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Renewal Theory Criteria of Evaluation of Water-Resource Systems: Reliability and Resilience

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The alternative concepts of reliability and resilience criteria of evaluation of water-resource systems are reviewed. The potential of the renewal theory is illustrated that covers many alternative criteria and suggests relations between them.

Introduction

Water-resource systems serve increasingly demanding and risk averse societies. Therefore, in addition to the traditional measures of quality of performance of water-resource systems like mean or variance of benefits or some operational variable, other criteria are also needed, that quantify various aspects of reliability and risk – recurrence, duration, severity and consequences of nonsatisfactory system performance.

In general, the term – satisfactory system performance means combination of several attributes. All variables of importance should remain within the permissible limits. If there exists at least one variable whose parameters are out of the permissible range, the system performs nonsatisfactorily. There is typically some degree of arbitrariness and subjectivity in assigning the permissible range.

Satisfactory system performance in the time instant t means

$$x(t) \in S_t$$

where

$x(t)$ – performance of the system in the time instant t; in general $x(t)$ can be a vector,

S_t – set of states of satisfactory performance of the system in the time instant t.

Similarly, satisfactory system performance in a time interval $(0,T)$ means that

$$x(t) \in S_t \quad \text{for each} \quad t \in (0,T)$$

The natural question arises – what is the meaning of x and S in the above formulation.

There may be single variable deciding upon the qualification of the system performance. This case of a scalar $x(t)$ will be considered in the present paper.

The relations (1) and (2) can be also expressed by the following inequality

load < resistance

introduced in mechanical engineering (Freudenthal 1961). The notion of load and resistance may be fairly broad. It may pertain to the structural load and resistance (*e.g.* of a levee, dam) as for example:

$h < h_M$ – water level should be lower than the top of levees. This notion may be also generalized to abstract load and resistance with relevance to the system target, rather than the structural load and resistance, as exemplified by the following inequalities:

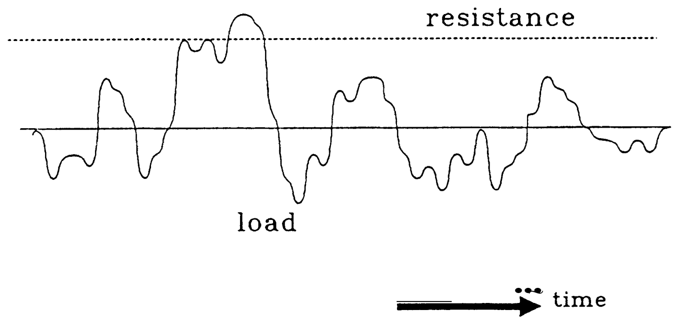
$Q < Q_M$ – flow rate should be less than some maximum flow rate,

$c_i < c_{iM}$ – concentration of pollutants (BOD, chemicals, nutrients etc.) should be less than some permissible limit,

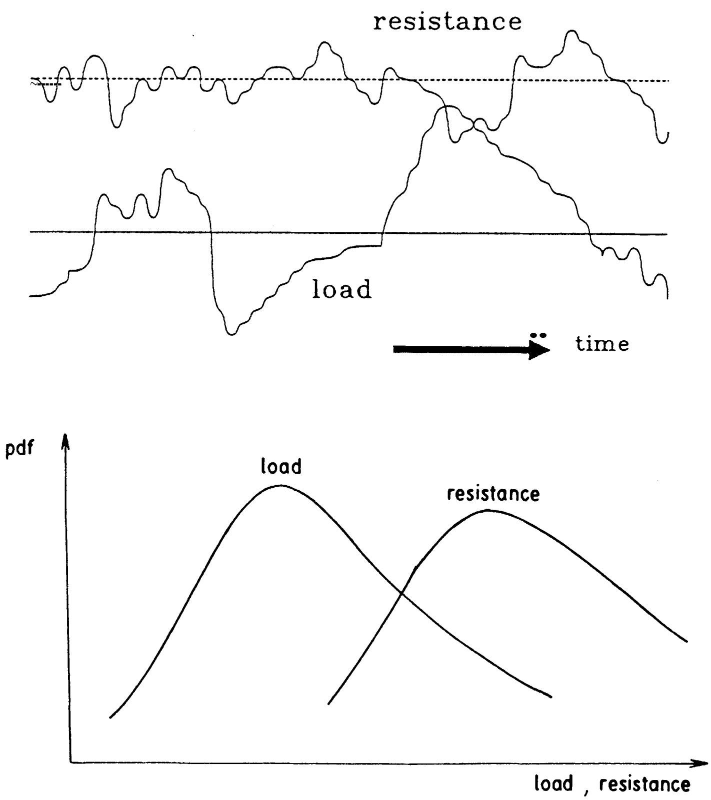
$v_d < v_s$ – water supply should not be lower than water demand.

Illustrations of load-resistance analogies in different situations (deterministic, random, stationary and nonstationary) are given in Fig. 1. Fig. 1a shows deterministic constant resistance and random stationary load fluctuating around the mean level. This can, for instance, correspond to the maximum permissible concentration of a pollutant (resistance) given as a rigid standard and fluctuating actual concentration of a pollutant (load). Both resistance and load shown in Fig. 1b (time series, *pdf*'s) are random and stationary. In this case the resistance can stand for the effective height of levees and the load for the water stage. Random load and resistance are depicted also in Fig. 1c, where the latter variable is not stationary. Water-resource illustration of this latter case is – reservoir volume (resistance) decreasing with time due to silting. The stationary load is the amount of water to be stored. The most complex case of both load and resistance being random and nonstationary is exemplified in Fig. 1d. Growing demand (load) accompanies decreasing supply (resistance). Therefore the *pdf*'s of both variables change with time. The masses of these *pdf*'s come closer to each other and the overlapping part grows.

Consider now a Boolean indicator variable corresponding to load and resistance

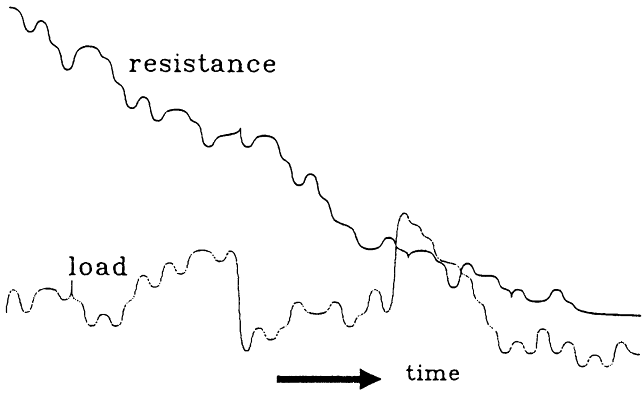


a) deterministic resistance (*e.g.* mandatory regulations) and stationary random load.

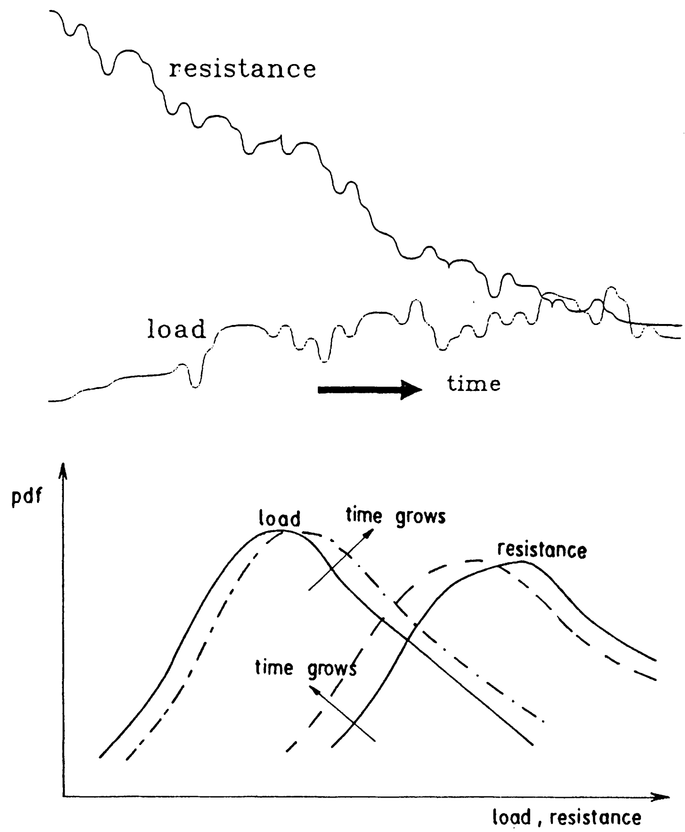


b) Load and resistance – stationary random processes.

Fig. 1. Load-resistance analogies.



c) Stationary random load – nonstationary random resistance.



d) Load and resistance – nonstationary random processes.

Fig. 1. Load-resistance analogies.

given in Fig. 2a and taking value of zero and one for satisfactory and non-satisfactory system performance, respectively (Fig. 2b see p. 222)

$$Z(t) \equiv \begin{cases} 0 & \text{for} \quad l(t) \leq r(t), \ \text{i.e.} \quad x(t) \in S_t \\ 1 & \text{for} \quad l(t) > r(t), \ \text{i.e.} \quad x(t) \notin S_t \end{cases} \tag{1}$$

where l and r denote load and resistance, respectively.

Based on the indicator variable one can evaluate different measures of risk and reliability. The measures analyzed in the present study are: reliability, *i.e.* probability of satisfactory system performance and resilience (or resiliency), *i.e.* system ability to return from the nonsatisfactory to the satisfactory state. The precise definition of the above criteria will be given in the sequel, following the discussion of many measures existing in the literature. The criteria that will not be considered herein are – risk understood as the complement to reliability, sometimes weighted by the consequences typically assessed in monetary terms, safety, *i.e.* a measure of the distance from the (instantaneous or mean) state of the system to the border between the satisfactory and nonsatisfactory states, vulnerability understood as the likely magnitude of the system entry to nonsatisfactory state and robustness conceived as a measure of insensitivity to changes of the border between the satisfactory and the nonsatisfactory states.

Reliability and Resilience – Review of Concepts

In the mathematical theory of reliability the basic notions are:

- Reliability function $\bar{F}(t)$, *i.e.* probability that the system has not entered the nonsatisfactory state till the time instant t, and
- Availability function, $A(t)$, *i.e.* probability that the system acts satisfactorily in the time instant t.

The latter concept embraces also the situation where one or more cycles of system sojourn in the state of nonsatisfactory performance and in the state of satisfactory performance have occurred. After the last return to the state of satisfactory performance the system stays in this state till the time instant t.

The classical notion of reliability in technical applications means – probability of satisfactory system performance (that is resistance greater than load) for a single time instant or for a time interval of interest (typically – design period).This former interpretation follows, among others, Freudenthal (1961) and Plate (1984). The latter interpretation has been used by Duckstein and Plate (1985) under the name of mission reliability (with T – fixed mission duration). Yet another notion of reliability is the probability of satisfactory system performance within a single event (Kaczmarek 1984). In this case not only the event starting time but also its duration may be random variables.

Three definitions of reliability due to Kritskiy and Menkel (1952) used also by Klemes (1969) read:

- Occurrence reliability, *i.e.* ratio of number of years in which the system did not enter the nonsatisfactory state to the total number of years considered. Attention: Kritskiy and Menkel considered also complementary measures like ratio of number of N-year periods (N=2,3...), in which the system did not enter the nonsatisfactory state to the total number of N-year periods considered.
- Temporal reliability, *i.e.* duration of time of system sojourn in the satisfactory state divided by the total time period considered and
- Volumetric reliability (water supply interpretation) understood as the portion of volume of water supplied in a time period considered with the rate less than or equal to the demanded rate, divided by the demanded volume.

As demonstrated above, there are several concepts labelled with one term – reliability. Even more ambiguity accompanies the term resilience (resiliency). The interpretation of this term should represent the system's ability to recover, to come back from the state of nonsatisfactory system performance to the state of satisfactory system performance. A resilient system should be able to accommodate a surprise (similar notions – insensitivity, robustness).

Many alternative criteria of resilience were considered by Fiering (1982) as measures of system ability to recover:

- Residence time in the state of satisfactory performance.
- Expected (undiscounted) outcome.
- Steady state probability of being in the state of satisfactory performance.

Several resilience criteria suggested by Fiering (1982) pertain to classification of the satisfactory system performance into a number of states. They read:

- Mean first passage time from the state of satisfactory performance to the state of nonsatisfactory performance (eventually with normalized marginal probabilities).
- Mean passage time between successive entries to the state of nonsatisfactory performance.
- Vector of γ % passage times from some or all states of satisfactory performance to the state of nonsatisfactory performance.
- Weighted vector sum, or scalar, of γ % passage times.

Yet another definition of resiliency was given by Hashimoto *et al.* (1982) as the inverse of the expected value of the length of system sojourn in the state of nonsatisfactory system performance.

In order to operationally use the criteria of reliability and resilience in a storage reservoir design and evaluation of decision rules, Moy *et al.* (1986), and Kindler and Tyszewski (1989) introduced some modifications. Reliability was measured as an estimate of the relative frequency of system sojourn in the state of satisfactory performance. Resilience was measured as the maximum number of consecutive periods of sojourn of system performance (water shortage) that occurred prior to recovery. Using the above definitions one could consider reliability and resilience as objectives in a programming model. Kindler and Tyszewski (1989) call for assessment of empirical probability distributions of these index values.

Ordering the many different measures of reliability and resilience can be attained basing on the level excursion (crossing) theory or the renewal theory. The former methodology is well established in hydrology (*cf.* Nordin and Rosbjerg 1970, Gottschalk 1976, Rosbjerg 1977, Bras and Rodriquez-Iturbe 1985). In particular, excursions above or below a specific flow rate or stage in a river were subject to several analyses. The present paper is concerned with the latter methodology, *i.e.* renewal theory in the continuous time framework.

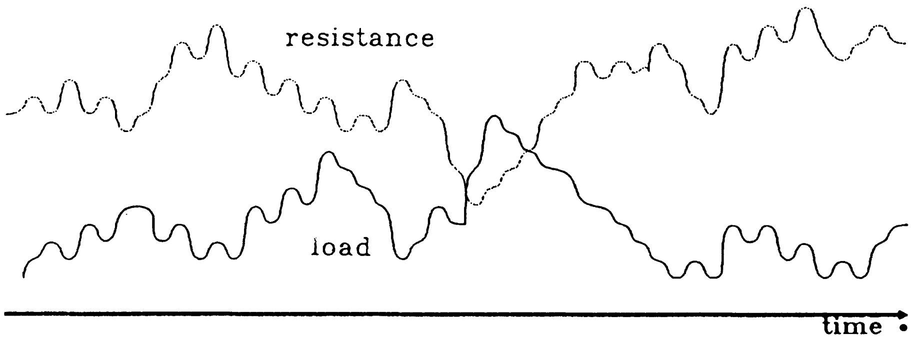
Renewal Theory Framework

Renewal theory renders a convenient framework embracing most measures of reliability and resilience criteria reviewed above. The term – renewal means a comeback of the system from a state of nonsatisfactory performance to a state of satisfactory performance. This notion that has been originally applied for repair or replacement of elements can be also used in a more abstract sense. Renewal theory deals with non-renewable (*i.e.* non-repairable) systems, immediately renewable systems (*i.e.* with zero renewal times) and renewable systems with non-zero renewal times. This last category is most interesting in water resources applications. Let us consider a renewable system whose average sojourn in the state of nonsatisfactory performace lasts longer than zero. An example of performance of such a system whose loads and resistances are illustrated in Fig. 2a is shown in Fig. 2b via an indicator variable. The situation can be also described by a renewal stream given in Fig. 2c. The notation used reads:

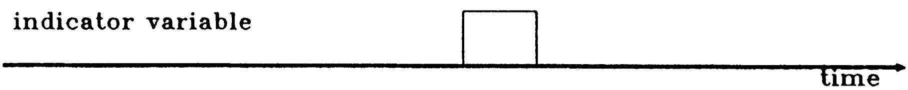
- τ'_i – Period of i-th system sojourn in the state of satisfactory system performance,
- τ''_i – period of i-th system sojourn in the state of nonsatisfactory system performance,
- t'_i – time instant of i-th system passage from the state of satisfactory system performance to the state of nonsatisfactory system performance,
- t''_i – time instant of i-th system passage from the state of nonsatisfactory system performance to the state of satisfactory system performance.

The assumptions made on such a renewal stream read:

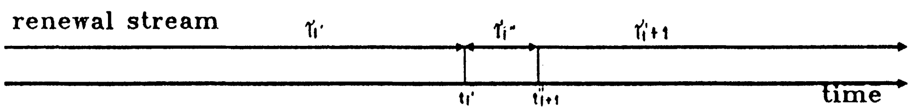
– All variables τ'_i and τ''_i are independent.
– All variables τ'_i are identically distributed



a) Resistance and load as functions of time.



b) Boolean indicator variable attaining zero and one for satisfactory and nonsatisfactory system performance, respectively.



c) Renewal stream.

Fig. 2. Time series of load, resistance and indicator variable, and renewal stream.

$$P(\tau_i' < t) \equiv F(t) \tag{2}$$

$$f(t) \equiv \frac{dF(t)}{dt} \tag{3}$$

$$E(\tau_i') = T_S \tag{4}$$

$$D^2(\tau_i') = \sigma_S^2 \tag{5}$$

– All variables τ_j'' are identically distributed

$$P(\tau_i'' < t) \equiv G(t) \tag{6}$$

$$g(t) \equiv \frac{dG(t)}{dt} \tag{7}$$

$$E(\tau_i'') = T_{NS} \tag{8}$$

$$D^2(\tau_i'') = \sigma_{NS}^2 \tag{9}$$

The following characteristics originating from the renewal theory are important in the assessment of various aspects of reliability:

- Mean time of system sojourn in state of nonsatisfactory system performance (reciprocal of resilience after Hashimoto *et al.* 1982).
- Mean time of system sojourn in state of satisfactory system performance.
- Distribution of *n*-th renewal.

Due to the assumptions taken on distributions of τ' and τ'', the renewal cycle durations

$$\tau_i \equiv \tau'_i + \tau''_i \tag{10}$$

are identically distributed, *i.e.*

$$\{\tau_i \quad i \equiv 1,2,\ldots\}$$

is a renewal process with instantaneous renewals, described by the following *p.d.f.* in Laplace complex domain

$$\bar{\phi}(s) = \bar{f}(s)\ \bar{g}(s) \tag{11}$$

where s is the complex variable and dashed superscipt denotes a Laplace transform, and, in the temporal domain

$$\phi(t) = f(t) * g(t) \tag{12}$$

where * denotes the convolution operation.

The variable φ allows one to determine the distribution of the *n*-th renewal

$$\Phi_{n+1}(t) = \int_0^t \Phi_n(t-x)\ \Phi(x)\ dx \tag{13}$$

$$\Phi_1(t) \equiv \Phi(t) \equiv \int_0^t \Phi(x)\ dx \tag{14}$$

- number $N(t)$ of renewal cycles in the interval $(0,t)$

$$P[N(t) \equiv n] \equiv P[t_n \le t \le t_{n+1}] \equiv \Phi_n(t) - \Phi_{n+1}(t) \tag{15}$$

- renewal function, *i.e.* expected value of number of renewals in the time interval $(0,t)$

$$M(t) \equiv E[N(t)] \equiv \sum_{n=1}^{\infty} \Phi_n(t) \tag{16}$$

- renewal density

$$m(t) \equiv \frac{dM(t)}{dt} \equiv \sum_{n=1}^{\infty} \Phi_n(t) \tag{17}$$

$$m(t) \equiv \Phi(t) + \int_0^t m(t-x)\ \Phi(x)\ dx \tag{18}$$

- cumulative time of system sojourn in the state of satisfactory performance

$$TS(T) \equiv \int_0^t Z(x)\, dx \tag{19}$$

where Z is the indicator variable defined in Eq. (1).

- time instant, when the cumulative time of sojourn in state of satisfactory system performance reaches x

$$\eta(x) \equiv \sup\{t: S_t \leq x\} \tag{20}$$

$$P\{S(t) \leq x\} = P\{\eta(x) \geq t\} \tag{21}$$

- instantaneous availability (most frequent understanding of the term reliability)

$$A(t) \equiv P\{X(t) \in S_t\} \tag{22}$$

$$A(t) \equiv \bar{F}(t) + \int_0^t \bar{F}(t-x)\, m(x)\, dx \tag{23}$$

- stationary (limiting) availability, that reads after manipulations in the complex domain of Laplace transforms

$$A(\infty) \equiv \lim_{t \Rightarrow \infty} A(t) \equiv \lim_{s \Rightarrow 0} s\, \bar{A}(s) \tag{24}$$

$$A(\infty) \simeq \lim_{s \Rightarrow 0} \frac{1-(1-s/T_S)}{1-(1-s/T_S)(1-s/T_{NS})} = \frac{1/T_S}{1/T_S+1/T_{NS}} = \frac{T_{NS}}{T_S+T_{NS}} \tag{25}$$

The above characteristics can be evaluated after having assumed the distributions of τ' and τ". If both distributions are exponential, it is very easy to derive the appropriate results. This case corresponding to the concept of Markov chain will be considered in the following section. However, it is also possible, though at the expense of more cumbersome algebra, to analytically derive the equations for the above characteristic for any distribution that may better fit the data in hand.

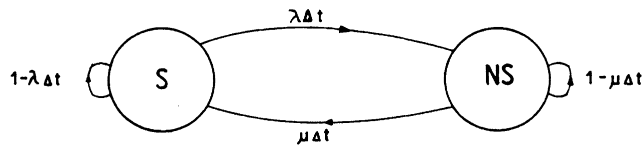
Markov Chains

A special case of renewal process with non-zero renewal time dwells on the concept of Markov chains, fulfilling the definition equation

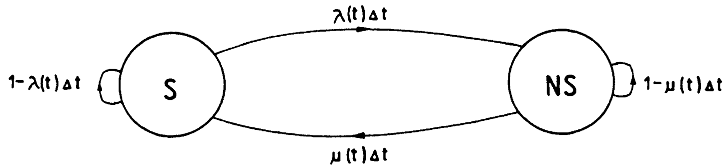
$$P[X(t+h) = j \mid X(t_1) = i_1'', X(t_2) = i_2'', \ldots, X(t) = i] =$$
$$= P[X(t+h) \equiv j \mid X(t) \equiv i] \tag{26}$$

where

$$t+h > t > \ldots > t_2 > t_1$$



a) Stationary case.



b) Nonstationary case.

Fig. 3. Two-state Markov chain.

Consider the stationary case, in which

$$P[X(t+h) = j \mid X(t) = i] = P[X(h) = j \mid X(0) = i] \tag{27}$$

It is clear that for a Markov chain the time that the system will spend in the state i is memory-less, *i.e.* does not depend on how much time the system has already spent in this state

$$P[\tau_i \le t+r \mid \tau_i \ge t] = P[\tau_i \le r] \tag{28}$$

This condition is met if and only if the distributions of the system sojourns in particular states are exponential. That is

$$f(\tau') = \lambda \exp(-\lambda\tau') \tag{29}$$

$$g(\tau'') = \mu \exp(-\mu\tau'') \tag{30}$$

The probabilities of change of state (transition probabilities) in a small time period Δt are illustrated in Fig. 3a. The mathematical description of this situation reads

$$p_{NS}(t+\Delta t) = p_S(t)\lambda\,\Delta t + p_{NS}(t)\,[1-\mu\,\Delta t] \tag{31}$$

After elementary manipulations and with $\Delta t \Rightarrow 0$ one gets

$$\frac{dp_{NS}(t)}{dt} + p_{NS}(t)[\lambda+\mu] = \lambda \tag{32}$$

with the solution

$$p_{NS}(t) = \exp[-(\lambda+\mu)t]\int_0^t \exp[(\lambda+\mu)t]\lambda\,dt = \frac{\lambda}{\lambda+\mu}\{1-\exp[-(\lambda+\mu)t]\} \tag{33}$$

For the case considered it is easy to arrive at various characteristics of interest via the Laplace transform techniques

$$\bar{f}(s) \equiv \frac{\lambda}{s+\lambda} \quad (34)$$

$$\bar{g}(s) = \frac{\mu}{s+\mu} \quad (35)$$

$$\bar{m}(s) = \frac{\lambda\mu}{s(s+\lambda+\mu)} = \frac{\lambda\mu}{(\lambda+\mu)s} - \frac{\lambda\mu}{(\lambda+\mu)^2}\frac{\lambda+\mu}{s+\lambda+\mu} \quad (36)$$

$$\bar{A}(s) = \frac{s+\mu}{s(s+\lambda+\mu)} = \frac{\mu/(\lambda+\mu)}{s} + \frac{\lambda/(\lambda+\mu)}{s+\lambda+\mu} \quad (37)$$

Back to the temporal domain

$$m(t) \equiv \frac{\lambda\mu}{\lambda+\mu} - \frac{\lambda\mu}{\lambda+\mu}\exp[-(\lambda+\mu)t] \quad (38)$$

$$\lim m(t) \equiv \frac{\lambda\mu}{\lambda+\mu} \equiv \frac{1}{1/\mu+1/\lambda} \equiv \frac{1}{T_{NS}+T_S} \quad (39)$$

$$A(T) \Rightarrow \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}\exp[-(\lambda+\mu)T] \quad (40)$$

$$\lim_{t\Rightarrow\infty} A(t) \equiv \frac{\mu}{\lambda+\mu} \quad (41)$$

The above general Markov chain formulation (discrete state, continuous time) embraces also special cases of:

– non-renewable system (death process with absorbing state), for which

$$\mu = 0$$

i.e. intensity of transition from nonsatisfactory state of system performance to satisfactory state of system performance is equal to zero, probability of such transition in any time interval is equal to zero and the expected duration of system sojourn in nonsatisfactory state, once this has occurred, is infinite,

– immediately renewable system

$$\mu = \infty$$

i.e. conditional intensity of transition from nonsatisfactory state of system performance to satisfactory state of system performance is infinite, conditional probability of such transition in any time interval is equal to one and the duration of system sojourn in nonsatisfactory state, once this has occurred, is equal to zero.

It may be problematic, whether the parameters λ and μ can be considered constant in time, what is necessary in order to justify the assumption of stationary exponential distributions, of the sojourns in particular states. One can find proposals of variable intensity in the literature on non-renewable systems (structural failure of a dam). Baecher *et al.* (1980) suggested that the intensity λ for a dam could be considered as a two-valued function $\lambda=10^{-3}$ for $t<5$ years and $\lambda=5\times10^{-5}$ afterwards. The proposal of Plate (1984) is the bath-tub function analogous to the curve resulting from mortality studies with distinct areas of increased juvenile and senile hazards.

Also for renewable systems it is likely that the number of entries to the state of nonsatisfactory system performance and the »depth« of these entries grow with time. That is, the load grows and the resistance goes down, so does also the safety in the sense of expected distance from the border between satisfactory and nonsatisfactory states of system performance.

Therefore it seems necessary to consider a nonstationary Markov chain shown in Fig. 3b and described by the following equation

$$p_{NS}(t+\Delta t) = p_S(t)\lambda(t)\Delta t + p_{NS}(t)[1-\mu(t)]\Delta t \tag{42}$$

and – for $\Delta t \Rightarrow 0$

$$\frac{dp_{NS}(t)}{dt} + p_{NS}(t)[\lambda(t) + \mu(t)] = \lambda(t) \tag{43}$$

with the solution (Ang and Tang 1984)

$$p_{NS}(t) = \exp\{-\int_0^t [\lambda(t)+\mu(t)]dt\} \int_0^t \exp\{\int_0^t [\lambda(t)+\mu(t)]dt\}\lambda(t)dt + c\exp\{-\int_0^t [\lambda(t)+\mu(t)]dt\} \tag{44}$$

If the system was in the state of satisfactory system performance in the time instant 0, than $p_{NS}(0) = 0$ and

$$p_{NS}(t) = \exp\{-\int_0^t [\lambda(t)+\mu(t)]dt\} \int_0^t \exp\{\int_0^t [\lambda(t)+\mu(t)]dt\}\lambda(t)dt \tag{45}$$

Consideration of one satisfactory and one nonsatisfactory state only may not suffice for proper characterization of the system performance. Therefore Markovian framework with more states can be used.

Conclusions and Remarks on Further Study

Numerous characteristics describing reliability and resilience of water-resource systems that were typically treated independently can be embraced by the renewal theory or the excursion (level crossing) theory. Within this framework one can determine relations between measures of different aspects of reliability and resilience. When particular distributions are assumed, analytical formulae expressing alternative measures can be developed. For instance, under the assumption of exponential distribution of times of sojourn in the states of satisfactory and nonsatisfactory system performance, all the criteria considered can be assessed with the help of two parameters (one for each exponential distribution).

One can easily see the analogy between the indices of reliability and resilience in water-resource systems and time series characteristics well known in hydrology in a number of contexts and frameworks (renewal theory, point processes, level crossings, excursions, runs, partial duration series). There are many two-valued aspects of hydrological variables, analogous to the dichotomy of satisfactory/nonsatisfactory system performance. Therefore similar concepts are used for description of floods, droughts, and precipitation events. However, it is the discrete formulation rather than the continuous one, that dominates in hydrology. The load and resistance variables, and also hydrological processes, are in fact continuous. There are several disadvantages of the discrete framework that disappear in the continuous approach. Within discrete framework one cannot directly transpose the results attained for one Δt_1 to another Δt_2. Markovian formulation assumes that no more than one transition can occur within one time interval. Overlapping of events of finite duration into several intervals needs be considered. All these deficiencies of the discrete approach do not exist in the continuous framework. The continuous formulation offers advantages of simplicity, generality, elegance and well developed theory.

In the present stage of research two case studies are being analyzed, pertaining to water supply systems in Poland. Hydrological records (flows in rivers) and projections of water demands are used to generate resistance and load variables. The indicator variables are analyzed for several conditions on resistance (without reservoir, with reservoir – in several variants) and on demand (several scenarios of demand growth, resulting from different forecasts of population growth, regional development and water consumption standards). It is foreseen that sensitivity studies will be performed (for departures from the projected trajectories).

Based on the above resistance-load analogies, studies of reliability and resilience are being performed, using renewal theory (Markovian methodology in stationary and nonstationary cases) and level crossings (excursions) theory.

It should be stressed, however, that reliability and resilience measures do not exhaust the set of important performance indices of water-resource systems. In fact, indices of vulnerability (measuring amplitude, *i.e.*, severity of system entry into the nonsatisfactory state) and robustness (measuring system sensitivity to the location of the border between the nonsatisfactory and the satisfactory system performance) are of great importance (*cf.* Hashimoto *et al.* 1982). Therefore the complete problem formulation requires a multi-objective framework.

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