

On Currents and Vertical Mixing in Lake Ontario during Summer Stratification

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Currents and vertical mixing characteristics were investigated on the basis of time series of current meter and temperature data from a summer-stratified period in Lake Ontario. The experimental set up consisted of seven current meters distributed in one vertical line from 12 m below the surface to 1 m above the lake bottom at a total depth of 143 m. The period considered for the analysis was from June to September, 1991.

The currents showed pronounced oscillations with two significant kinetic energy peaks, one at about 17 hours due to inertial motions, and one at 10 days, probably due to meteorological forcing. The current shear in the hypolimnion was strong enough to overcome stability and generate turbulence (Richardson numbers below 0.25) and there was probably turbulence enough available to keep the matter (almost neutral buoyant particles) in the whole Nepheloid bottom layer in suspension. In the thermocline region the turbulence was mainly damped (Richardson numbers above 1), but some events with lower Richardson numbers were also calculated indicating increased mixing during these events. By analysing filtered and unfiltered current meter data it was found that the shear-generated turbulence in the hypolimnion was mainly due to the meteorologically forced currents. In the thermocline region, however, the vertical shear associated with the inertial oscillation had a greater impact on the mixing.

Introduction

Lakes in mid latitudes exhibit a characteristic annual thermal cycle with two convective overturns, one during spring and one during autumn, when the lakes are mixed from top to bottom. In the summer season a well mixed warm surface layer (epilimnion) is formed on top of the colder deep water (hypolimnion). The transition, the thermocline, forms a stable stratified layer between the two different water masses. In the coastal zone, the deep water may interact with the surface layer through upwelling, but in the main part of the Great Lakes the thermocline acts as a layer inhibiting deep water to be in contact with the atmosphere. After the autumn overturn re-stratification starts and lighter colder water is formed on top of denser water, which may freeze and form ice during the winter season. The deep water is then again decoupled from the atmosphere.

As the hypolimnion is protected from the direct influence of winds and as there generally is a weak but stable temperature gradient, one would expect small vertical mixing in the hypolimnion. From scaling arguments Imberger and Ivey (1991) argued that the current shear should be of the same order of magnitude as the buoyancy frequency, implying a Richardson number of order one. The authors also stated that the mixing in the hypolimnion is intermittent and extremely patchy. The energy source for isolated mixing events is uncertain but probably associated with breaking internal wave motions, where mean or wave kinetic energy is transformed into potential energy. The importance of breaking internal waves for deep water mixing in oceans and fjords has also been discussed by for example Gargett (1984), Stigebrandt and Aure (1989) and Sjöberg and Stigebrandt (1992). Close to the bottom, in the boundary layer region, the flow is directly modified by the bottom friction and the current shear is an important mixing process.

In Lake Ontario the mixing in the hypolimnion is believed to be weak (Boyce *et al.* 1991) but important for different aspects of the lake ecosystem. On the basis of fluorescent tracer experiments, Kullenberg *et al.* (1974) calculated vertical mixing coefficients in the thermocline and hypolimnion regions giving low values. The experiments indicated that the mixing is controlled by the density stratification, the vertical current shear, and the fluctuating kinetic energy. The hypolimnion mixing in Lake Ontario is, however, in general poorly known.

An indirect evidence of mixing in the deeper layers of Lake Ontario is the presence of the Nepheloid layer. The study of the origin and characteristics of the Nepheloid layers in the Great Lakes began in the early 1980s, following the reconnaissance of the two battleships, Scourge and Hamilton, which sank in Lake Ontario during the war of 1812. Jacques Cousteau's colleagues viewed the two shipwrecks from a submersible and commented that the water at the bottom of the lake looked like "pea soup". This discovery prompted scientists to study the Nepheloid layer in the Great Lakes.

The Nepheloid bottom layer is a well characterized feature of ocean systems.

Many oceanographers have observed light scattering maxima in specific regions throughout the water column and near the ocean floor due to suspended particles. They referred to the regions of increasing in light scattering as the Nepheloid (or cloud or scatter) layer. Results of numerous studies of the Nepheloid layer indicates several possible sources of particles in the Nepheloid layer, such as material from the continental slopes, suspended matter from large river plumes, sediments eroded from the bottom and organic matter settling from surface water (Jerlov 1976). The temporal variability in a deep-ocean Nepheloid layer has also been observed to be considerable due to rapid changes near the sea floor (Gardner and Sullivan 1981).

Sandilands and Mudroch (1983) and Mudroch and Mudroch (1992) investigated the development and geochemical composition of the Nepheloid layer in Lake Ontario. In general, the formation of the Nepheloid layer starts in July, and the thickness of the layer increases and attains 40-50 m in October. The origin of the particles are caused by several sources, as bottom sediments and primary production. The vertical mixing generates the turbulent energy that keeps the matter in suspension and controls the thickness of the Nepheloid layer.

The sedimentation in Lake Ontario has been investigated by Rosa (1985). From weekly data it was found that the sedimentation rates were highly variable, a factor of 10 for the open sea station. The variability was mainly due to the amount of particulate matter in the epilimnion. Thus one can expect that both the turbulent mixing as well as the amount of matter in the Nepheloid layer are variable in time.

The purpose of the present work was to investigate vertical mixing characteristics of Lake Ontario during summer stratification. The analysis is based upon time series of temperature and current meter data from different depths at one location during a three-month summer period. Based upon calculated Richardson numbers and eddy viscosities, a clear indication of turbulence was found both in the bottom boundary layer and the hypolimnion. When particles are available one could thus expect that they could easily be held in suspension and form the Nepheloid layer. The turbulent mixing in the bottom boundary layer and the hypolimnion were mainly due to currents changing over a 10-day period. In the thermocline region, the turbulence was damped owing to the temperature gradient and mainly associated with inertial motions.

Experiments and Methods

The experiments were conducted at one station, 50 km east of Niagara River in Lake Ontario (Fig. 1). A string of ten Neil-Brown current meters were distributed starting from 12 m below the free surface to 1 m above the lake bottom. The total depth at the measuring site was 143 m. In addition to the current meter mooring, two meteorological buoys were located close to the station. The measuring pro-

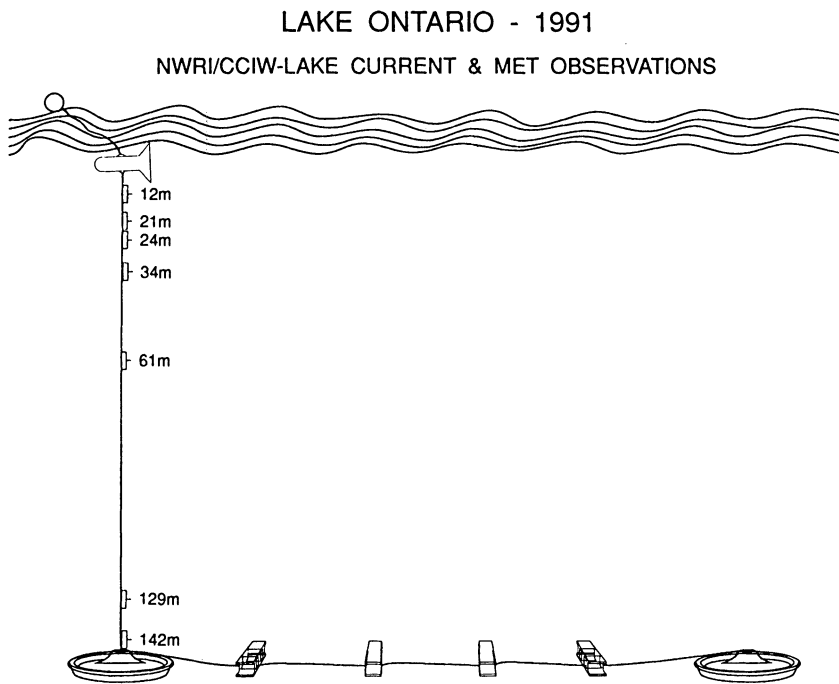
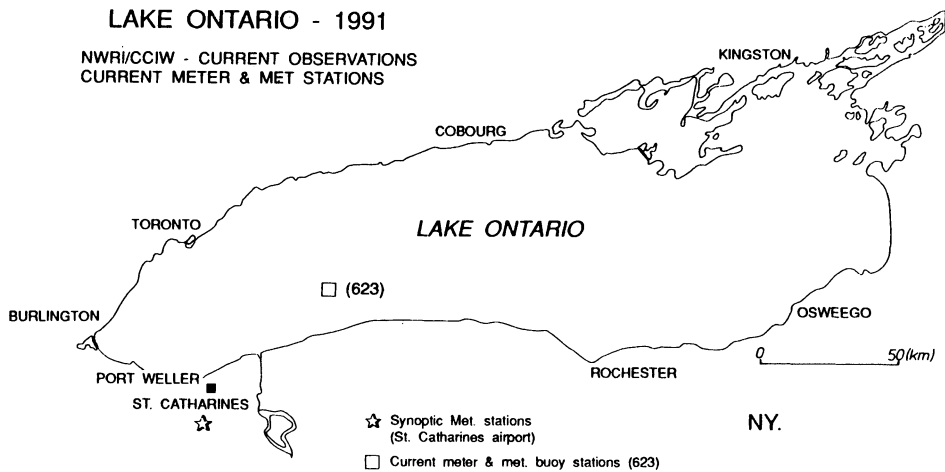


Fig. 1. Lake Ontario with measuring site.

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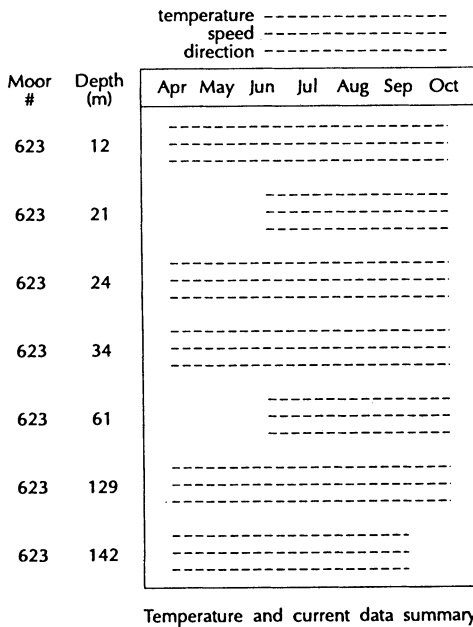


Fig. 2. Data summary.

gram was conducted from April to November, 1991, to coincide with the intensive surveys for toxic contaminants in the water column and sediments. Detailed vertical temperature and transmission profiles were also collected throughout the period. Except for three current meters all other instruments performed well giving detailed data. In the present work we will concentrate on the measurements of currents and temperature during the summer period from June to September, 1991. During that period seven current meters and two meteorological buoys were measuring simultaneously. Fig. 2 shows the tabular summaries of all physical data used in this analysis.

This experimental data base forms the basis for analysing vertical mixing characteristics in Lake Ontario.

The calculations were done in a vertical grid, with gradient calculations centered around each measuring level. The vertical density gradient, the vertical current shear, the Richardson number and the eddy viscosity coefficients were calculated according to the equations given in the next section. Energy spectra were also calculated (Blackman and Tukey 1959), and the current data were filtered using a Graham filter (Graham 1963), with a 18 to 24 hour cut off. The energy spectra and filter calculations were made by using a standard statistical package at the Canada Centre for Inland Waters. The results will be presented later in "Results and Discussion".

Theoretical Considerations

Dynamic processes in shallow seas are closely related to meteorological forcing and the basin geometry. The observed motions in general show pronounced oscillations due to transient forcing, variable topographic and Earth rotation effects. The origin of turbulence could be due to several mechanisms as current shear, breaking internal waves and convective overturn. Stable vertical stratification damps the turbulence. The strong influence that vertical shear and stability have on turbulence can be estimated from the Richardson number

$$R_i = -\frac{g}{\rho_0} \frac{\partial \rho}{\partial z} \left[\left(\frac{\partial U}{\partial z} \right)^2 + \left(\frac{\partial V}{\partial z} \right)^2 \right]^{-1} \quad (1)$$

where z is the vertical coordinate positive upwards, U and V are the currents towards east and north respectively, g is the gravity constant, ρ_0 a reference density and ρ the density.

Some implications of the Richardson number on turbulence can be studied from the conservation equation for turbulent kinetic energy, k , which under the assumption of horizontal homogeneous flows reads

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial z} \left(\frac{\nu_T}{\sigma_k} \frac{\partial k}{\partial z} \right) + P_s \left(1 - \frac{1}{\sigma_T} R_i \right) - \varepsilon \quad (2)$$

where ν_T is the eddy viscosity, σ_k the Schmidt number, P_s the production due to shear, σ_T the turbulent Prandtl number and ε dissipation rate of k . From the equation, neglecting the vertical diffusion term, one can notice that the turbulence is damped if the Richardson number becomes larger than 0. For Richardson numbers less than 0, the turbulent kinetic energy may increase. These observations are also in accordance with theoretical and laboratory studies suggesting that when the Richardson number is less than 0.25, laminar flow changes into turbulent flow. On the other hand, if the Richardson number is greater than 1, the turbulence damps out and the flow becomes essentially laminar (Stull 1988, p. 176).

The stability of lake water is due to a rather complicated equation of state. Lake water is by no means pure water. Instead, dissolved substances may affect the thermodynamic properties. Also pressure effects need to be considered in deep lakes. The approach taken is to apply equations for sea water to lake water. The equation of state and the equation for the temperature of maximum density in deep lakes have been examined by Farmer and Carmack (1981). In this study re-stratification near the temperature of maximum density was examined on the basis of data from Babine Lake, Canada. The estimated salinity in Babine Lake was 65 mg/kg or less than 0.065 per mil. Of more importance to the temperature of maximum density was, however, the pressure effect, which decreased with pressure at a rate of about 0.02 °C/10 m.

The salinity content in Lake Ontario, based upon data from 1981, was estimated

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Table 1 – Constants used in the equation of state

Constant	Value	Unit
C_o	$4.9388 \cdot 10^{-10}$	P_a^{-1}
T_o	3.983	$^{\circ}C$
α	$3.3039 \cdot 10^{-12}$	$^{\circ}C^{-1} P_a^{-1}$
β	$8.2545 \cdot 10^{-6}$	$^{\circ}C^{-2}$
ρ_o	999.975	$Kg \cdot m^{-3}$

(Dobson 1984, p. 10) to about 0.2 per mil. For surface water this implies a lowering of the temperature of maximum density from 3.98 to 3.94 °C, illustrating that also in Lake Ontario the pressure effects are more important than the salinity. Following Farmer and Carmack (1981) and neglecting salinity, the equation of state reads

$$\rho = \rho_o \{ 1 + P(C_o - \alpha(T - T_o)) - \beta(T - T_o)^2 \} \quad (3)$$

where P is the pressure, T the temperature and C_o , α , β , T_o , ρ_o are constants given in Table 1.

To calculate the stability of water one needs to consider the potential gradient in density, defined as the measured minus the adiabatic gradient. The latter arises from the adiabatic expansion or compression of a rising or sinking water volume. For fresh water close to the temperature for maximum density, the adiabatic effect can, however, safely be neglected (Farmer 1975).

The stability of lake water can thus be calculated from the *in-situ* temperatures according to

$$\frac{\partial \rho}{\partial z} = \frac{\partial \rho}{\partial T} \frac{\partial T}{\partial z} \quad (4)$$

$$\frac{\partial \rho}{\partial T} = -\rho_o \alpha P - 2\beta \rho_o (T - T_o) \quad (5)$$

From Eq. (5) one can notice that for temperatures close to T_o , pressure increases in importance.

The turbulent mixing can be calculated on the basis of different models. Instead of applying a more advanced turbulence closure model, which involves transport equations for turbulent quantities as Eq. (2), we will only apply a simple empirical formula. Pacanowski and Philander (1981) related the eddy viscosity to the Richardson number and studied the modelling of the temperature in the Tropical Ocean. The same formula has been applied in several other studies as, for example, by Semtner and Chevin (1992). The eddy viscosity (ν_T) then reads

$$\nu_T = \frac{\nu_o}{(1 + 5R_i^*)^2} + \nu_b \quad (6)$$

where v_o and v_b are equal to 10^{-2} (m^2/s) and 10^{-6} (m^2/s) respectively. In the present study the background eddy viscosity (v_b) is reduced by a factor of 100 (corresponding to molecular values) to account for the low eddy viscosities measured by Kullenberg *et al.* (1974). The parameterization in Eq. (6) relates the eddy viscosity to the Richardson number in a realistic way but is quite incomplete, as, for example, the eddy viscosity becomes a constant during homogeneous conditions. Also, the equation indicates turbulent conditions even for Richardson number values above 1. In the present paper we will, however, calculate the eddy viscosity according to Eq. (6) and use them as an alternative way of interpreting the Richardson number calculations.

Results and Discussion

Currents

The measured currents are illustrated as filtered data in Fig. 3. The filtered currents show oscillations at all depth with a typical period of 10 days. The currents, even at 1 m above bottom, were considerably high.

In Fig. 4, the energy spectra for the current meter data are given. Two energy peaks, one at about 17 hour period and one at about 10 days period, are well pronounced at all depths. The main energy is due to oscillation with a time period of 10 days, which also gives rise to the dominant currents. The short time oscillation rides on top of the longer ones and was more pronounced in direction than speed. The short time oscillation peak (16.7 hours) was slightly less than the theoretical inertial time period which is equal to 17.3 hours in Lake Ontario. The reason of a slightly lower period is probably due to standing internal Poincare waves, which have oscillations close but less than the inertial time period, see further discussion in Boyce *et al.* (1989). Inertial currents in Lake Ontario based upon winter data have been investigated earlier by Marmorino (1978).

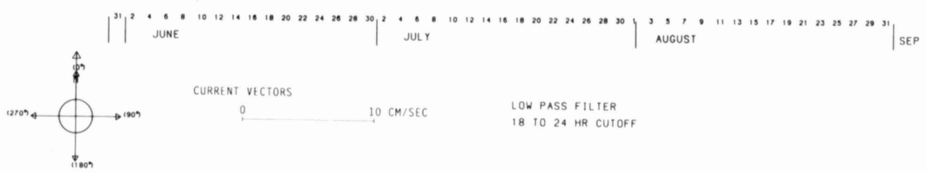
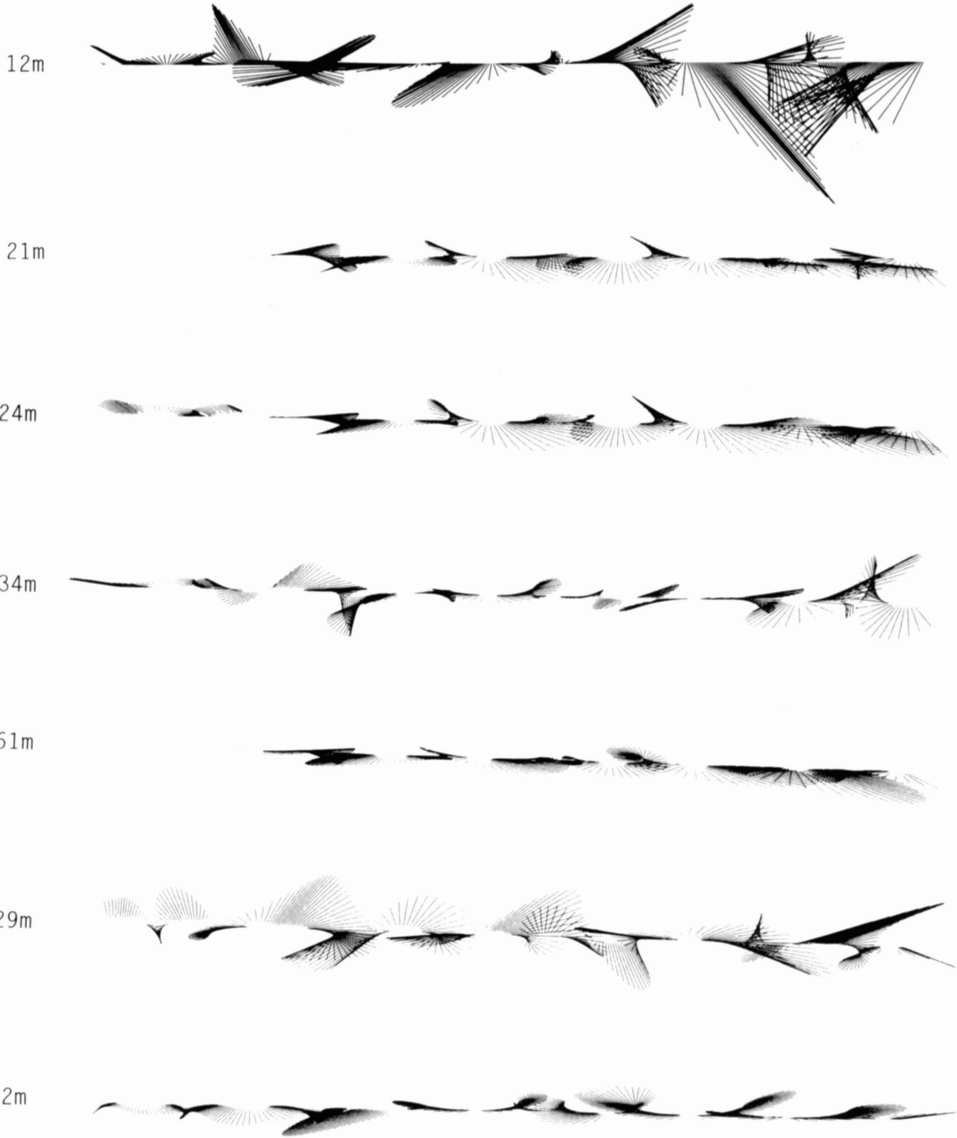
The vertical variations of the two spectra peaks are given in Fig. 5. In the 12 m current meter data, the inertial energy was larger than at deeper layers, probably due to winds, and in the deepest layer the inertial energy was slightly lower than above, probably due to bottom friction. In large lakes, the wind forces the coastal water in the wind direction and in the central parts the flow is reverse (Csanady 1982). This generates two rotating cells, that travels around the basin in a counter clockwise direction. From numerical simulation Simons (1980, p. 129) estimated that this topographically controlled rotational mode of oscillation should have a period of about eight days. The circulation may reverse also due to variable winds.

Fig. 3. Current stick plots from all depths. All current data are filtered using an 18 to 24-hour filter.

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DEPTH FROM SURFACE (m)

31 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31
JUNE JULY AUGUST SEP



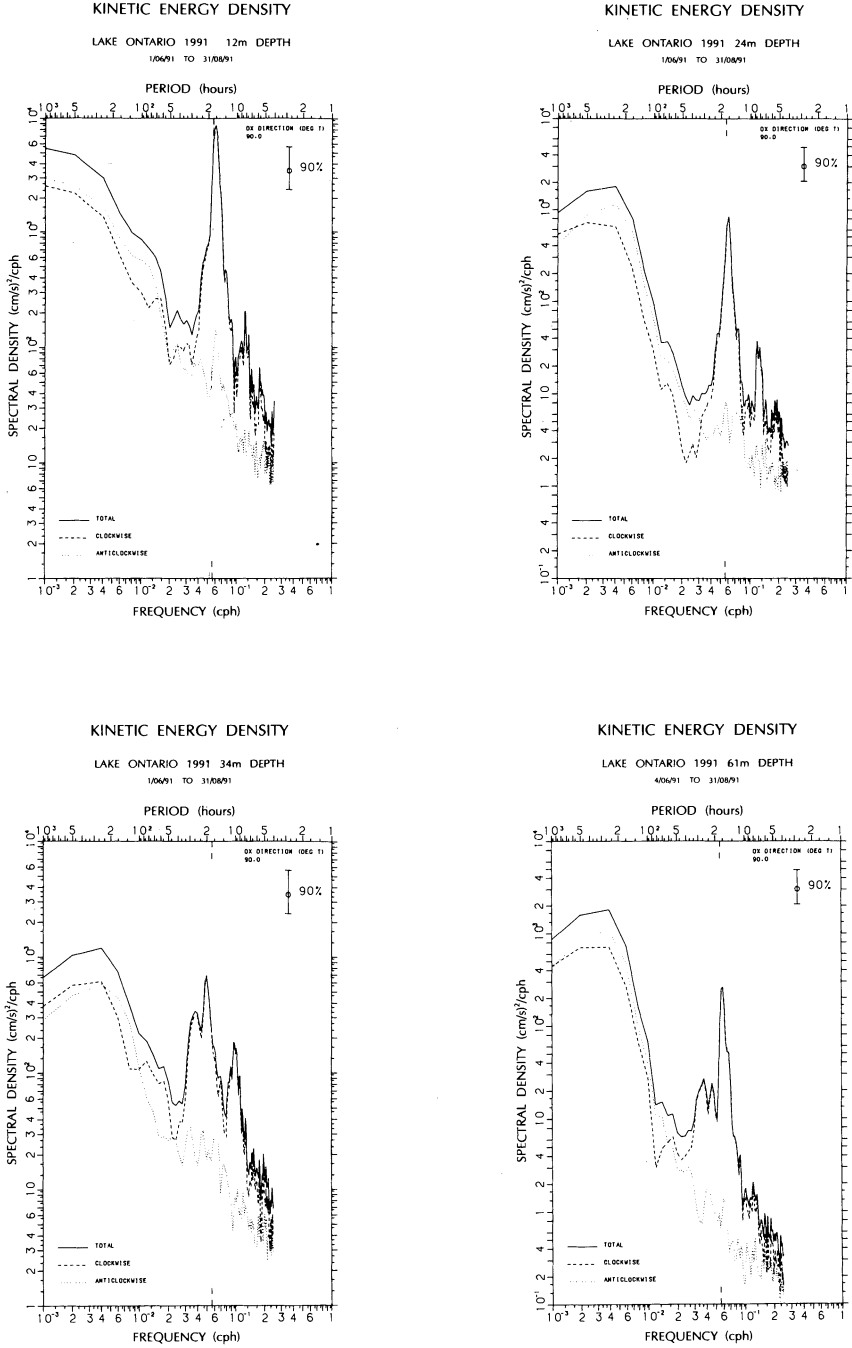


Fig. 4.

Vertical Mixing in Lake Ontario

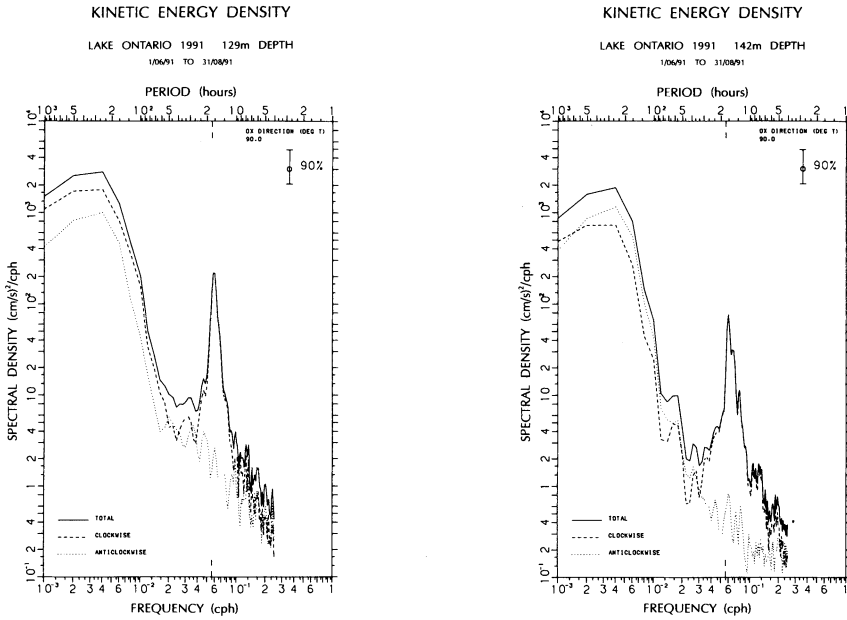


Fig. 4. Current spectra from different depths.

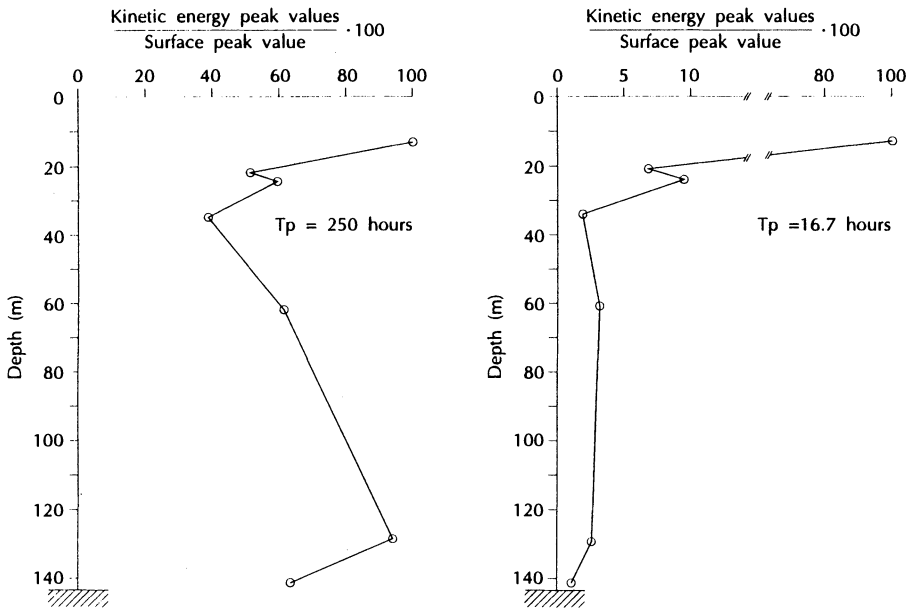


Fig. 5. The vertical variation of the normalized kinetic energy peak values at 250 hours (a) and 16.7 hours (b).

KINETIC ENERGY DENSITY

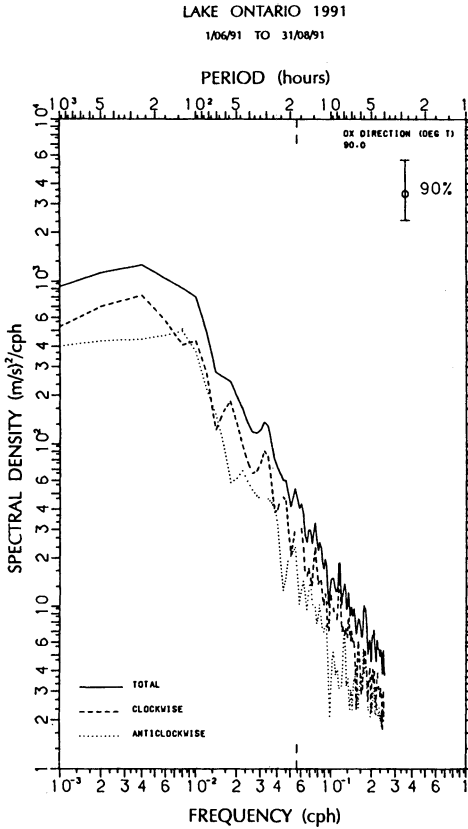


Fig. 6. Wind spectra.

KINETIC ENERGY DENSITY

LAKE ONTARIO 1991 VERTICAL MEAN CURRENTS

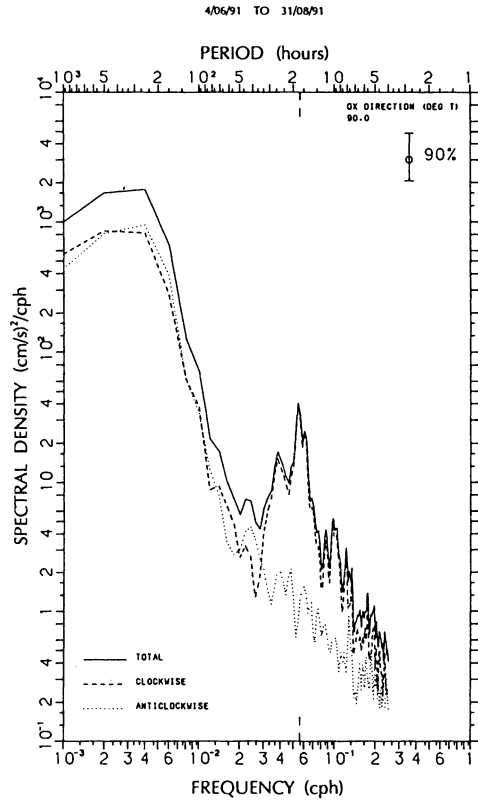


Fig. 7. Vertical mean current spectra.

Winds from west force the coastal currents towards east and winds from east force the coastal currents towards west with corresponding currents in the central part. The strong influence the wind has on coastal currents in Lake Ontario has, for example, been used by Murthy *et al.* (1986) in the development of models for pollutant transports in the coastal zone. During the studied period the reversal of winds was probably the main reason for the oscillation peak at 10 days, which therefore was mainly associated with the meteorological forcing, as can be confirmed in the wind spectra for the period (Fig. 6).

The spectrum from the vertical averaged current meter data is shown in Fig. 7. The spectrum gives a structure similar to the individual spectra for the different depths, indicating that currents had a strong barotropic (constant with depth) component. The deviation between the different depths is illustrated in Fig. 5.

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6 HOUR MEANS

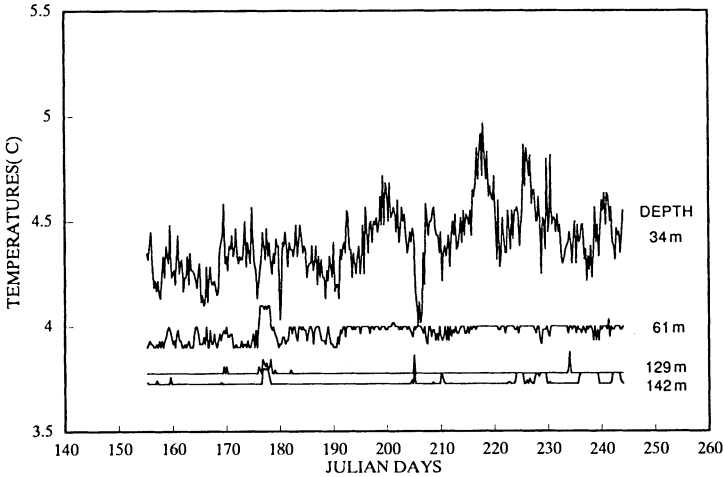


Fig. 8. Measured temperatures in the hypolimnion.

6 HOUR MEANS

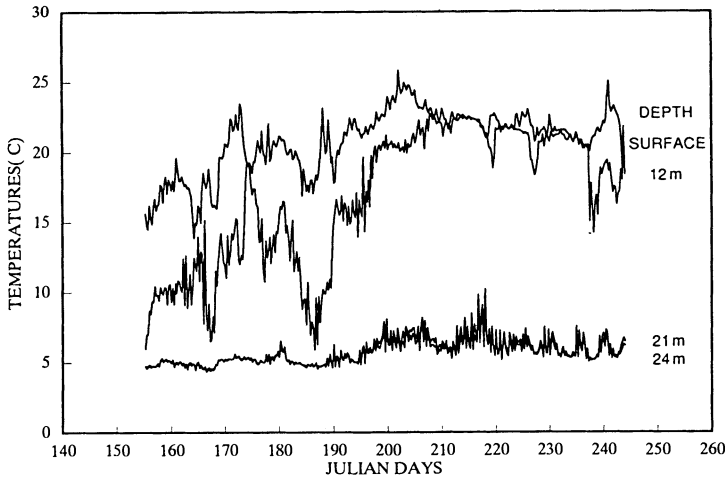


Fig. 9. Measured temperatures in the epilimnion and the thermocline region.

Thermal Structure

The temperature variations in the hypolimnion are illustrated in Fig. 8. All temperatures are slightly greater than the temperature for maximum density and show quite small variations during the summer period. The horizontal lines in the figure indicate that the temperature variations were less than the instrument accuracy ($\pm 0.05^\circ\text{C}$). The measurements at 34 m depth indicate a slight increase due to mixing with the thermocline. In Fig. 9, temperatures from the thermocline and epilimnion

regions are given. After Julian Day 210, the temperatures at the surface and 12 m below the surface are the same, indicating a well mixed surface layer deeper than 12 m after that date. Below the well mixed surface layer, the stable thermocline region decreases the temperatures from about 20 °C to 5 °C. The thickness of the thermocline region varies but after Julian Day 210, the thickness was estimated to about 25 m.

Vertical Mixing

The previous sections have illustrated that rather strong currents were observed at all levels and also that the temperature stratification was stable, with weak stable gradients in the hypolimnion and stronger in the thermocline region. As the thermocline protects the deeper layers from direct disturbances from the wind, the question addressed in this section is whether the current shear was strong enough to overcome the stability and generate turbulence or not.

The calculated Richardson numbers are illustrated in Fig. 10. Close to the bottom at 142 m depth, the Richardson numbers are small and mainly below 0.25, indicating turbulent mixing during the whole summer period. Above the bottom boundary layer the Richardson numbers are higher but during several episodes below 0.25. The calculations thus indicate turbulence in the whole hypolimnion during several periods. In the thermocline region, the Richardson numbers are above 1 during most of the summer period. However, events with lower Richardson numbers can be noticed, particularly at 24 m depth. Thus some generation of turbulence was probably also taking place in the thermocline region.

To examine the dynamics in some more detail, calculations based on unfiltered and filtered data were performed. The filter used was a Graham filter with a 18 to 24 hour filter. This filter damped out all inertial motions in the time series. The difference between calculated eddy viscosities using filtered and unfiltered time series could then be used as a check of the importance of inertial to meteorologically forced currents. From the calculations it was clear that mixing in the hypolimnion was mainly due to the meteorologically forced currents (10-day period). Inertial motions were, however, the main reason for mixing in the thermocline (Fig. 11). In the figure, calculations from three different levels are illustrated. Close to the bottom the difference between calculations based upon filtered and unfiltered data are small, indicating that almost all turbulence in the bottom boundary layer was due to meteorologically forced currents. At 61 m depth the calculations based upon filtered data are reduced by about 25%, indicating that inertial oscillation added 25% to the vertical mixing. Further up at 34 m, the calculations based upon filtered data are reduced to about 96%, indicating that almost all mixing at the level was due to inertial motions.

The implications for the mixing in the Nepheloid layer can be estimated by scaling arguments. Since the eddy viscosity has the dimension of (L^2/τ), the characteristic vertical length scale is given by

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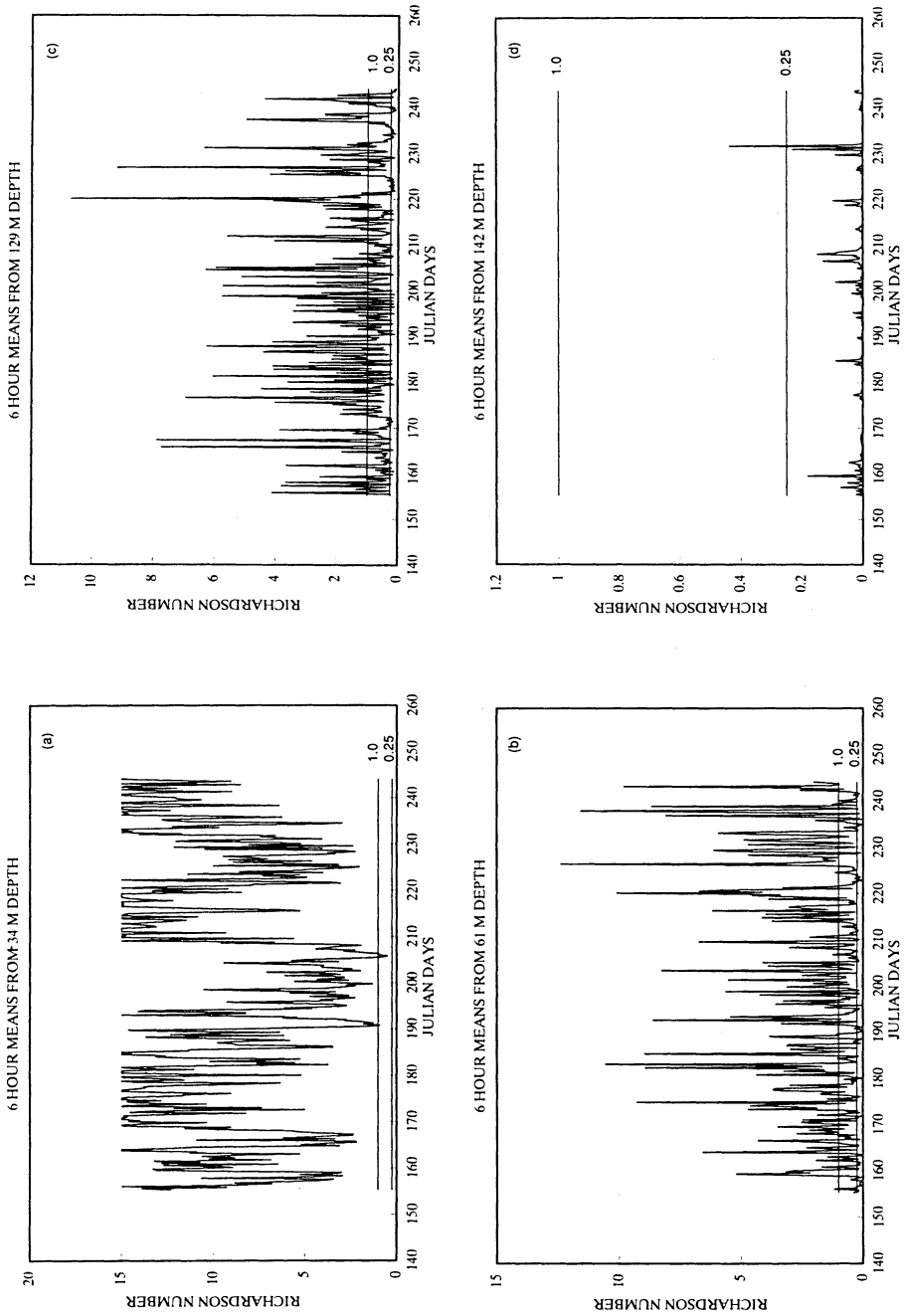


Fig. 10. Calculated Richardson numbers at different depths.

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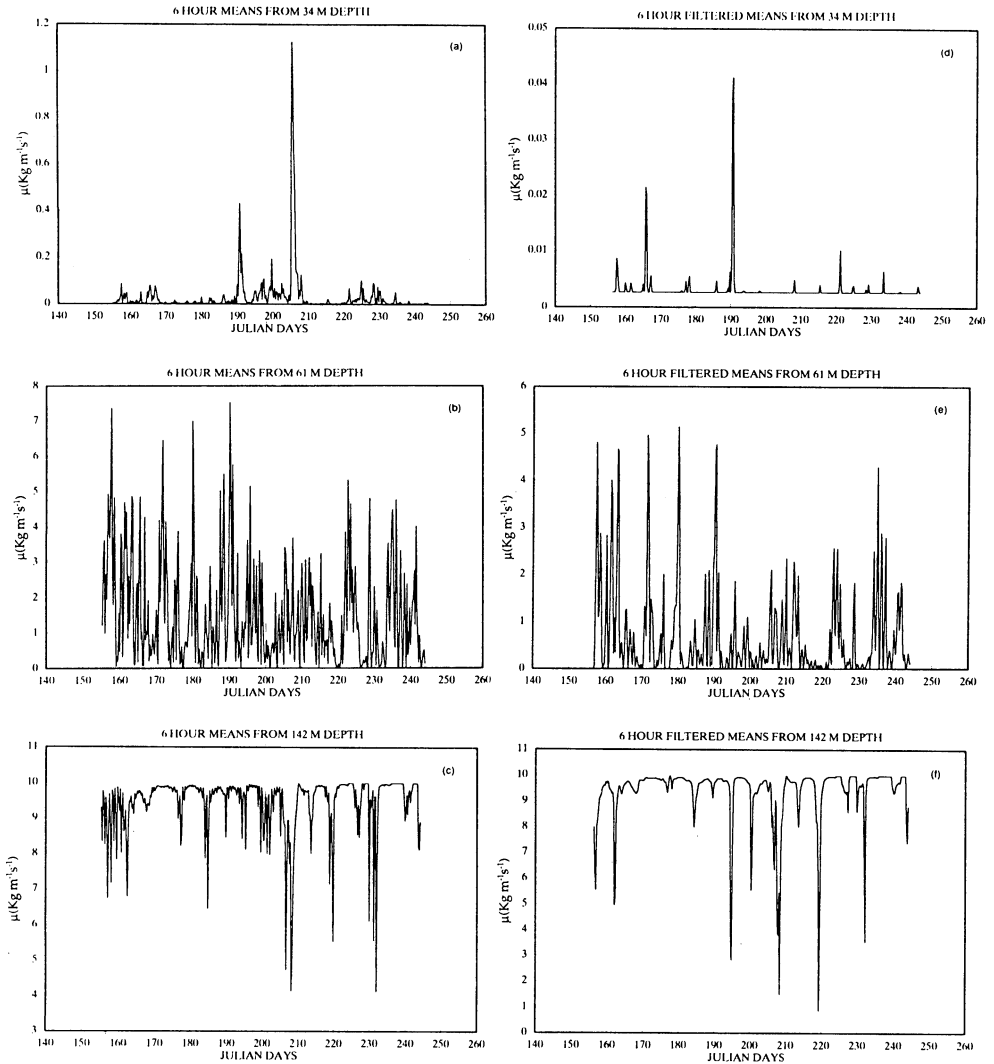


Fig. 11. Calculated dynamic eddy viscosities ($\mu = \nu_T \rho$) using filtered and unfiltered data. Note the different scales on the vertical axes.

$$L \sim \sqrt{\frac{\nu_T \tau}{\sigma_c}} \quad (7)$$

where τ is the characteristic time scale and σ_c a Schmidt number for suspended matter.

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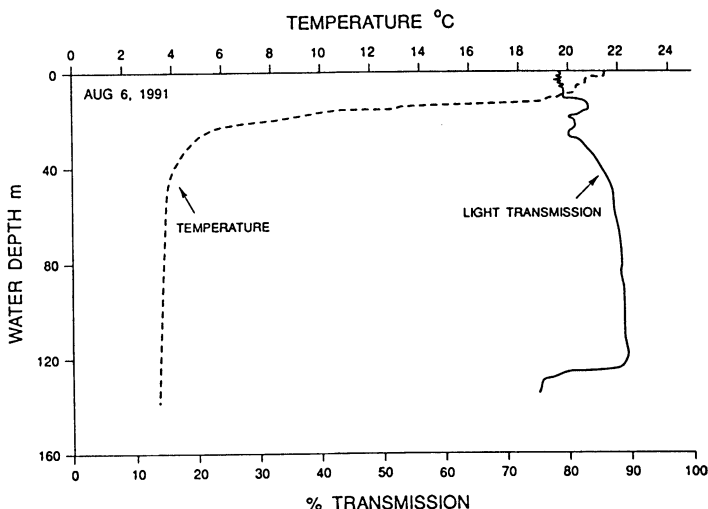


Fig. 12. Transmission and temperature profiles from August 1991.

In the hypolimnion, a typical value of the dynamical eddy viscosity ($\mu = \rho \nu_T$) was $3 \text{ (Kg m}^{-1} \text{ s}^{-1})$. For a mixing event of about 10 days and a Schmidt number in the order of 1, the estimated length scale becomes 50 m. This is larger than indicated by the transmission profiles during the same period (Fig. 12). However, from earlier studies already mentioned, it is well documented that the bottom Nepheloid layer in Lake Ontario increases up to 40-50 m until October. The calculations thus indicate that enough turbulence was available for mixing but probably the amount of suspended matter was not yet large enough to fill up a 50 m thick layer during the studied period. An underlying assumption is, of course, that the settling velocities of the suspended matter are small.

Summary and Conclusions

The vertical structure of currents and vertical mixing characteristics under summer conditions in Lake Ontario have been investigated on the basis of current meter and temperature data. The experimental set up consisted of seven Neil-Brown current meters distributed in one vertical from 12 m below the surface to 1 m above the bottom. The total depth at the measuring site was 143 m and the period considered was from June to September 1991. In addition to the current meter data, meteorological buoy data were used in the analysis. Also kinetic energy spectra characteristics were studied in the analysis together with calculations of the Richardson number and the vertical eddy viscosity.

The conclusions from the study may be summarized as:

- 1) The currents at all depths show pronounced transient characteristics with two kinetic energy peaks, one at a 17-hour and one at a 10-day period. The periodicity corresponded to inertial oscillation and meteorological forcing.
- 2) The kinetic energy peak due to inertial motions was increased in the surface layer, probably due to winds, and decreased in the bottom boundary layer, probably due to friction. The kinetic energy peak due to meteorological forcing was strong at all depths, indicating strong vertical shear in the bottom boundary layer.
- 3) Even though the temperature stratification showed weak stable gradients in the hypolimnion, current shear was strong enough to overcome stability and generate turbulence (Richardson numbers below 0.25). In the thermocline region the turbulence was damped (Richardson number above 1). Events with lower Richardson numbers in the thermocline were also calculated, indicating periods with shear generated turbulence in the thermocline.
- 4) The shear generated turbulence in the hypolimnion was mainly associated with the meteorologically forced currents. In the thermocline region on the other hand, the currents associated with inertial oscillation were the main reason for the turbulence.

The study indicates that the deep water mixing is due to turbulence even during summer time in Lake Ontario. The calculated Richardson numbers in the hypolimnion indicated that enough current shear was available to keep the matter (almost neutral buoyant particles) in the whole Nepheloid bottom layer in suspension. The present study was, however, only based on measurements from one location and had a rather low vertical resolution. It is therefore of great interest to analyse data with higher vertical resolution and also from other sites in Lake Ontario. Particularly it would be interesting to know whether the meteorologically forced currents have similar strong influence on the hypolimnion mixing, as the data from the present studied location indicate.

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