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Table 1 Surface parameters for magnetic disk (Ganti and Bhushan, 1995)

Scan size (μm × μm)	σ (nm)	D	C (nm)
1 (AFM)	0.7	1.33	9.77e-4
10 (AFM)	2.1	1.31	7.59e-3
50 (AFM)	4.8	1.26	1.74e-2
100 (AFM)	5.6	1.30	1.38e-2
50 (NOP)	3.5	1.27	1.58e-2
250 (NOP)	2.4	1.32	2.75e-4
4000 (NOP)	3.7	1.29	7.89e-5

AFM–Atomic force microscope
NOP–Noncontact optical profiler

the fractal dimension D and the vertical intercept corresponds to the G parameter. These lines are fairly constant (not precisely) but G is a strong function of the measuring instrument and the scan sizes for a given instrument.

Recently we have developed a generalized fractal analysis for a fractional Brownian motion (fBm), an example of a nonstationary process with stationary increments (NSPSI) which has been shown to be fractal and all fractal analyses for characterization of surface roughness (including M-B model) assume NSPSI. The new structure function exhibits the power law behavior which is similar to that in the M-B model except that the new function includes a "measuring length unit." The lateral resolution of the instrument is used as a measuring length unit (similar to the length unit in the calculation of fractal dimension of surfaces using the length-number relation proposed by Mandelbrot) in the analysis. The structure and power functions with lateral resolution taken into account are given by (G-B model)

$$S(\tau) = C\eta^{(2D-3)}\tau^{(4-2D)} \tag{1}$$

$$P(\omega) = \frac{c_1\eta^{(2D-3)}}{\omega^{5-2D}} \tag{2}$$

where

$$c_1 = \frac{1}{\int_0^\infty (1-\cos x)x^{-(5-2D)}dx}C$$

$$= \frac{\Gamma(5-2D)\sin(\pi(2-D))}{2\pi}C$$

D relates to the relative power of the frequency contents and C relates to the amplitude of all frequencies. The difference between this structure function and the one obtained from M-B model is that it takes into account the measuring length unit (in this case the lateral resolution of the measuring instrument) which accounts for the lateral shift observed in the structure functions of experimental data.

Calculated values of D and C for the disk data of Fig. 11, are presented in Table 1. We note that D is approximately independent whereas C still varies. Though we have made improvements in the fractal analysis, we still need a better measure of measuring length unit before one can obtain scale-independent parameters for the measured roughness.

I believe that until we develop a methodology to obtain scale-independent characterization of surface roughness, fractal analysis will remain mathematical curiosity with little practical value!

Additional Reference

Ganti, S., and Bhushan, B., 1995, "Generalized Fractal Analysis and Its Applications to Engineering Surfaces," *Wear*, Vol. 180, pp. 17–34.

Authors' Closure

The authors would like to thank Professor B. Bhushan for his discussion. The discusser raises the issue of the practical utility of surface characterization using the fractal theory, in view of experimental data demonstrating that the fractal parameter G depends on the measuring instrument and the scan size (Fig. 11), and proposes a fractal model in which the lateral resolution of the instrument is used as a measuring length unit to improve the estimation of the fractal parameters. However, Table 1 shows that the calculated values of the modified fractal roughness parameter, C, differ by one to two orders of magnitude, depending on the measuring instrument and scan size.

The present theory is based on the fractal characterization of the surface topography using the commonly accepted W-M function which contains the fractal parameters D and G (refer to Eq. (1)). For the measurement of a real surface, whether unique values of D and G can be obtained from surface height data does not raise any question about the validity of the W-M function as a fractal model, but rather presents a problem with the scheme used to obtain these parameters. Hence, schemes for obtaining fractal parameters from experimental data may require further improvement, and new schemes may be proposed with different sets of fractal parameters. Thus, it is not surprising that a given fractal surface can be characterized by different sets of fractal parameters. Given the diversity of parameter choices, the present analysis is still applicable to engineering surfaces, provided the relationship between the parameters of the W-M function and those used for a real surface is determined. It is shown below that such a relationship can always be determined for any parameter set.

For given values of the fractal parameters D and G, a surface profile can be defined using the W-M function given by Eq. (1). This simulated surface profile can be analyzed at any length scale, and, thus, can be "numerically measured" at any precision. A numerical measurement is defined as a process of taking N function values $z(x_i)$, where $i = 0, 1, 2, \ldots, N-1$, from the W-M function $z(x)$. If the sampling interval, $\Delta x = x_{i+1} - x_i$, is kept constant for each data set, the measured heights $z(x_i)$ will resemble the surface data acquired with an atomic force microscope or an optical profiler.

Since the W-M function is fractal, any valid scheme used to analyze the surface profile given by this function based on the digitized data points, $z(x_i)$, should yield consistent values of fractal parameters at all scales. If any fractal parameter set other than D and G (such as the set of D and C suggested by the discusser) is chosen for surface characterization, the relationship between this set of parameters and that used in the W-M function can be determined by performing a parametric study of various values of D and G. For each pair of D and G values, the W-M function can be used to define a fractal surface. Numerical measurements of this surface at different length scales using a proper scheme will yield corresponding values of the new fractal parameter set. Consequently, the present analysis for a given pair of D and G values can be applied to a surface characterized by the values of the corresponding set of fractal parameters.

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