Kirchhoff diffraction mapping in media with large velocity contrasts

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ABSTRACT

Finite-difference methods for calculating traveltimes are superior to ray-tracing methods in inhomogeneous media. However, when these techniques are applied to Kirchhoff migration, a severe problem occurs in the presence of large velocity contrasts. If finite-difference traveltime methods are used to calculate first arrivals, an incomplete image is created because substantial subsurface information is often carried by direct body waves. We propose a solution to this problem by developing a new method of calculating later arrival times and applying both first and later arrival times to a Kirchhoff diffraction mapping algorithm. A comparison shows that the implementation of both first arrivals and later arrivals in Kirchhoff migration can substantially improve the images in media with large velocity contrasts.

INTRODUCTION

Kirchhoff migration is flexible in dealing with media with arbitrarily varying velocities, large velocity contrasts, and overturned reflectors. It can handle both vertical seismic profiling (VSP) and crosshole data. Important to the success of Kirchhoff migration is the calculation of traveltimes for applying the Green’s functions in inhomogeneous media. Currently, two kinds of methods exist for traveltime calculations: (1) ray tracing and (2) eikonal solution to the wave equation.

In media with large velocity contrasts, ray tracing encounters severe problems that may cause ray divergence, producing artificial shadow zones. Such problems can be avoided by using the eikonal solution with recently developed techniques for wavefront calculations by finite-difference methods (Reshef and Kosloff, 1986; Vidale, 1988; Podvin and Lecomte, 1991; Schneider et al., 1992; Zhao, 1996). Most of these methods of calculating traveltimes can effectively handle large velocity contrasts and irregularly sharp boundaries, and shadow zones are no longer a problem.

When the eikonal solution is used for Kirchhoff migration, problems arise if only the first arrival times are calculated. In media with large velocity contrasts, the first-arrival time may differ substantially from the arrival time of the direct body wave. The first-arrival frequency has lower amplitudes than later arrivals, and if only the first arrival time is used in the migration, the image of the diffractor may be incomplete. This problem of using only the first-arrival eikonal solution is well known, but later arrivals also satisfy the eikonal equation. Better methods are needed to compute traveltimes and amplitudes to improve the performance of Kirchhoff migration.

We review some of the problems with traveltime calculations in Kirchhoff migrations. We then propose a solution with a method for numerical traveltime calculation that includes later arrivals; this solution is applied to our Kirchhoff diffraction mapping method for better imaging. A comparison is made of the new Kirchhoff traveltime technique, a widely used commercial method of Kirchhoff depth migration using ray tracing, and a method using the eikonal solution.

ANALYSIS OF PROBLEMS OF TRAVELTIME CALCULATIONS IN DEPTH MIGRATION

In diffraction mapping, all points in the subsurface are regarded as point scatterers or diffractors. For every subsurface diffraction point, traveltimes are calculated from the sources (or receivers). Direct body waves involve the shortest travel path from a source (or receiver) to a diffractor consistent with Snell’s law and Fermat’s principle, but not necessarily involving critical refractions or head waves. The first arrival may be
the only arrival in simple structural situations. When media are complex with large velocity contrasts, traveltimes for the direct body waves may be greater than first-arrival times created by refracted or locally diffracted waves. Later arrivals may carry more subsurface information than first arrivals, and they may be of greater amplitude.

Figure 1a shows a very simple model with a large velocity contrast in which a low-velocity medium (1.0 km/s) intrudes a constant-velocity medium with a considerably higher velocity (3.0 km/s). Traveltime isochrons are from a diffractor at source A. Such traveltimes are used in the Kirchhoff migration analysis described below.

Figure 1b is a shot gather for a source at point B with fifty receivers located on the surface at 2-m intervals. The seismogram was generated by a commercially available finite-difference program for the velocity model shown in Figure 1a. To emphasize the reflections and other later arrivals in Figure 1b, direct S-wave arrivals have been muted. The first event on the shot

**FIG. 1.** Comparison of arrival times. (a) Velocity model and traveltime isochrons emanating from a diffraction point at A. (b) Shot record from a shot at B with 50 receivers on the surface. The location of the zero-offset trace is shown by the line through B. Twice the traveltime from A to B on (a) corresponds to time C' on (b), rather than the reflection event at time A' on (b).
The two-way traveltime for the first event from point A to point B (Figure 1a) is at 53 ms. The arrival time of C', at 53 ms, is much earlier than that for event A' reflected from the bottom of the low-velocity medium, including point A (Figure 1b).

The images shown in Figure 2 are the result of Kirchhoff migration based on the eikonal solution using first-arrival times. The image is reconstructed using seven shot gatherers measured at the surface at equal spatial intervals. Figure 2a shows the image generated using a constant velocity of 2.6 km/s; Figure 2b shows the result for a velocity model with three horizontal layers with velocities of 3.0, 1.0, and 3.0 km/s. The top and bottom of the middle layer are at the same positions as the top and bottom of the low-velocity layer. Figure 2c shows the image using the correct velocity model (Figure 1a). The upper boundary of the low-velocity layer is properly imaged, but the upper-right boundary is only partially imaged because of a limited migration aperture. Since only the first arrivals were used, the bottom of the low-velocity layer is not imaged.

Although ray-tracing methods can be used to calculate multiple arrivals, severe problems exist because of shadow zones. Figure 3 shows three images of the low-velocity layer using a Kirchhoff migration method incorporating ray tracing. The same velocities were used as for Figure 2. In Figure 3c, the upper boundary and upper-right boundary of the low-velocity layer are now properly imaged. The lower-right side is incomplete because of the limited data aperture. The bottom of the low-velocity layer is again absent because the ray-tracing technique fails to adequately handle the low-velocity layer.

These examples show that Kirchhoff migration, using either ray tracing or the eikonal solution for first-arrival time calculations, may fail to handle media with large velocity contrasts.

A METHOD OF KIRCHHOFF DIFFRACTION MAPPING WITH LATER ARRIVALS

The correct calculation of traveltimes is crucial to the success of Kirchhoff migration. Traveltime mapping using a finite-difference method has the potential to handle large velocity contrasts. It also avoids the shadow zone problem if more than first-arrival times can be calculated. Our method of traveltime calculation with later arrivals generalizes the techniques of Podvin and Lecomte (1991). Their technique of a nearest neighbor approximation to Huygens' construction is only used to compute first-arrival times. Our method includes the computation of later arrival times using a curved wavefront approach (Schneider et al., 1992), which is an improvement on the accuracy of the method of Podvin and Lecomte.

The first

The two differences between this new proposed Kirchhoff diffraction mapping method and existing methods of Kirchhoff depth migration:
1) Conventional methods of Kirchhoff migration image structures by reconstructing the waves using the Kirchhoff integral. Our approach is a small step toward a true reflectivity reconstruction because it evaluates the reflectivity function using the amplitudes of both downward- and upward-propagating waves.

2) Existing methods of Kirchhoff migration are based either on ray tracing, which may face problems of shadow zones, or on finite-difference solutions to the eikonal equation, which account only for first arrivals and do not yield the correct amplitude for the Green's function.

The method proposed uses a Green's function constructed from a traveltime calculation using later arrivals and an amplitude calculation that does not require ray tracing. This method of traveltime mapping overcomes the problem of shadow zones.

Fig. 2. Images produced by migration using the eikonal solution using (a) a constant velocity, (b) a velocity model with three horizontal layers, and (c) the true velocity function.

Fig. 3. Images produced by Kirchhoff migration using ray tracing using (a) a constant velocity, (b) a velocity model with three horizontal layers, and (c) the true velocity function.
in conventional ray tracing. It also avoids the problems that currently exist with the eikonal solution in accounting for later arrivals. The mathematical derivation of the method is given in the Appendix.

A TEST OF THE METHOD IN A MEDIUM WITH A LARGE VELOCITY CONTRAST

We migrated the input data (seven shot gathers, including the gather shown in Figure 1b) using the new method. The images in Figure 5 were produced using Kirchhoff diffraction mapping accounting for both first and later arrivals. In addition to the same input data, the same three velocity models were used in this migration. The image in Figure 5c was created using the correct velocity model, and both the upper and lower boundaries of the low-velocity layer are properly imaged. The upper-right side is also imaged, but the lower-right side is still incomplete because of the limited migration aperture.

Comparing the image obtained by Kirchhoff migration using the eikonal solution (Figure 2c) with the image obtained by Kirchhoff migration using ray tracing (Figure 3c) and the image obtained by the new method, we can see that the image of the bottom of the low-velocity layer is substantially improved (Figure 5c). Clearly, later arrivals emanate from the lower boundary of the low-velocity layer and, when migrated, yield an image of that boundary.

APPLICATION TO A GEOLOGICALLY REALISTIC MODEL

A numerical model simulating low-velocity gas sands (1.2 km/s) interfingering higher velocity (3.2 km/s) country rock was developed (Figure 6). The model is 200 m wide and 200 m deep. Fifty receivers were placed on the surface at 4-m intervals, and seven shots were fired through the spread at 30-m intervals, starting at shot 1 shown by the star (*) in Figure 6.

Seven shot gathers (Figure 7) were generated using a commercially available finite-difference algorithm for acoustic wave equation modeling, and these data were used as input to the migration comparisons. The first arrivals have been muted, and the first event on each gather is from the top of the shallowest finger (Figure 6). Diffractions from the corner of the shallowest finger are also present. The second event is reflected from the base of the shallowest finger. The third event is a reflection from the top of the second finger and diffractions from its corner. Reflections from the bottom of the second finger are...
indistinct, although reflections from the base of the structure are seen clearly on shots 4 through 7.

MIGRATION COMPARISONS

Comparisons of the results of the three migration methods are shown in Figures 8 through 10. Figure 8 shows the image obtained with a commercially available Kirchhoff migration package using ray tracing. The tops of the interfingered low-velocity layers are clearly imaged, although the base of the layers and the overturned structures are not.

Figure 9 shows the result of migration with a 70° finite-difference algorithm. The wave equation solution preserves data observed on the shot records and can handle high-velocity contrasts without a problem with shadow zones. For example, the base of the shallowest finger is imaged, but the method fails with steeply dipping and overturned features.

Figure 10 shows the results obtained with the new migration method. The upper surfaces of the three fingers are clearly imaged, as are the overturned boundaries. The base of the shallowest finger is imaged; the base of the second finger is absent, presumably because data were not present in the original shot gathers. However the steeply dipping interfaces below each of the fingers are clearly imaged, unlike the images obtained using Kirchhoff and finite-difference migration methods (Figures 8 and 9).
FIG. 8. Imaging of the numerical model using a commercially available Kirchhoff migration module with ray tracing. The top boundaries of the model fingers are imaged; the bottom boundaries and overturned structures are not.

FIG. 9. Imaging using a finite-difference migration module. Wave equation migration preserved all the information, and there is no problem with shadow zones. However, steep dips and overturned structures are not imaged.
CONCLUSIONS

We have developed a new method of Kirchhoff diffraction mapping that uses both first and later arrivals. This new method is capable of overcoming shadow zone problems that exist in conventional Kirchhoff migration using ray tracing and the problem of incomplete imaging in Kirchhoff migration using the eikonal solution. The method does not give an explicit interpretation as to what type of waves matches the first or later arrivals. However, when this technique is applied to a realistic model of an interfingered gas sand, there are substantial improvements in mapping subsurface structures with large velocity contrasts.

As the industry moves to seismic mapping in areas with more complex geology, techniques are needed to generate images in the presence of large velocity contrasts. Our results suggest that this new method of Kirchhoff diffraction mapping using both later arrivals and first arrivals provides a possible approach to dealing with media with large velocity contrasts.

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REFERENCES

DERIVATION OF A NEW KIRCHHOFF DIFFRACTION MAPPING TECHNIQUE

Green's secondary formula (Morse and Feshbach, 1953) states that

\[ \int_{\Omega} (U \nabla G - G \nabla U) \, ds = \int_{V} (U \nabla^2 G - G \nabla^2 U) \, dv, \quad (A1) \]

where \( \Omega \) is the surface of integration and \( V \) is the volume enclosed by \( \Omega \).

Let \( G = G(\mathbf{r}, \omega) \) be the Green's function solution for the Helmholtz equation:

\[ (\omega^2/c^2 + \nabla^2) G = -\delta(\mathbf{r} - \mathbf{r}_0), \quad (A2) \]

where \( \omega \) is angular frequency, \( c \) is velocity, \( \mathbf{r}_0 \) is the position of any point on surface \( \Omega \), and \( \mathbf{r} \) is the position of any point in volume \( V \).

The wavefield \( U = U(\mathbf{r}, \omega) \) satisfies the equation

\[ (\omega^2/c^2 + \nabla^2) U = 0, \quad (A3) \]

where \( \nabla^2 \) is the Laplacian operator. Let \( L = \omega^2/c^2 + \nabla^2 \); then \( \nabla^2 = L - \omega^2/c^2 \).

Applying the Laplacian operator \( \nabla^2 = L - \omega^2/c^2 \) to the right side of equation (A1) and applying the reciprocity principle for Green's function \( G(\mathbf{r} | \mathbf{r}_0) = G(\mathbf{r}_0 | \mathbf{r}) \), we obtain

\[ \int_{\Omega} [U(\mathbf{r}_0, \omega) \nabla G - G \nabla U(\mathbf{r}_0, \omega)] \, ds = \int_{V} [U(\mathbf{r}, \omega)L G - GL U(\mathbf{r}, \omega)] \, dv \]
\[ = -\int_{V} U(\mathbf{r}, \omega) \delta(\mathbf{r}_0 - \mathbf{r}) \, dv. \quad (A4) \]

Integrating the right side of equation (A4), we get the Kirchhoff integral

\[ U(\mathbf{r}, \omega) = -\int_{\Omega} [U(\mathbf{r}_0, \omega) \nabla G - G \nabla U(\mathbf{r}_0, \omega)] \, ds. \quad (A5) \]

French (1975) shows that if the data are collected on a horizontal plane by introducing an image source, the second term in the integral of equation (A5) is the negative of the first term. Under these circumstances equation (A5) reduces to

\[ U(\mathbf{r}, \omega) = -2 \int_{\Omega} U(\mathbf{r}_0, \omega) \nabla G \cdot ds. \quad (A6) \]

In inhomogeneous media, Green’s function in the frequency domain, obtained by the high-frequency approximation, has the following form (Bleistein, 1986):

\[ G(\mathbf{r}, \mathbf{r}_0, \omega) = A(\mathbf{r}, \mathbf{r}_0, \omega) e^{i \omega \tau}, \quad (A7) \]

where \( \tau \) is traveltime from \( \mathbf{r}_0 \) to \( \mathbf{r} \). Equation (A7) is obtained for each frequency using the definition of the Fourier transform:

\[ F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{i \omega t} \, dt. \quad (A8) \]

The derivative of the Green's function given in equation (A7) is then

\[ \nabla G = \nabla A e^{i \omega \tau} + i \omega \nabla A e^{i \omega \tau}. \quad (A9) \]

The Kirchhoff integral for forward wave modeling using this Green's function in general velocity media can be obtained by substituting equation (A9) into equation (A6):

\[ U(\mathbf{r}, \omega) = -2 \int_{\Omega} \left[ \left( U(\mathbf{r}_0, \omega) \nabla A + \right. \right. \]
\[ + i \omega U(\mathbf{r}_0, \omega) \nabla \tau A e^{i \omega \tau} \left. \right] ds. \quad (A10) \]

Equation (A10) states that wavefield \( U(\mathbf{r}, \omega) \) at any point \( \mathbf{r} \) in the medium can be calculated by using the data \( U(\mathbf{r}_0, \omega) \) on the integral surface \( \Omega \) for each frequency.

To achieve an image of the reflectivity at any point \( \mathbf{r} \) in the medium, a forward wave must be propagated from the source to point \( \mathbf{r} \) and a backward wave propagated to extrapolate the observed data at the observation surface to point \( \mathbf{r} \).

The value \( U_i(\mathbf{r}, \omega) \) represents the incident wave from the source arriving at any point \( \mathbf{r} \) by forward propagation, and \( U_s(\mathbf{r}, \omega) \) represents the scattered wave extrapolated by backward propagation using the surface data \( U_s(\mathbf{r}_0, \omega) \) from the observation surface at \( \mathbf{r}_0 \) to point \( \mathbf{r} \). Also, \( A_i \) and \( \tau_i \) represent amplitude and traveltime of the Green's function for the incident wave. The values \( A_s \) and \( \tau_s \) represent amplitude and traveltime of the Green's function for the scattered wave.

To use the forward equation (A10) for backward wave extrapolation, a retarded term in time is needed to extrapolate the waves to the scatterers at any point \( \mathbf{r} \) in the medium. The value \( U(\mathbf{r}, \omega) \) on the left of equation (A10) is the scattered field at any point \( \mathbf{r} \) in the medium. Using \( U_i(\mathbf{r}, \omega) \) instead of \( U(\mathbf{r}, \omega) \), a reverse term \( -\tau_i \) instead of \( \tau_i \) and an amplitude \( A_s \) instead of \( A \) in equation (A10), where \( \tau_i \) and \( A_s \) represent Green's function for the scattered wave, the backward wave extrapolation relation for the scattered waves can be written as

\[ U_s(\mathbf{r}, \omega) = -2 \int_{\Omega} \left[ U_i(\mathbf{r}_0, \omega) \nabla A_s e^{-i \omega \tau_s} \right. \]
\[ \left. + i \omega U_i(\mathbf{r}_0, \omega) \nabla \tau_s A_s e^{-i \omega \tau_s} \right] ds. \quad (A11) \]

To achieve an image of reflectivity \( R(\mathbf{r}) \) at any point \( \mathbf{r} \) in the medium, the imaging principle at any point \( \mathbf{r} \) (Claerbout, 1976) for Kirchhoff backward mapping was used:

\[ R(\mathbf{r}) = \int_{\omega} \frac{U_i(\mathbf{r}, \omega)}{U_i(\mathbf{r}, \omega)} \, d\omega. \quad (A12) \]

The incident wave \( U_i(\mathbf{r}, \omega) \) can be calculated by a high-frequency approximation as

\[ U_i(\mathbf{r}, \omega) = A_i(\mathbf{r}) e^{i \omega \tau_i}, \quad (A13) \]

where \( A_i \) is the ray amplitude and \( \tau_i \) is the traveltime of the incident wave from the source to the scattering point \( \mathbf{r} \).
Kirchhoff Diffraction Mapping

Substituting equations (A11) and (A13) into equation (A12), we obtain the reflectivity function:

\[ R(\mathbf{r}) = -\frac{2}{A_i} \int \left\{ \int_{0}^{\infty} \left[ U_s(\mathbf{r}, \omega) \nabla A_s - i\omega U_s(\mathbf{r}, \omega) \nabla \tau_s A_s \right] e^{-i\omega(\tau_s + \tau_i)} d\omega \right\} d\mathbf{s}. \quad (A14) \]

Equation (A14) is an inverse Fourier transform related to \((\tau_s + \tau_i)\). Applying the operator \((d/dt) \equiv i\omega\), equation (A14) then has the form

\[ R(\mathbf{r}) = -\frac{2}{A_i} \int \left[ u_s(\mathbf{r}, \tau_s + \tau_i) \nabla A_s - \nabla \tau_s A_s \left( u_s(\mathbf{r}, \tau_s + \tau_i) \right) \right] d\mathbf{s}. \quad (A15) \]

Equation (A15) is the formula for Kirchhoff diffraction mapping introduced in this paper. It uses true amplitudes, arbitrary velocities, and arbitrary observation surfaces.

The ray amplitudes of both the incident wave, \(A_i\), and the scattered wave, \(A_s\), can be calculated using any numerical method mentioned in the text. Traveltimes \(\tau_s\) and \(\tau_i\) can be calculated using the new travelt ime mapping method by a finite-difference method, which can cope with large velocity contrasts. The value \(\nabla \tau_s\) can be easily obtained once these traveltimes are mapped.

In the special case where the velocity is constant and the observation data \(u_s(\mathbf{r}, \tau_s + \tau_i)\) are collected on the surface at position \(\mathbf{r}_0\), the amplitude of the incident wave from the source to the scattering point at \(\mathbf{r}\) is \(A_i = 1/4\pi R_i\), where \(R_i\) is the distance from the source to the scattering point at \(\mathbf{r}\). The amplitude for the Green's function from the scattering point at \(\mathbf{r}\), measured at the receiver, is \(A_s = 1/4\pi R_s\cdot n = -\cos \theta/4\pi R_s^2\), where \(R_s\) is the distance from the receiver to the scattering point at \(\mathbf{r}\). The travelt ime from the diffractor to the receiver is \(\tau_s = R_s/c, \nabla \tau_s \cdot n = \cos \theta/c\), where \(n\) is the direction of the integral surface \(d\mathbf{s} = n ds\) and \(\theta\) is the angle of incidence.

Substituting these parameters into equation (A15), we obtain

\[ R(\mathbf{r}) = \frac{R_i}{2\pi} \int_{\mathbf{n}_0} \frac{\cos \theta}{c R_s} \left[ \frac{d u_s(\mathbf{r}, \tau_s + \tau_i)}{d t} + \frac{c}{R_s} u_s(\mathbf{r}, \tau_s + \tau_i) \right] d\mathbf{s}. \quad (A16) \]

Equation (A16) is the exact Kirchhoff migration formula proposed by Schneider (1978) apart from a coefficient \(R_i\), which appears because of the imaging principle being used.