DERIVATION OF SPREADING PARAMETERS OF OIL AT SEA FOR OIL SPILL MODELING

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ABSTRACT

Oil spill models are commonly used to simulate the large-scale (tens to hundreds of kilometers) transport of oil spills in the oceans. The values of the spreading parameters of these models are obtained empirically by fitting to observed slicks, thus they do not account explicitly for the effects of waves. In addition, there is little success in using these values to predict the spread at smaller scales (tens of meters to a few kilometers). This works attempts to better understand the physics of oil movement in the ocean by focusing on the small-scale mechanisms. The investigation also leads to evaluation of small-scale spreading parameters.

The Random Walk Method is used in a Monte Carlo simulation framework to track the transport of oil due to the effects of waves, buoyancy, and turbulent diffusion. The small-scale spreading parameters are then calculated using the Method of Moments. Our results indicated that the approach for using a spreading coefficient becomes after a time equal to about 30 wave periods. This corresponds to a travel of the centroid of about two wave lengths. At larger scales, the longitudinal spreading coefficient increased with distance from the initial location and the lateral spreading coefficient became equal to the turbulent diffusion coefficient. The vertical spreading coefficient reached a value that is smaller than half of the turbulent diffusion coefficient, which is due to the presence of the upper boundary (the free surface) and buoyancy.

INTRODUCTION

Oil spill models are commonly used to simulate the large-scale (tens to hundreds of kilometers) transport of oil spills in the oceans (Al Rabeh *et al.*, 1989; Chao *et al.*, 2001; Reed *et al.*, 1995). Existing modeling approaches account for the intensity of waves (and their breaking) on dispersion, but not on spreading. The spreading coefficients used in oil spill models were estimated by fitting to observed large slicks. While these coefficients are useful for simulating large spills, there is no assurance that they are capable to simulating the spreading at small scale (hundreds to a few kilometers). In addition, the buoyancy of dispersed oil droplets opposes the downward motion due to turbulent diffusion. We derive in this work spreading coefficients that account for wave properties (wave length and height) and the oil droplet sizes. We limit our investigation to regular waves.

OIL SPILL MODEL

Governing Equation

If one assumes that the transport can be simulated by the convection-diffusion equation, then it can be theoretically proven that (Fischer et al., p 40, 1979):

$$E_x = \frac{1}{2}\frac{d}{dt}\left(\sigma_x^2\right) \quad (1a)$$

$$E_y = \frac{1}{2}\frac{d}{dt}\left(\sigma_y^2\right) \quad (1b)$$

$$E_z = \frac{1}{2}\frac{d}{dt}\left(\sigma_z^2\right) \quad (1c)$$

Where σ_X^2, σ^2, σ_Z^2 are the variances of the plume in the x, y, and z directions, respectively.

The variances of the plume could be obtained from an actual spill or based on the solution of Eq. 1. We adopt the latter approach in this work, because our goal is to establish theoretical relations between the spreading coefficients and wave and oil properties. Validation by comparison to actual (small scale spills) is left for future work.

Assuming regular waves propagating in the x direction, an oil droplet is transported by the velocities U, V, and W. The velocity U is in the horizontal along the direction of wave propagation; V is the horizontal velocity perpendicular to wave propagation; and W is the vertical velocity. These velocities are given by:

$$U = u + u_t \quad (2a)$$

$$V = 0 + v_t \quad (2b)$$

$$W = w + w_b + w_t \quad (2c)$$

Where u and w are the velocities due to wave motion. They are obtained in this work based on the second order wave theory,

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which is also known as Stokes theory (Ippen, 1966). For deep water waves, these velocities are given by:

$$u = -\frac{\partial \Phi}{\partial x} = \frac{H}{2}\frac{g k}{\sigma}\frac{\cosh k(h+z)}{\cosh k}\cos(k \; -\sigma t) + \frac{3}{16}\frac{H^2 \sigma k \cosh 2k(h+z)}{\sinh^4 k}\cos 2(k \; -\sigma t) \quad (3a)$$

$$w = -\frac{\partial \Phi}{\partial z} = \frac{H}{2}\frac{g k}{\sigma}\frac{\sinh k(h+z)}{\cosh k}\sin(k \; -\sigma t) + \frac{3}{16}\frac{H^2 \sigma k \sinh 2k(h+z)}{\sinh^4 k}\sin 2(k \; -\sigma t) \quad (3b)$$

where

H= wave height, g= acceleration due to gravity, k= $\frac{2\pi}{L}$ is the wave number, and _= $\frac{2\pi}{L}$ is wave frequency.

The velocity w_b is a rise velocity of droplets due to their buoyancy. It is approximated as the terminal velocity given by:

$$w_b = \sqrt{\frac{4g(\rho - \rho_d)d}{3C_d \rho}} \quad (4)$$

ρ = density of seawater; ρ_p= density of oil; d= diameter of oil particles; C_d = drag coefficient.

The velocities u_t, v_t, and w_t represents the random velocities of particles due to turbulence. They are typically simulated by writing:

$$u_t = \frac{X_{diff}}{\Delta t} = \frac{R\sqrt{2D_x \Delta t}}{\Delta t} \quad (5a)$$

$$v_t = \frac{Y_{diff}}{\Delta t} = \frac{R\sqrt{2D_y \Delta t}}{\Delta t} \quad (5b)$$

$$w_t = \frac{Z_{diff}}{\Delta t} = \frac{R\sqrt{2D_z \Delta t}}{\Delta t} \quad (5c)$$

Where X_{diff}, Y_{diff}, Z_{diff} are distances traveled in time Δt, and D_x, D_y, D_z are the turbulent diffusion coefficients in the three directions. The term R represents a Normal random number (i.e. obtained from a Gaussian distribution with zero mean and a variance equal to 1).

Numerical Implementation

The Random Walk Method is used to simulate the transport, the plume can be observed as a large but finite ensemble of small discrete quantities of mass. A collection of particles is moved from one position to another position under the effect of velocities. The new positions are expressed as:

$$x^{n+1} = x^n + U(x^n, y^n, z^n)\Delta t \quad (6a)$$

$$y^{n+1} = y^n + V(x^n, y^n, z^n)\Delta t \quad (6b)$$

$$z^{n+1} = z^n + W(x^n, y^n, z^n)\Delta t \quad (6c)$$

Where x^{n+1}, y^{n+1}, z^{n+1} = particle positions at new time level n+1, x^n, i^n, z^n = particle positions at old time level n, and U ,V, and W are the velocities of the particle at the time level n. Fig. 1 shows the simulation domain, which consists of a regular grid (i.e., the spatial intervals Δx, Δy and Δz are constant). Our model also supports irregular grids.

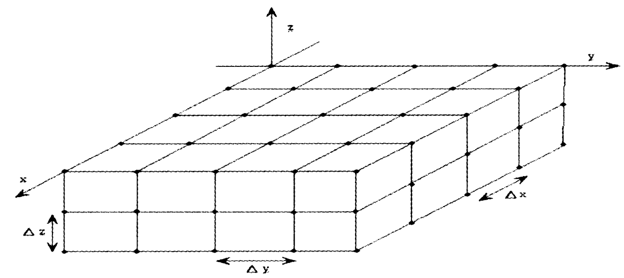

FIGURE 1: SPATIAL DISCRETIZATION OF THE STUDY AREA.

The time step was selected based on the Courant condition:

$$\Delta t = \epsilon \; * \min\left(\frac{\Delta x}{u_{max}}, \frac{\Delta y}{v_{max}}, \frac{\Delta z}{w_{max}}\right) \quad (7)$$

Where Cr= Courant number, representing the number of cells a particle will be allowed to move in any direction in one time interval, and the subscript "max" indicates the maximum velocity. We integrated Eq. 8 in time using a fourth-order Runge-Kutta scheme (Kreyszig, p 948, 1999).

Derivation of Transport and Spreading Parameters

In each simulation, the plume was considered as consisting of a high number of oil particles. For each simulation, the following was conducted. First the moments of orders q are computed in each direction. We show below those in the x direction for brevity. The equations for the moments in the y and z directions can be obtained by substituting y or z for x:

$$M_q(t) = \sum_{i=1}^{N}(X_i(t))^q m_i = \frac{4\pi}{3}\sum_{i=1}^{N}\rho_i (X_i(t))^q d_i^3 \quad , q=0,1,2 \quad (8)$$

where m_i, ρ_i, d_i are the mass, density, and diameter of an oil particle. The centroid in the x direction is:

$$X_c = \frac{M_1(t)}{M_0(t)} \quad (9)$$

The variance in each direction was then computed. In the x direction it is:

$$\sigma_x^2(t) = \frac{M_2(t)}{M_0(t)} - \left(\frac{M_1(t)}{M_0(t)}\right)^2 \quad (10)$$

The dispersion coefficient is computed next by:

$$E_x(t) = \frac{1}{2}\left(\frac{\sigma_x^2(t+\Delta t) - \sigma_x^2(t)}{\Delta t}\right) \quad (11)$$

After applying Eqs. 8-11 on each simulation, the average parameters were obtained by averaging the left hand quantities over all simulations:

$$X_{c,avg}(t) = \frac{1}{N_s}\sum_{j=1}^{N_s} X_{c,j}(t) \quad (12a)$$

$$\sigma_{x,avg}^2(t) = \frac{1}{N_s}\sum_{j=1}^{N_s} \sigma_{x,j}^2(t) \quad (12b)$$

$$E_{x,avg}(t) = \frac{1}{N_s}\sum_{j=1}^{N_s} E_{x,j}(t) \quad (12c)$$

where $X_{c,avg}(t)$ is the average centroid of all plumes at time t, $E_{x,avg}$ is the average spreading coefficient at time t, and N_S is the total number of simulations.

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RESULTS AND DISCUSSION

The waves were assumed regular and propagating in the x direction. The wave period was 1.5s and the wave height was 0.35m. The wave length was 3.5m. For each simulation, 650 oil particles were placed at the depth 0.01 m from the mean water level (essentially at MWL). Half of the particles had a diameter of 250_m and the other half had a diameter of 350_m. The oil density was taken as 750 kg/m^3. The turbulent diffusion coefficient, assumed isotropic and uniform, was taken to be 0.01m^2/s. Five hundred simulations were conducted.

Figure 2 shows the plume at select times. As time progresses, the plume appears to spread deeper in the water column. Fig. 3a reports the average centroid location as function of time. The figure shows that Xc increases rapidly initially, but later continues to increase at apparently a constant rate, as evidenced by the linear variation of X_c with time at times greater than about 50 seconds. The centroid travels about 12 meters in about 125 seconds (recall the wave period is 1.5 s). Thus the average horizontal motion speed of the centroid was about 0.1 m/s, which is about 5% of the wave speed m/s. Figs 3b and 3c report the variation of the spreading coefficients (Eq. 12c) as function of the average centroid (Eq. 12a). Noting that the wave length was about 3.5 m, one can conclude from this figure that a meaningful spreading coefficient occurs after the centroid of the plume travels about two wave lengths. From Fig. 3a, one sees that this correspond to about 50 seconds (about 30 wave periods). The variation of the horizontal spreading coefficient with scale afterwards appears to be a linear function of the travel distance of the centroid. The vertical spreading coefficient appears to level off after about two wave lengths to a constant value of about 0.004 m^2/s. Such a value is less than half of the turbulent diffusion coefficient. This could be explained by the fact that turbulence and buoyancy were the only mechanisms causing a net transport in the vertical direction (note that even according to the second order wave theory, there is no net displacement in the vertical after passage of a wave). The presence of the free surface would restrict the particles from going upward 50% of the time. Buoyancy has the net effect of opposing the downward motion, hence the spreading coefficient it is understandable that the spreading coefficient is smaller than half of the turbulent diffusion coefficient.

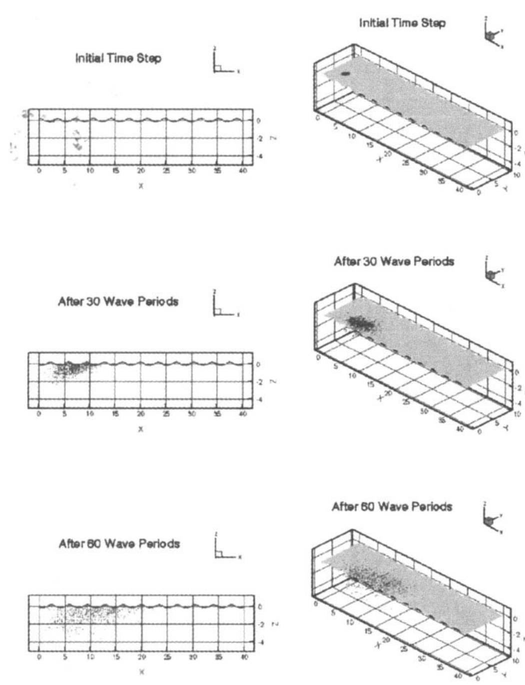

FIGURE 2: PARTICLE POSITIONS AT THREE TIMES

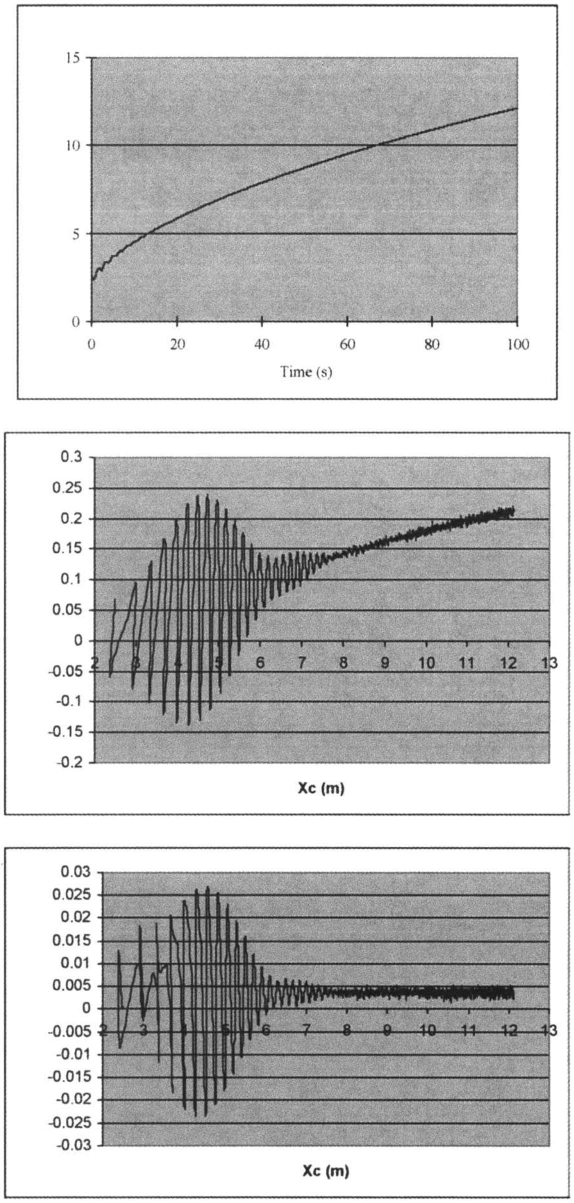

FIGURE 3: (A) VARIATION OF THE CENTROID WITH TIME AND VARIATION OF THE (A) THE LONGITUDINAL AND (B) VERTICAL SPREADING COEFFICIENTS WITH THE CENTROID LOCATION.

CONCLUSION

A three-dimensional particle tracking model was used to understand the transport of oil particles subjected to wave motion, buoyancy, and turbulence. The method of moments was used to derive the spreading parameters of oil under regular waves. It was found that the approach of using spreading coefficients becomes valid after the plume travels about two wave lengths, after which, the horizontal spreading coefficient varies linearly as function of the travel distance. The vertical spreading coefficient appears to reach a plateau at a value that is smaller than the turbulent diffusion coefficient. This work represents a first attempt for deriving small-scale spreading parameters based on wave properties. In the future, irregular waves will be considered along with breaking waves and various particle size distributions.

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