A RISK-ANALYTIC APPROACH TO CONTROL OF LARGE-VOLUME OIL SPILLS

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ABSTRACT

The frequency of large-volume oil spills is considerably greater than is consistent with prediction based upon traditional methods. The reason for this phenomenon is that standard probability distributions of magnitude of spills do not have the flexibility to admit of very large coefficients of variation, especially for distributions which are highly skewed to the right. Hence, distributions which have large means relative to the median and which have long thick tails are prerequisites for an appropriate treatment of the problem. The class of stable laws provides a convenient method for investigating the empirical oil spill experience: several large spills dominate the total volume of spillage in virtually all accounting periods; e.g., quarterly. Our methodology involves a statistical assessment of "accident-proneness component"; if one exists, the data is further examined to identify insofar as is possible the genesis of the component (s); if none exists, we assess the frequency and severity of discharge for various geographic areas. A new approach has been utilized to fit these long, thick-tailed probability distribution to a U.S. Coast Guard data file on oil spills, the pollution incident reporting system (PIRS), with considerable success. We pay particular attention to the fitted upper tail vis-a-vis the actual upper tail. The agreement, where our methodology is deemed applicable, is very good. We also indicate improvements to methodology and applications.

INTRODUCTION

The extent to which the environment—as well as property, health, etc.—is damaged by oil spills depends jointly, *ceteris paribus*, on the frequency and the severity of the spills in a given area. Damages incurred may be due to cumulative effects of previous coupled with current discharges or to the effect of individual discharges. Individual spills of greatest interest are those of extremely large magnitude. Also of considerable interest is the aggregate (or total) spill over some accounting period; e.g., quarterly or yearly. It turns out, as we shall show, that individual spills of large magnitude can and do dominate the aggregate spill over an accounting period. Thus, in order to properly examine oil spill phenomena we must treat individual and aggregate spills in a hand-in-hand manner. The location of a spill is also relevant to a proper analysis. Thus, we shall also examine oil spills according to whether they occur in inland, coastal, or high-seas waters.

Whatever the setting, there is valuable information for management of cleanup operations contained in data which records spill incidents by time of occurrence and magnitude (or severity). An insurance company which insures homogeneous risks against loss accurately records both frequency and severity of claims against policies in order to assess the amount of resources it must keep in reserve to protect policyholders and the company against ruin. Similarly, the accurate recording of frequency and severity of oil spills such as is done by the U.S. Coast Guard affords control agencies an opportunity to assess the amount and location of resources which must be marshalled in spill situations to achieve the objectives of alternative control policies. We shall show that a risk-analytic approach provides a viable methodology for prediction and forecasting of both large individual spills and aggregate spills and thus the extent to which the appropriate agencies must be prepared to exercise effective control.

The frequency and severity of oil spills data which were used in this study were obtained from the United States Coast Guard data file on oil spills, the pollution incident reporting system (PIRS). Some editing and transcription was necessary to put the file into usable form.

The data

We have done more extensive analyses than are reported in this paper; for brevity we present results obtained from the analysis of data from District 8, New Orleans, and District 13, Seattle. The results are typical and thus indicative of the generality of the approach we employ.

The frequency of oil spills per month, District 8, is given in figure 1. There are apparent seasonal components in the 1971-1972 data, and there appears to be an upward trend from 1971 to 1972. The latter is due, however, to increased Coast Guard patrol activity which resulted in a larger number of reported spills and does not necessarily imply an increase in the frequency of the spills.

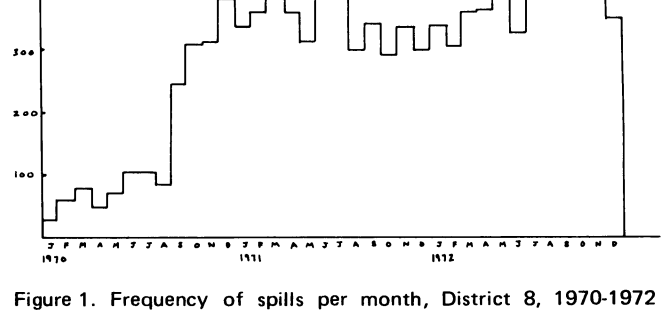

Figure 1. Frequency of spills per month, District 8, 1970-1972

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The 1970 data are of a start-up nature and will not be analyzed in detail. The spillage incidents are recorded sequentially in time by volume (in gallons) of spill and location (inland, coastal, high seas).

Since the spills are recorded sequentially in time, the interspill time intervals were computed for all the data for which the spill times were sufficiently finely recorded. If the spills occurred at random instants in time, the interspill times would be exponentially distributed with density

$$f(t) = \lambda e^{-\lambda t}$$

and mean interspill time $1/\lambda$. An exponential interspill time distribution is equivalent to a spill-number process:

$$P(n \text{ spills in a period of length } t) = \Pr(n, t)$$

$$= \left[\frac{(\lambda t)^n}{n!}\right] e^{-\lambda t}, \qquad (1)$$

where λ is the mean number of spills per unit time. Analysis of the District 8 and District 13 interspill data by the usual chi-square goodness-of-fit test shows that the spill numbers may be adequately regarded as being generated by the Poisson process expressed in (1). The physical interpretation of this is that statistical analysis fails to uncover an accident-proneness component in the data. Thus, without a further, more refined analysis of the data, we may reasonably suppose that the spill-number process follows (1). In this case the standard deviation associated with the random variable n is $\lambda^{1/2}$.

At this stage, we make the further assumption that each spill magnitude is mutually independent of every other spill. There will, of course, be exceptions, but these should not have any influence on the analysis or conclusions since the large number of independent spills will almost certainly swamp the effect of the nonindependent spills. The spill data have without exception the following characteristics: the sample means are large relative to the sample medians; the ratio of the sample standard deviations to the sample means (the coefficients of variation) are very large relative to unity; there are a preponderance of low-volume spills, but sufficiently many large-volume spills to render the empirical frequency distribution of spill volume radically skewed to the right with a markedly long, fat tail. Figure 2 provides a typical depiction of the phenomenon. Table 1 provides some summary statistics for Districts 8 and 13.

Table 1 together with figure 2 indicate that distributions which have long, fat tails; are extremely skew; and admit of very large

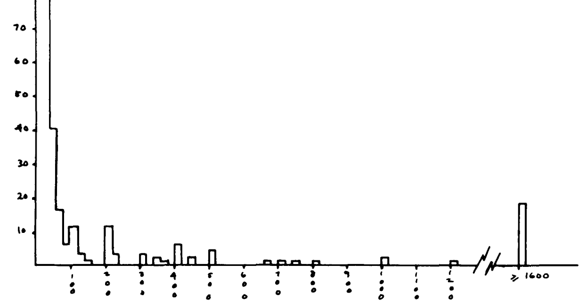

Figure 2. Frequency of spills of given magnitude, District 13, 1970-1972

Table 1. Number of spills n, mean spill m, standard deviations s, coefficient of variation CV, and maximum spill M for districts 8 and 13

District	Year	n	m	s	CV	M	Total
8	1971	3143	1087	36005	33.1	2,000,000	3417698
8	1972	3411	2602	91074	35.0	5,000,000	8876104
13	1971	136	2819	21540	7.6	220,000	383424
13	1972	346	28000	162966	5.8	9,600,000	9688450

coefficients of variation are prerequisites for the spill-volume distribution function

$$S(x) = P\,(\text{individual spill volume} \leqslant x).$$

The spill-volume density is $(d/dx)[S(x)] = s(x)$. It is at this point that we encounter a serious problem. Commonly used statistical densities are not appropriate for fitting data as exhibited in figure 2 [1]. One distribution which seems appropriate in the current context is the Pareto, with density

$$g(x) = \frac{\alpha}{\gamma}\left[1 + \frac{(x-\mu)}{\gamma}\right]^{-(\alpha+1)}, \qquad (2)$$

where $\alpha > 0$, $\gamma > 0$, $x > \mu$. The density function (2) has the additional property of admitting the possibility of extremely large spills. Unfortunately, the estimates of the parameters α, γ, and μ pose problems which have to date not been satisfactorily resolved. There is an additional problem posed by the use of (2) which is even more serious in its implications. Data of the sort represented by figure 2 are most likely not generated by a process which has a density expressible in a form so simple as (2). Spill-volume distributions for given districts, such as 8 or 13, may in some instances be represented by a "single distribution," but are more likely represented by mixtures of distributions due to the variety of causes and types of spills. We shall see that if we choose $S(x)$ to be a member of the class of stable laws, we can perform analyses of a risk-analytic nature which can be useful in the control of oil spills.

The Stable laws

The stable laws are defined in terms of their characteristic functions. We say that a random variable X is distributed according to a stable law with distribution function $S(x;\ \alpha,\ \beta,\ \gamma,\ \delta)$ is it has characteristic function [2]

$$\phi(u) = E[\exp(iuX)] = \exp\left\{iu\delta - \gamma|u|^{\alpha}\left[1 + i\beta\left(\frac{u}{|u|}\right)\tan\left(\frac{\pi\alpha}{2}\right)\right]\right\}, \qquad (3)$$

where $0 < \alpha \leqslant 2$, $|\beta| \leqslant 1$, $\gamma > 0$, and $-\infty < \delta < \infty$. The distributions defined by (3) are not expressible in terms of elementary functions except for the cases $\alpha = 2, 1, 1/2$. The case $\alpha = 2$ corresponds to the normal distribution; the case $\alpha = 1$ corresponds to the Cauchy distribution; the reciprocal of a χ^2_1 variate gives the case $\alpha = 1/2$. The parameter α is called the characteristic exponent of the distribution: it is a measure of peakedness and of thickness of the tails. As α decreases, the density $s(x;\ \alpha,\ \beta,\ \gamma,\ \delta)$ becomes more pronouncedly peaked and the tails of s become longer and fatter. The extent of the length and fatness of the tails is partially reflected in the fact that all absolute moments of order $\geqslant \alpha$ are infinite. The parameter β is an index of skewness. When $\beta < 0$ are density s is skewed to the right; when $\beta = 0$ the density s is symmetric about the location of parameter δ. The scale parameter γ determines the spread of the distribution. When $\alpha \neq 2$, one or both the tails of $s(x;\ \alpha,\ \beta,\ \gamma,\ \delta)$ satisfy asymptotically the Pareto distribution given in (2). Of even greater importance for our purposes is the stable laws' unique property of being the only possible limiting laws for sequences of independent, identically distributed, random variables—in the appropriate sense of being the complete generalization of the central limit theorem. If $X_1, X_2, ..., X_n$ constitute a random sample from $S(x; \alpha, \beta, \gamma, \delta)$ then

$$\sum_{j=1}^{n}$$

X_j has distribution $S(x; \alpha, \beta, n\gamma, n\delta)$. Furthermore, if $Y_1, Y_2, ..., Y_m$ constitute a random sample from the distribution function $F(y)$, where $F(y)$ is in turn a mixture given by

$$F(y) = \sum_{j=1}^{j} p_j F_j(y),$$

$p_j > 0$, $\Sigma p_j = 1$, then for m sufficiently large, the distribution of

$$\sum_{j=1}^{m}$$

Y_j is approximately stable [3]. It is this latter result which we shall make considerable use of. For the general appearance of stable densities see figures 3 and 4.

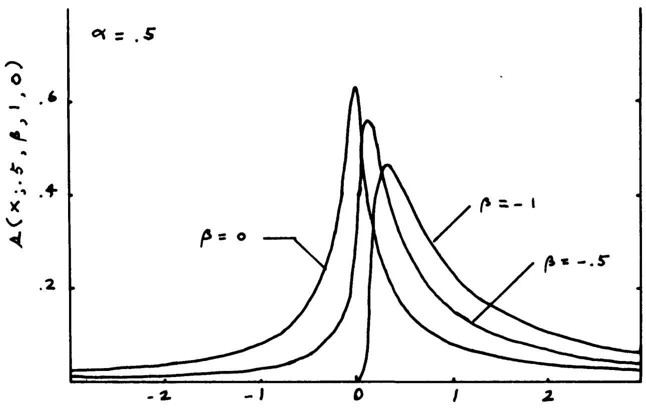

Figure 3. Stable density functions s

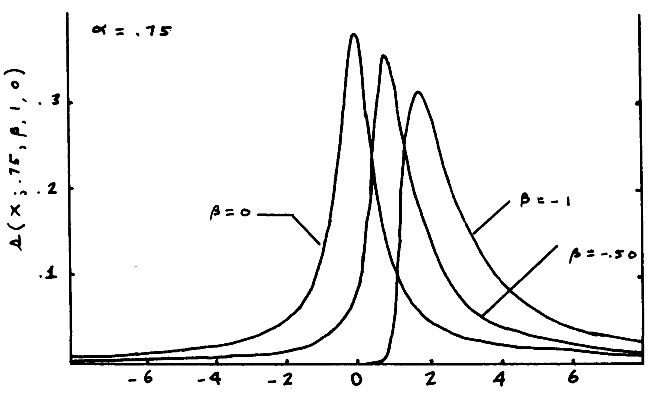

Figure 4. Stable density functions s

The methodology

We have seen that the number of spills in a given district and accounting period is a random variable. The magnitude of the oil spill is also a random variable. Thus all quantities of interest—magnitude of spills, frequency of spills, and total spill in an accounting period of length t—are governed by probability distributions and hence can exhibit considerable variability. The total spill in a period of length t is given by

$$T(t) = \sum_{j=1}^{N(t)} X_j$$

where $N(t)$ is the number of spills governed by (1) and X_j is the magnitude of the spill, which we will suppose to be governed by distribution function $V(x)$. By the usual risk analytic argument [4], $T(t)$ has distribution function

$$F_T(x, t) = \sum_{n=0}^{\infty} \Pr(n, t) V^{n*}(x), \quad (4)$$

where V^{n*} is the n-fold convolution of $V(x)$ with itself. We may, for example—and do initially—take $V(x)$ to be a member of the stable class as given by (3). Afterward, we make use of the "generalized" central limit theorem.

The estimation of the parameters α, β, γ, δ of (3) involves essentially the minimization of an integral of the form

$$\int_{-\infty}^{\infty} |\phi(u) - \hat{\phi}(u)|^2 e^{-u^2} du$$

with respect to α, β, γ, δ, where $\hat{\phi}(u)$ is the empirical characteristic function $\phi(u) = (1/n) \Sigma \exp iux_j$, and $x_1, ..., x_n$ constitute a random sample from (3) [5]. Our purpose at this juncture is to determine the applicability of (3) to oil spill data. Our measure will be the agreement between the empirical distribution function and the fitted distribution function. Figure 5 depicts the agreement between the empirical and the fitted distributions for District 8, inland water area, 1971. The fit is visibly bad. In fact, the agreement between fitted and empirical distributions for all the original data is not generally good, especially in the upper tail where it is imperative that the fit be reasonably good. As we have seen from table 1, the few largest observations dominate the average spill—and hence the total spill. There are a few interesting facets which may be gleaned from figure 5, however. First, the upper tail of the fitted distribution is too thin. This means that the resulting density would provide an insufficient number of spills of very large magnitude. Secondly, since $\alpha = .95$ the resulting density has infinite mean! The implication is that spills of fantastic magnitude can be expected to occur [6].

Rather than here conclude that the stable laws are inapplicable as a spill-volume distribution to data of this sort, we investigate the possibility, as noted above, that if the spill data is generated by a mixture of distributions, the nonoverlapping k-sums

$$(X_1 + X_2 + ... + X_k) + (X_{k+1} + ... + X_{2k}) + ...$$

might be sufficiently close to a member of the class $S(x; \alpha, \beta, \gamma, \delta)$ *a la* generalized central limit theorem. For k sufficiently large, each summand of the k-sum will have approximately an $S(x; \alpha, \beta, \gamma, \delta)$ distribution if a limiting distribution exists. There are not sufficiently many observations in District 13 to allow a detailed k-sum analysis, so we shall henceforth focus our attention on District 8. Since the spill data are time sequenced, we can form 10-sums by taking the observations in the natural order. But since we have observed that the frequency of spills is consistent with the Poisson process in (1), the formation of k-sums by randomly permuting the indexes of the sample observations 1, 2, ..., n is also a reasonable approach. This latter procedure will also be an indication of departure of spill frequency from the Poisson process to a certain extent. Agreement of the two types of k-sums indicates consistency of our approach and also is a check that the data are truly generated by a stable k-sum spill-volume law. Figure 6 gives the plot of the time-sequenced 10-sum empirical data from District 8, 1971, compared with the fitted distribution function. The fit is not especially good. Note in particular that the upper tail of the fitted distribution is well above the empirical upper tail. This implies that predicted large-volume spills would consistently understate those which would actually occur. Compare the randomly assigned 10-sum situation in figure 7. The fit is better but the upper tail of the fitted distribution is still above the upper tail of the empirical distribution.

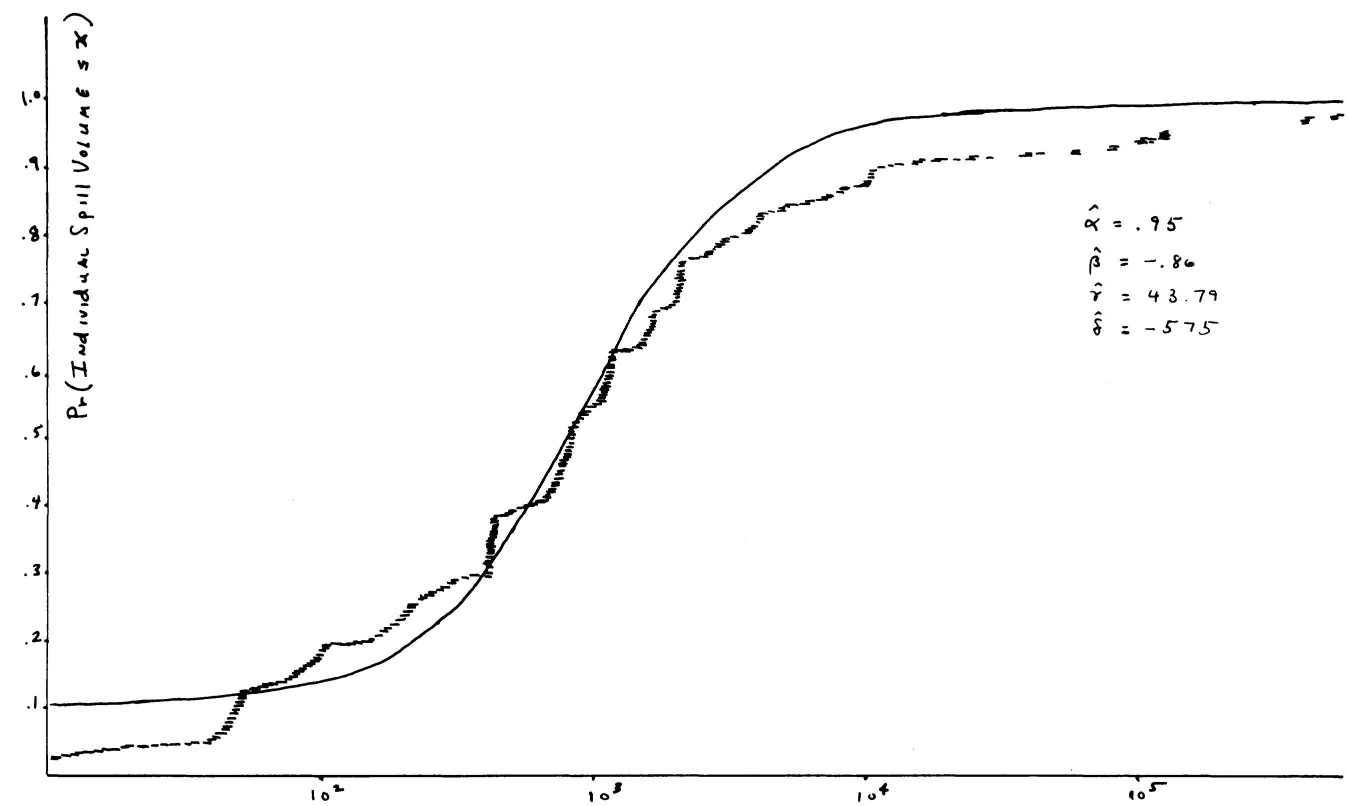

Figure 5. Comparison of fitted and actual spill volumes, District 8, inland waters, 1971

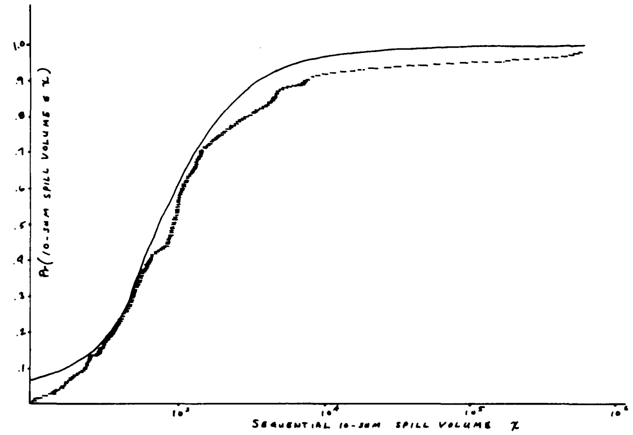

Figure 6. Comparison of fitted and actual sequential 10-sum spill volume, District 8, 1971

Figures 8 and 9 correspond exactly to figures 6 and 7 for District 8, 1972. The fits are appreciably better. A possible reason for this is the increased Coast Guard patrolling which resulted in a larger number of detected spills, presumably those which were not the large-volume spills (see figure 1). Notice in particular the good agreement between the α and β values across figures 8 and 9. Values of β of such magnitude indicate a very marked skewness to the right. Values of α of the magnitude shown indicate a fantastically risky situation as regards oil spills, since the mean is infinite. Again, the fitted distributions would understate the magnitude of very large spills for the reasons provided above. The Cramer-von Mises statistic, which measures the goodness of fit [7], is 0.27, 0.11, 0.14 for figures 7, 8, 9, respectively. This indicates reasonably good fits. Figure 10 depicts the agreement between empirical and theoretical distribution functions for 10-sums for District 8, 1972, inland waters. The Cramer-von Mises statistic is 0.06, a very good fit. Again, there is an understatement of the probability of very large-volume spills.

Let us indicate the effect of this understatement on the agreement between total actual spill and that which would result from utilization of the fitted distribution. Table 2 delineates the agreement. Observe that the actual spill (see table 1) falls between the 85th and 90th percentile of the fitted total-spillage distribution. One would prefer to see the actual spill fall around the 50th percentile. Thus, one readily sees that the discrepancy between the actual spill and the 50th percentile represents—and mangifies—the discrepancy between the upper tails of the empirical and fitted distributions. It is clear that the lack of fit in the lower tail is of little significance. One solution to this problem of understatement of large-volume spills and total spillage is to constrain (1) an upper fitted percentile to match an upper empirical percentile (e.g., 95th) in the k-sum approach or (2) the total actual spillage to match the "predicted" spillage at the 40th, 50th, 60th percentiles. The latter approach seems, from implementation with insurance data experience, to be superior. More effort in this vein is needed and is a subject of further research.

Discussion

We have seen that the risk-analytic approach results in reasonable predictions of large-volume oil spills and total oil spillage. With a little more research it should be possible to produce even better fits to empirical data describing oil spills so as to provide a convenient quantitative prediction of individual or total spillage and the ranges thereof. Data from the augmented PIRS file containing 1973-1974 spill records will be utilized in validating our results. Since $\alpha < 1$, the maximum probable spill and the total spill are subject to fantastic fluctuations (see table 1, District 13 data). This phenomena, if validated, must be considered in formulating policies

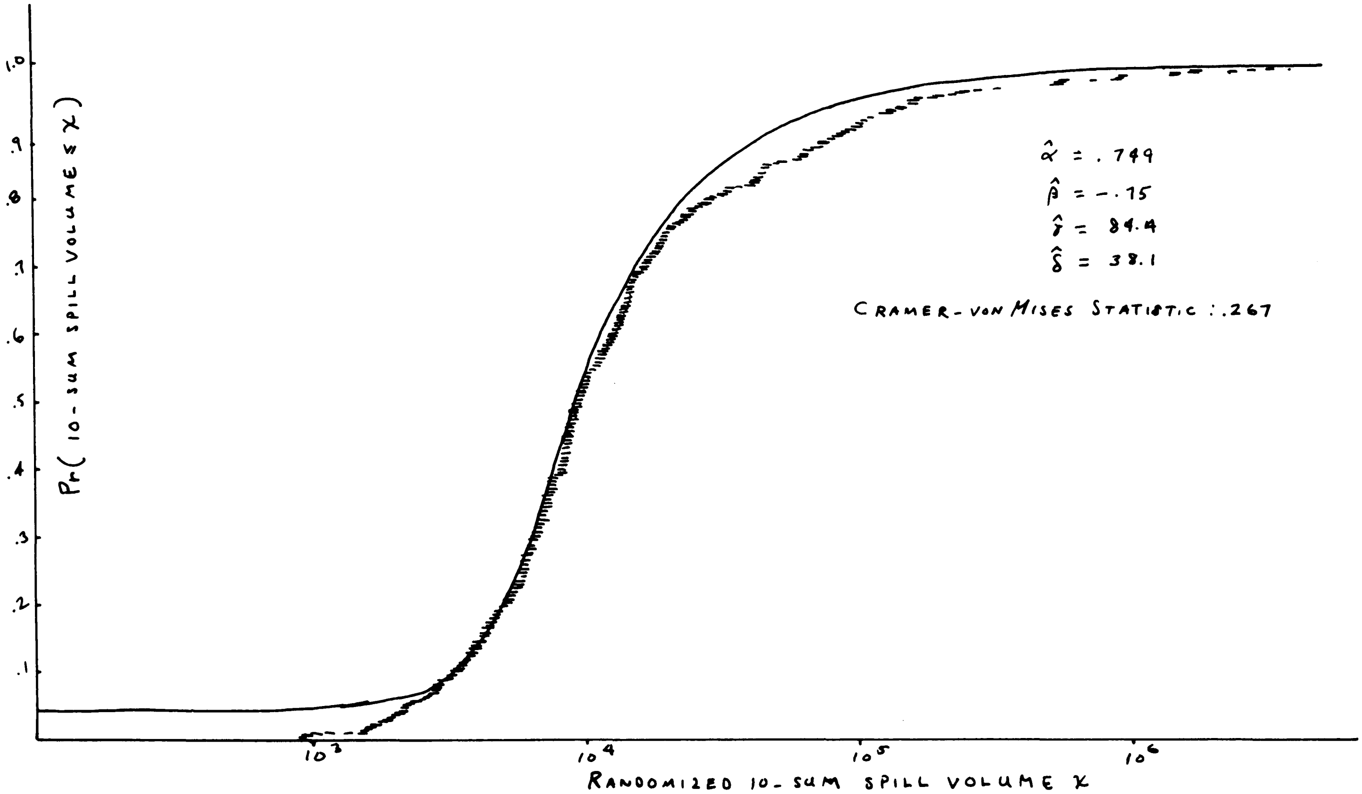

Figure 7. Comparison of fitted and actual randomized 10-sum spill volume, District 8, 1971

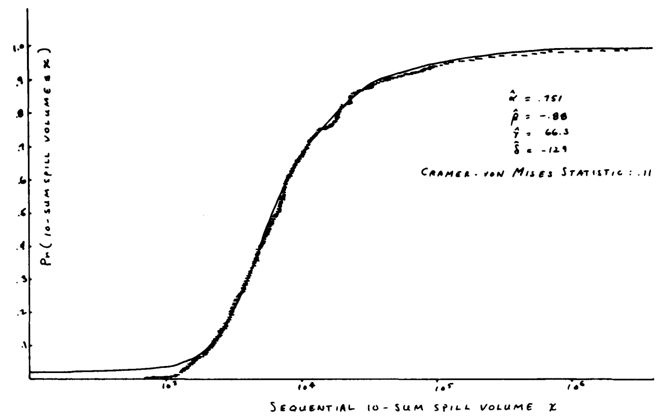

Figure 8. Comparison of fitted and actual sequential 10-sum spill volume, District 8, 1972

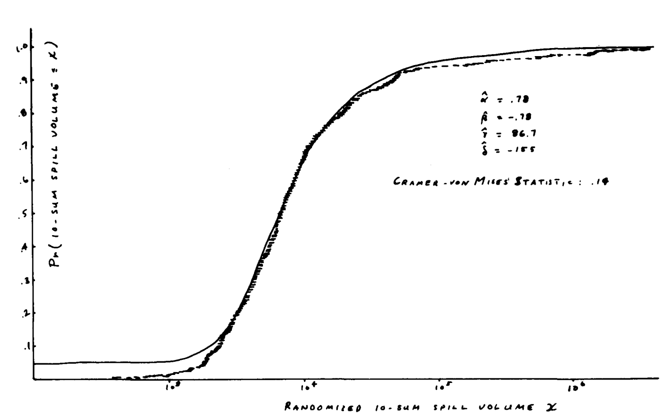

Figure 9. Comparison of fitted and actual randomized 10-sum spill volumes, District 8, 1972

for the control and cleanup of oil spills since catastrophic results have a relatively large probability of occurrence. In order to reduce these large fluctuations, it is necessary to reduce the frequency of spills or to increase, *ceteris paribus,* the value of α, or both. See table 3 which provides a range of possibilities based on the fact that (1) has standard deviation $\lambda 1/2$, thus implying that the number of spills can vary considerably.

Let us consider the managerial implications of this analysis for one agency concerned with oil spills, the U.S. Coast Guard. This agency was given the responsibility and authority for the protection on the marine environment from discharges of oil and other hazardous materials, and formed the marine environmental protection (MEP) division in 1971 to meet these requirements. Based upon limited data, MEP program management promulgated the guidelines for activities such as harbor patrols and monitoring of transfer operations in an attempt to prevent discharges; i.e., reduce the frequency of spills. In addition, a national response center and a work force were established in part to allow for the large-volume oil spill. Accurate predictions of total oil spillage and large-volume oil spills as given in this paper will enable MEP program management to more effectively allocate resourses and fit operational guidelines for the field units. For example, those locations with greater total spill volume could be assigned more personnel to monitor vessel transfer operations, while determination of the number and location of the U.S. Coast Guard teams that constitute the national strike force can consider the probability of large-volume oil spills at various locations. More importantly, the risk-analytic approach described herein permits trade-offs to be made among various managerial strategies, given finite resourses.

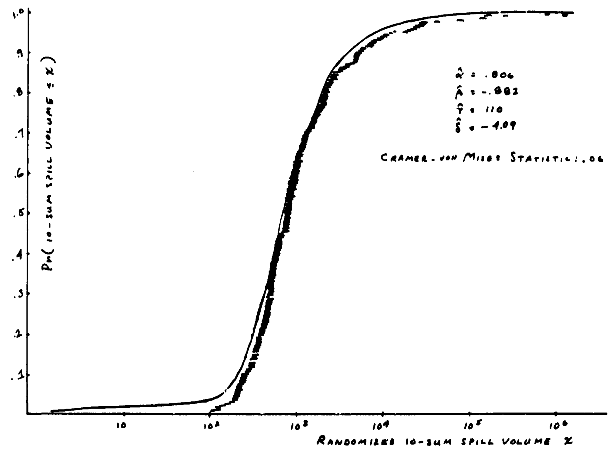

Figure 10. Comparison of fitted and actual randomized 10-sum spill volumes, District 8, inland waters, 1972

Table 2. District 8, 1972: values[a] T such that the probability of total spill volume in calendar year 1972 not exceeding T is f, given that the total number of spills is n = 3411

f	T
.10	759,353
.20	963,756
.30	1,164,145
.40	1,399,374
.50	1,706,836
.60	2,153,529
.70	2,898,204
.80	4,449,127
.90	9,692,958
.95	22,318,493

[a]The procedure describing computation of these values is given in Paulson et al. [5]. The actual observed spill falls at approximately f = .87.

ACKNOWLEDGMENT

We are grateful to the U.S. Coast Guard and the Department of Transportation for making available the PIRS file without which this research could not have been undertaken.

Table 3. District 8, 1972: values[a] T such that the probability of total spill volume in calendar year 1972 not exceeding T is f, given that the total number of spills is $N = n + k\sqrt{n}$, n = 3142, $k = 0, \pm 1, ..., \pm 5$[b]

k	-5	-4	-3	-2	-1	0
f/N	2860	2920	2970	3030	3080	3142
.10	621,215	638,580	653,142	670,723	685,461	703,240
.20	875,463	900,203	920,757	945,573	966,377	991,474
.30	1,088,313	1,118,794	1,144,354	1,175,216	1,201,088	1,232,300
.40	1,329,159	1,366,409	1,397,635	1,435,344	1,466,958	1,505,095
.50	1,640,165	1,686,142	1,724,697	1,771,248	1,810,275	1,857,356
.60	2,090,466	2,149,087	2,198,245	2,257,599	2,307,361	2,367,391
.70	2,841,902	2,921,622	2,988,475	3,069,193	3,136,866	3,218,506
.80	4,412,988	4,536,823	4,640,671	4,766,058	4,871,181	4,998,000
.90	9,760,039	10,034,016	10,263,771	10,541,183	10,773,765	11,054,349
.95	22,712,245	23,349,910	23,884,652	24,530,316	25,071,641	25,724,693

k	1	2	3	4	5
f/N	3200	3250	3310	3360	3420
.10	721,154	736,159	754,264	769,435	787,050
.20	1,016,763	1,037,943	1,063,502	1,084,920	1,110,759
.30	1,263,750	1,290,091	1,321,878	1,348,514	1,380,650
.40	1,543,524	1,575,712	1,614,554	1,647,101	1,686,370
.50	1,904,797	1,944,533	1,992,485	2,032,666	2,081,146
.60	2,427,881	2,478,547	2,539,688	2,590,922	2,652,737
.70	3,300,771	3,369,675	3,452,827	3,522,505	3,606,575
.80	5,125,334	5,232,830	5,362,001	5,470,241	5,600,840
.90	11,337,087	11,573,908	11,859,700	12,099,185	12,388,139
.95	26,382,758	26,933,954	27,599,132	28,156,531	28,829,070

[a]Computational procedures are given in Paulson, et al. [5].

[b]The maximum likelihood estimate of λ given one year's data in (1) is n; the standard deviation of n is $\lambda^{1/2}$. We estimate $\lambda^{1/2}$ by $n^{1/2}$. Thus $N = n + k\sqrt{n}$ coupled with f gives us a range of possible values for total spills in a calendar year. Noteworthy is the fact that if N were to be as large as 3420 (k=+5), as it is in 1972 (see table 2), then corresponding to the observed spill is the value f of approximately .87.

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