\[
-\frac{\beta T_h}{c_p(T_h - T_a)} = 5\sqrt{\pi} \frac{t}{3} \text{ at low values of } \rho_i \left( T_m - T_a \right) \rho_o \left( T_h - T_a \right).
\]
An examination of equations (29), (30), and (31) reveals that

\[\beta \text{ is positive, if } T_h < T_m\]

\[\beta \text{ is zero, if } T_h = T_m\]

and

\[\beta \text{ is negative, if } T_h > T_m\]

Therefore the injection of helium gas into liquid oxygen results in the growth of gas bubbles and consequently cooling of the liquid oxygen is attained. Negative \(\beta\) may be obtained in case a steam bubble (although it is condensable) is injected into water.

Concluding Remarks

Approximate asymptotic solutions, including the influence of radial convection and diffusion, have been obtained for the growth and collapse of spherical bubbles in a nonisothermal system of varying composition. The results apply to growth and collapse controlled by heat and mass transport, which is characterized by uniform pressure throughout the system and by the movement of the phase boundary asymptotically proportional to the square root of time.

For bubble growth in boiling pure liquids and boiling binary mixtures, the results predicted by the source theory are in fair agreement with the exact solution of Scivion [1]. Other analyses or experimental results are not presently available to check the prediction for the bubble growth and collapse of the noncondensing, nonsoluble gas in liquids. This problem is currently being studied by the authors and their students. The accuracy of prediction may be improved by carrying more terms in equation (27) and by using more precise \(T_h - T_a\) relationship than that expressed by equation (30).

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References


Additional References

10. G. Horvay, "The Tension Field Created by a Spherical Nuclear Freezing Into a Less Dense Undercooled Melt," to be submitted shortly to a technical magazine.

Authors' Closure

The authors appreciate the contributions to this discussion by Dr. Horvay. As mentioned in the first paragraph of the Introduction and in the statement leading to equation (11), the results are restricted to the asymptotic stage of bubble development, where the growth has been initiated and is mainly governed by the transport of heat and mass from the surrounding liquid. The time intervals of the initial and intermediate stages which precede the asymptotic stage are very short for the types of bubble growth which we have studied (see references [1, 3, and 4]).

The initial conditions were not mentioned, although they probably should have been, because the use of both Green's function and equation (11) implies the assumption of uniform initial temperature distribution. (See chapter 14 of reference [5].)

In regard to Dr. Horvay's question "b," we assume he means equation (7) by "assumption (7)." The mathematics employed in the analysis is, of course, necessary for the reduction of the problem as defined by equations (1), (2), and (3). The limitation on the use of the analysis to small times, however, is basically a question of physics since the governing differential equations would not then be the same.

Dr. Horvay's reference to small-time behavior is most significant and does constitute an important consideration. The systems at which we have directed our present attention involve physical processes in which the time corresponding to the initial and intermediate stages of bubble growth is a negligible fraction of the total time under consideration.