Hist	−0.43	−0.82	−0.77	−0.50	−0.64	−0.19	−0.88	−0.31	−0.34	−0.33	−0.81	−0.51	−0.81
Sydney	5	−0.51	−0.87	−0.70	−0.73	−0.56	−0.19	−0.87	−0.44	−0.03	−0.24	−0.69	−0.72	−0.67
	3	−0.29	−0.78	−0.72	−0.32	−0.77	−0.22	−0.78	−0.05	−0.31	+0.04	−0.80	−0.65	−0.79
	Hist	+0.27	−0.72	−0.65	−0.50	−0.38	+0.24	−0.58	−0.19	−0.75	−0.28	−0.52	−0.69	−0.63
Monto	5	+0.20	−0.40	−0.67	−0.58	−0.39	−0.27	−0.34	−0.17	−0.42	−0.29	−0.45	−0.71	−0.20
	3	−0.05	−0.57	−0.66	−0.42	−0.51	+0.15	−0.46	−0.08	−0.58	−0.32	−0.54	−0.63	−0.69
	Hist	−0.64	−0.90	−0.35	−0.73	−0.52	−0.08	−0.16	−0.59	−0.70	−0.83	−0.47	−0.74	−0.93
Cowra	5	−0.40	−0.91	−0.46	−0.64	−0.49	+0.20	−0.36	−0.75	−0.59	−0.42	−0.45	−0.40	−0.59
	3	−0.46	−0.62	−0.66	−0.50	−0.73	−0.59	−0.40	−0.41	−0.37	−0.64	−0.37	−0.55	−0.92
	Hist	−0.59	−0.47	−0.71	−0.47	−0.33	−0.28	−0.43	−0.54	−0.80	−0.16	+0.73	−0.68	−0.27
Brisbane	5	−0.68	−0.50	−0.65	−0.74	−0.26	−0.76	−0.77	−0.67	−0.25	−0.12	+0.45	−0.55	−0.53
	3	+0.19	−0.17	−0.63	−0.41	−0.45	−0.04	−0.31	−0.46	−0.64	+0.04	+0.38	−0.55	−0.32
	Hist	−0.67	−0.58	−0.55	−0.64	−0.65	+0.13	−0.52	−0.30	−0.49	+0.26	−0.08	−0.44	−0.73
Broome	5	−0.48	−0.48	−0.63	−0.71	−0.12	+0.47	−0.31	−0.05	−0.25	+0.12	−0.17	−0.22	−0.49
	3	−0.59	−0.38	−0.61	−0.47	−0.55	−0.02	−0.36	−0.41	−0.22	−0.17	−0.15	−0.59	−0.66
	Hist	−0.06	−0.25	+0.09	−0.33	−0.44	+0.37	−0.07	−0.44	−0.39	−0.67	−0.21	−0.14	−0.09
Perth	5	−0.15	−0.30	+0.02	−0.59	−0.63	+0.35	−0.10	−0.57	−0.30	−0.55	−0.35	−0.48	−0.30
	3	−0.07	−0.23	+0.01	−0.36	−0.52	+0.03	−0.04	−0.21	−0.22	−0.52	−0.21	−0.21	−0.05
	Hist	−0.68	−0.72	−0.49	−0.22	−0.59	−0.26	−0.56	−0.58	−0.84	−0.71	−0.39	−0.63	−0.83
Alice Springs	5	−0.52	−0.48	−0.32	−0.36	−0.64	−0.23	−0.25	−0.47	−0.47	−0.54	−0.34	−0.27	−0.45
	3	−0.51	−0.59	−0.62	−0.38	−0.61	−0.46	−0.45	−0.55	−0.70	−0.51	−0.58	−0.62	−0.86
	Hist	−0.57	−0.52	−0.57	−0.78	−0.64	−0.39	−0.83	−0.36	−0.70	−0.72	−0.62	−0.16	−0.81
Kalgoorlie	5	−0.75	−0.47	−0.32	−0.76	−0.59	−0.47	−0.70	−0.35	−0.66	−0.58	−0.66	−0.21	−0.63
	3	−0.44	−0.52	−0.50	−0.63	−0.78	−0.21	−0.82	−0.33	−0.60	−0.52	−0.57	−0.48	−0.79

Daily Evaporation Model

Daily evaporation has been found to depend on the state of the day (dry or wet); evaporation on a rainy day is less than that on a dry day. From the daily data for each station, moments of daily evaporation on dry and wet days and the correlation between daily rainfall depth and daily evaporation on wet days were calculated for each month. Unlike the annual and monthly cases, the correlation was found to be small for most months and stations. However, mean daily evaporation on wet days was consistently lower than that on dry days. Hence, wet and dry days were considered separately, so that the relationship with rainfall could be taken into account.

Model 6: First order Markov model dependent on the type of day
Four types of wet days were considered. They were:

(a) dry day followed by a dry day,
(b) dry day followed by a wet day,
(c) wet day followed by a dry day,
(d) wet day followed by a wet day.

A first order Markov model (Eq. (4)) was used to generate daily evaporation on each type of wet day. This takes into account the lag one autocorrelation and the skewness is modelled through the Wilson-Hilferty transformation (Eq. (5)).

Wet days are determined by a transition probability matrix up to seven states in size. Our previous studies have shown that this performs better than a two-state Markov model in reproducing the number of wet days (Srikanthan and McMahon 1982).

Generation of Daily Evaporation Data

From the daily rainfall data, a set of transition probability matrices was calculated using the state limits and the number of states given in Tables 7 and 8. However, it should be noted that if the number of states is k, the k^{th} state is unbounded. The whole of Table 9 applies only to months with seven states. From a concurrent set of evaporation and rainfall data, means, standard deviations, coefficients of skew-

Table 7 – State limits for daily transition probablity matrices

State number	Upper state limit in mm
1	0
2	1
3	3
4	7
5	15
6	31
7	∞

Table 8 – State numbers for each month

Station	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Melbourne	6	6	6	6	6	6	6	6	6	6	6	6
Sydney	7	7	7	7	7	7	7	7	7	7	7	7
Monto	6	6	6	6	6	6	6	6	6	6	6	6
Cowra	6	6	6	6	6	6	6	6	6	6	6	6
Brisbane	7	7	7	7	7	7	7	7	7	7	7	7
Broome	7	7	7	3	3	3	3	3	3	3	3	4
Perth	6	6	6	6	6	6	6	6	6	6	6	6
Alice Springs	4	4	4	4	4	4	4	4	4	4	4	4
Kalgoorlie	5	5	5	5	5	5	5	5	5	5	5	5

Table 9 – Comparisons of historical and generated daily parameters for Melbourne

Parameter	Model	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean	Hist	6.82	6.28	4.50	3.05	2.03	1.36	1.58	2.11	3.05	4.11	5.02	6.33
(mm)	Gen	6.96	6.25	4.48	3.10	2.02	1.36	1.60	2.10	3.04	4.14	5.03	6.28
Standard deviation	Hist	2.47	2.32	1.93	1.64	1.08	0.86	0.87	1.08	1.50	1.79	1.98	2.33
(mm)	Gen	2.54	2.34	1.91	1.67	1.09	0.87	0.91	1.12	1.45	1.83	1.97	2.33
Coeff. of	Hist	0.43	0.56	0.75	1.10	0.96	1.54	1.26	1.21	1.33	0.65	0.67	0.55
skewness	Gen	0.46	0.50	0.77	1.14	0.91	1.20	1.41	1.23	0.89	0.59	0.63	0.57
Maximum	Hist	17	16	14	10	7	7	6	7	12	11	12	16
(mm)	Gen	17	16	14	12	7	5	7	8	10	11	13	16
Minimum	Hist	2	5	5	2	2	2	2	2	2	2	8	10
(tenths of mm)	Gen	2	2	0	1	0	0	0	0	1	0	6	2

ness and lag one autocorrelation coefficients were calculated, corresponding to each type of day. A normal distribution was used whenever the absolute value of skewness was less than 0.1, otherwise the Wilson-Hilferty transformation was used. Five replicates were generated for each station.

Discussion of Daily Results

In order to evaluate the performance of the daily model, daily means, standard deviations, coefficients of skewness, maximum and minimum evaporation were calculated, along with monthly and annual means and standard deviations for each replicate; values for each replicate were averaged. These averaged values were compared with the corresponding historical values. Comparisons are presented in Tables 9 and 10 for Melbourne only, as the other stations exhibited

Table 10 – Comparison of historical and generated monthly and annual parameters for Melbourne from daily model

Parameter	Model	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Ann
Mean	Hist	212	179	138	90	62	41	47	65	91	127	153	192	1397
(mm)	Gen	216	178	139	92	62	41	50	65	91	128	151	195	1407
Standard deviation	Hist	26	19	17	14	7	11	8	12	15	16	12	28	96
(mm)	Gen	21	14	15	12	7	5	7	7	9	13	12	15	39

similar characteristics to those of Melbourne. Table 9 indicates that the daily parameters are satisfactorily preserved, although the standard deviations are smaller than the corresponding historical values (Table 10). This inadequacy usually occurs when values at a higher level (monthly) are obtained by aggregating generated values at a lower level (daily).

Conclusions

If one wishes to generate evaporation data only, a first order Markov model (model 1) and a two-tier model (model 4) are satisfactory for annual and monthly evaporation respectively. With an adequate length of historical data, one could use the method of fragments to generate monthly evaporation. This method was not used in this study as the two-tier model performed satisfactorily.

If evaporation data are required along with rainfall, both should be generated simultaneously so that the observed cross correlations are preserved. For annual data, a multisite type model (model 2) is satisfactory. For monthly data where an adequate length of evaporation data is available, model 3 may be used to generate monthly evaporation and rainfall simultaneously. Otherwise, model 5 may be used, as it satisfactorily generates monthly evaporation from previously generated monthly rainfall. This allows one to use all available rainfall data which is usually much longer than evaporation.

Finally, a first order Markov model dependent on the type of day was found to perform satisfactorily in generating daily evaporation data.

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References

Boughton, W. C. (1966) A mathematical model for relating runoff to rainfall with daily data. *Civil Eng. Trans., I. E. Aust., Vol. CE8(1),* pp 83-93.

Chapman, T. G. (1968) Catchment Parameters for a Deterministic Rainfall-Runoff Model. In *Land Evaluation* (G. A. Stewart, ed.) (Macmillan, Melbourne).

Crawford, N. H., and Linsley, R. K. (1966) Digital simulation in hydrology: Stanford watershed model IV. Stanford University Tech. Report No. 39.

Harms, A. A., and Campbell, T. H. (1967) An extension to the Thomas-Fiering model for the sequential generation of streamflow. *Water Resour. Res., Vol. 3(5),* pp 653-661.

Hounam, C. E. (1961) Evaporation in Australia. Bulletin No. 44, Bureau of Meteorology, Melbourne.

Hoy, R. D. (1977) Pan and lake evaporation in nothern Australia. Hydrology Symposium, I. E. Aust., Brisbane, pp 57-61.

Jones, J. W., Colwick, R. D., and Threadgill, E. D. (1972) A simulated environmental model of temperature, evaporation, rainfall and soil moisture. *Trans. ASAE, Vol. 15(2),* pp 366-372.

Mejia, J. M., Rodriguez Iturbe, I. and Cordova, J. R. (1974) Multivariate generation of mixtures of normal and log normal variables. *Water Resour. Res., Vol. 10(4),* pp 691-693.

Nicks, A. D., and Harp, J. F. (1980) Stochastic generation of temperature and solar radiation data. *J. Hydrol. Vol. 48,* pp 1-17.

Ritchie, J. T. (1981) Integrating climatic, soil, management and genetic factors into a model for grain yields. Abstract XIII, Int. Bol. Congress.

Saxton, K. E., and Bluhm, G. C. (1982) Regional prediction of crop water stress by soil water budgets and climatic demand. *Trans. ASAE, Vol. 25(1),* pp 105-110, 115.

Singh, K. P. (1979) Synthetic hydrologic information – a case study. *J. Hydraul. Div., ASCE, Vol. 105* (HY10), pp 1257-1275.

Srikanthan, R., and McMahon, T. A. (1982) Simulation of annual and monthly rainfalls – a preliminary study at five Australian stations. *Jour. App. Meteorology, Vol. 21*(10), pp 1472-1479.

Srikanthan, R., and McMahon, T. A. (1982) Stochastic generation of daily rainfall at twelve Australian stations. Agricultural Engineering Research Report No. 57/82, Department of Civil Engineering, University of Melbourne, Australia.

Srikanthan, R., McMahon, T. A., Codner, G. P., and Mein, R. G. (1983) Theory and application of multisite streamflow generation models. Sent to Civil Eng. Trans., I. E. Aust., for possible publication.

Van Dijk, M. H. (1982) Reduction in evaporation due to the bird screen used in the Australian class A pan evaporation network. Australian Meteorological Magazine.

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