

## DISCUSSION

### B. F. Langer<sup>9</sup>

As is pointed out by the authors, considerable thought has been given by many investigators to the question of the relationship between the theoretical stress concentration factor and the actual fatigue strength reduction factor. Some promising approaches to this problem have been proposed, and the " $\delta$ -concept" used by the authors appears to be one of the best. Since all approaches are at least partly empirical, the final evaluation will have to depend on experimental verification. So far this experimental verification has been somewhat haphazard, due largely to the lack of a framework on which to base a program which will cover the required range of the parameters. One of the chief values of the present paper is that it provides this framework.

The curves in this paper provide the best information now available for estimating the actual reduction in fatigue strength which will be produced by a very sharp notch or crack-like defect in a cyclically stressed member. It is to be hoped that this paper will also be used as a guide for the formulation of experimental programs to validate the concept on which it is based.

### L. U. Rastrelli<sup>10</sup>

The authors are to be commended on undertaking a discussion of a complex and oftentimes ambiguous problem. It is difficult to convey in any one paper the many diverse facets associated with the phenomenological effect of cracks on the fatigue strength of materials. The authors begin by wisely adopting a fundamental concept, and on this basis, and with the help of such experimental data as are available, proceed to develop a reasonable and convincing rational argument.

On the other hand, we cannot agree with basic concepts which neglect the plastic region adjacent to the crack and the associated deformation-stress history that evolves with each load cycle. In particular, the statement in the middle of p. 206 "... average strain in yielded area is equal to the average strain calculated in an elastic basis. . .," in our opinion should read, "The average strain in the yielded area may be *many times* the average elastic strain." The discussion preceding equation (1) seems to say that the value of  $K$  is an *elastic* stress concentration factor entailing the calculated stress at a point  $\delta$  which has already been defined as a fictitious value.

These and other statements clearly point out the weakness of approaching the problem from the elastic viewpoint. It is much more natural to take plasticity into account, thereby enabling one to consider the possibility of locked-in stresses during each cyclic event.

### Authors' Closure

In the present state of the art in fatigue design analysis, the elastic stress distribution is used as a basis for estimating fatigue notch factors. The authors have used this approach to evaluate theoretical fatigue notch factors for cracks.

Mr. Rastrelli points out the need to include the effects of yielding when calculating the strains in the vicinity of the crack, even though these strains are controlled by the surrounding elastic material. The authors agree that this represents the next step in the continuing improvement of the general fatigue design method not only for cracks but for all notches.

However, the authors do not agree that "The average strain in the yielded area may be many times the average elastic strain." Safe design practice requires that nominal stresses be kept below yield in order to prevent gross deformations. For example, most design criteria limit nominal tension to  $2/3$  of the

yield strength of the material, and the large strain concentrations which accompany gross yielding are not found when these nominal stress limits are met. Structures designed to safe allowable nominal stress limits tend to "shakedown" to elastic behavior after a small number of cycles due to the residual stresses developed during cycling and to the important difference between cyclic stress-strain properties and conventional monotonic tensile properties.

Dr. Stowell<sup>11</sup> calculated the strain concentration at a hole in an infinitely wide aluminum alloy plate subjected to monotonic tension. His results show that at a nominal stress equal to  $2/3$  of yield, the peak strain is 1.5 times the value calculated assuming elastic behavior. Measurements by Griffith<sup>12</sup> indicate that the strain is actually closer to 1.75 times the elastic value at this load. The effect of yielding is, of course, smaller for lower loads, there being no effect for a nominal stress less than  $1/3$  of the yield strength. Peterson<sup>13</sup> presents some experimental results obtained by Johnson in a monotonic tension test on an aluminum alloy plate containing a hole with a diameter equal to  $1/6$  of the plate width. The measured strain at a nominal stress equal to  $2/3$  yield was 30 percent higher than the elastically calculated value. Since all of these results were obtained for monotonic loads, they do not include the effects of residual stresses created during each cycle on the subsequent cycles, nor do they account for the fact that aluminum alloys strain harden when cycled.<sup>14</sup> These factors tend to reduce the significance of yielding still further.

Peterson<sup>13</sup> has proposed an analytical method for evaluating the strain concentration that occurs during cycling using the cyclic stress-strain properties of the material.<sup>15</sup> The authors have used this method to calculate the strain concentration in SAE 4130 normalized steel for notches with  $K_T$ -values from 3 to 14. Strain concentration was found to be from 11 to 56 percent higher than the elastically calculated value as nominal stress varied from 25 to 100 percent of yield strength.

Blatherwick and Olson<sup>16</sup> present some experimental results of plastic behavior under cyclic conditions. Constant load-amplitude tests were run on flat specimens of low carbon steel containing  $K_T = 1.85$  edge notches. When the nominal cyclic stress was 70 percent of yield, the measured strain at the surface of the notch was 20 percent higher than the value calculated assuming elastic behavior.

The authors believe that the results described indicate the magnitude of the maximum error that could be introduced by neglecting the effects of local yielding. Recall that the  $\delta$ -values used in the analysis were obtained by correlating the theory with experimental fatigue data. Hence, the error introduced by neglecting the effects of yielding tends to make the theoretical fatigue notch factors given in the paper somewhat conservative in the high-cycle regime and unconservative in the low-cycle regime.

The authors are in complete agreement with Mr. Langer's remarks and are aware of an extensive test program formulated from their paper. The preliminary experimental results are very encouraging.

The authors would also like to thank each of the discussers.

<sup>11</sup> E. Z. Stowell, "Stress and Strain Concentration at a Circular Hole in an Infinite Plate," NACA Tech Note 2073, 1950.

<sup>12</sup> George E. Griffith, "Experimental Investigation of the Effects of Plastic Flow in a Tension Panel With a Circular Hole," NACA Tech Note 1705, 1948.

<sup>13</sup> R. E. Peterson, "Fatigue of Metals in Engineering & Design," Edgar Marburg Lecture, 1962, American Society for Testing & Materials.

<sup>14</sup> L. F. Coffin, Jr., and J. F. Tavernelli, "The Cyclic Straining and Fatigue of Metals," *Trans. Metallurgical Society of AIME*, vol. 215, 1959.

<sup>15</sup> This approach appears to neglect residual stresses introduced by cycling on the behavior during subsequent cycles.

<sup>16</sup> A. A. Blatherwick and B. K. Olson, "Stress Redistribution in Notched Specimens Under Cyclic Stress," ASD Tech Report 61-451, 1961. Published by Wright-Patterson Air Force Base, Dayton, Ohio.

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