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Interpretation of the Positive-Degree-Days Factor by Heat Balance Characteristics – West Greenland

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W. Ambach

University of Innsbruck, Austria

From heat balance measurements in the ablation area of West Greenland (EGIG-profile, 1,013 m a.s.l., 69°40'N, 49°38'W) the positive-degree-days-factor is interpreted by the heat balance characteristics. The positive-degree-days-factor depends on the heat transfer coefficient and is a function of the share of the sensible heat flux density to the heat of melt. Using simple approximations the calculated positive-degree-days-factor is in good agreement with the measured value.

Introduction

The positive-degree-days method is successfully used in snow- and ice hydrology in order to determine the runoff from snow-covered and glacierized catchment areas. Some efforts have been made to improve this method by introducing the "Temperature-Snow-Function" (TS-function), where summer snowfalls were taken into account (Hoinkes and Steinacker 1975). However, the great advantage of the original method is that only a minimized number of parameters is to be measured for the evaluation. Recently this method was applied to ablation areas in SW Greenland (Braithwaite and Olesen 1984, 1984 a).

In the present paper, the dependence of the positive-degree-days factor on the heat balance characteristics is investigated. It is shown that the positive-degree-days factor is strongly influenced by the ratio of sensible heat to heat of melt. This ratio significantly varies with altitude and with distance from the ice margin. Therefore, the positive-degree-days factor can only be extrapolated to regions with similar heat balance characteristics.

Positive-Degree-Days-Factor

It is the intention of the present paper to exhibit a basic relationship between the positive-degree-days factor and the heat transfer coefficient. The numerical examples serve to underline the applied concept.

The heat balance equation reads

$$W_M = W_R + W_C + W_L + W_S \tag{1}$$

where W (kgm^{-2}) are water equivalents originating from the respective heat flux densities (MJm^{-2}). The subscripts mean M = melt, R = net radiation balance, C = conduction of heat, L = latent heat, and S = sensible heat. Positive signs are attributed to increased melt.

W_M is subdivided into two parts

$$W_M = W_T + W_{NT} \tag{2}$$

where W_T is proportional to the positive-degree-days value and W_{NT} is independent of air temperature. The approximation yields

$$W_T = W_S \tag{3}$$

$$W_{NT} \equiv W_R + W_C + W_L \tag{4}$$

However, this simplification cannot be accepted without criticism because of the possible existence of a positive feedback mechanism between W_S and W_R. At high values of air temperature and respective high values of W_S the albedo is low and therefore W_R is high. This feedback relation is not taken into account.

Per definition the following quantities are introduced

i) The positive-degree-days factor

$$\beta \equiv \frac{W_M}{PDD} \quad \text{kgm}^{-2}\text{K}^{-1}\text{d}^{-1} \tag{5}$$

ii) The heat transfer coefficient

$$\alpha = \frac{W_S}{PDD} \quad \text{kgm}^{-2}\text{K}^{-1}\text{d}^{-1} \tag{6}$$

iii) The non-dimensional number n

$$n = \frac{W_{NT}}{W_T} \tag{7}$$

PDD is the positive-degree-days value.

Note that β is proportional to W_M and α is proportional to W_S at the same positive-degree-days value. By introducing α, β, n into Eqs. (1), (3) and (4) the positive-degree-days factor yields

$$\beta = \alpha\,(1+n) \tag{8}$$

The positive-degree-days factor is proportional to the heat-transfer coefficient and is a function of the n-number (Ambach 1988).

Calculation of *n*-number from Heat Balance Characteristics

Calculation of the *n*-number from W_M- and W_S-values

From Eqs. (1), (3), (4) and (7) the n-number is

$$n = \frac{W_M - W_S}{W_S} \tag{9}$$

For $0 < W_S/W_M < 1$ a n-number of $\infty > n > 0$ results (Table 1). In general, W_M is the measured quantity whereas W_S can be estimated by applying Prandtl's boundary layer concept of adiabatic stratification. From Ambach and Kirchlechner (1986) it results

i) For ice surfaces:

$$W_S \equiv 6.34\ 10^{-6}\ b\ v_{200}\ T_{200} \quad (\mathrm{kgm^{-2}\,d^{-1}}) \tag{10}$$

$$\alpha \equiv 6.34\ 10^{-6}\ b\ v_{200} \quad (\mathrm{kgm^{-2}\,K^{-1}d^{-1}}) \tag{11}$$

ii) For snow surfaces:

$$W_S \equiv 4.42\ 10^{-6}\ b\ v_{200}\ T_{200} \quad (\mathrm{kgm^{-2}\,d^{-1}}) \tag{12}$$

$$\alpha \equiv 4.42\ 10^{-6}\ b\ v_{200} \quad (\mathrm{kgm^{-2}\,K^{-1}d^{-1}}) \tag{13}$$

Table 1 – n-number as a function of W_S/W_M and W_R/W_M.

W_S/W_M	n	W_R/W_M	n
0.0	∞	0.0	0.00
0.1	9.0	0.1	0.09
0.2	4.0	0.2	0.19
0.3	2.3	0.3	0.32
0.4	1.5	0.4	0.47
0.5	1.0	0.5	0.67
0.6	0.67	0.6	0.92
0.7	0.43	0.7	1.27
0.8	0.25	0.8	1.78
0.9	0.11	0.9	2.57
1.0	0.00	1.0	4.00

where b is the atmospheric pressure (Pa), v_{200} the wind velocity (m/s) and T_{200} the air temperature (°C) at a height of 200 cm above the surface. The different numerical factors for ice- and snow surfaces reflect the difference of the roughness parameters of those surfaces. The following numerical data are used (see notation):

$z = 200$ cm, $c_p = 1.01$ kJ/kgK, $\varkappa = 0.42$, $b_o = 101.3$ kPa, $\varrho_o \equiv 1.29$ kg/m^3, $z_{oW} = 0{,}2$ cm (ice), $z_{oW} = 0.01$ cm (snow), $z_{oT} = 6.10^{-4}$ cm (ice and snow).

The calculation of W_S is based on measurements or estimations of the averaged wind velocity v_{200}, the averaged air temperature T_{200}, and the atmospheric pressure b, corresponding to the altitude above sea level.

Calculation of the *n*-number from W_M- and W_R-values

In cases where no data of v_{200} and T_{200} are available, the n-number can be calculated by the ratio W_R/W_M. The following simplification is applied

$$W_C + W_L = -0.2\, W_R \qquad (14)$$

where $W_C + W_L$ is introduced as a share of W_R. The numerical factor -0.2 is obtained from heat balance investigations at the Western profile of the International Glaciological Greenland Expedition (EGIG-profile) at 1,013 m a.s.l. (Ambach 1977, Tab. 9b, p. 41).

With Eqs. (1), (3), (4) and (7) the n-number is

$$n = \frac{0.8\, W_R}{W_M - 0.8\, W_R} \qquad (15)$$

However, Eqs. (14), (15) may not be valid in general, rather with restriction in other regimes than the Western EGIG-profile. For $0 < W_R/W_M < 1$ a n-number of $0 < n < 4$ results (Table 1).

Numerical Evaluation

Numerical evaluation of the positive-degree-days factor is carried out for the following sites:

i) *Camp IV - EGIG*

Ablation area West Greenland, EGIG-profile, 69°40'N, 49°38'W, 1,013 m a.s.l.

The field data are (Ambach 1977, Tab. 9b, p. 41):

$$v_{200} = 7.4 \text{ m/s}, \; b = 0.9.10^5 \text{ Pa}, \; W_S/W_M = 0.262$$

The figure $W_S/W_M = 0.262$ is obtained by $W_S = 3.10$ MJm^{-2}d^{-1} and $W_M = 11.82$ MJm^{-2}d^{-1}.

From Eqs. (8), (9), (11) it results

$$\alpha = 4.22 \quad \mathrm{kgm^{-2}K^{-1}d^{-1}} \tag{16}$$

$$n = 2.8 \tag{17}$$

$$\beta = 16.1 \quad \mathrm{kgm^{-2}K^{-1}d^{-1}} \quad \text{(calculated)} \tag{18}$$

$$\beta \equiv 18.6 \quad \mathrm{kgm^{-2}K^{-1}d^{-1}} \quad \text{(measured,Ambach 1988,Fig.1,p 60)} \tag{19}$$

The measured β-value is based on the 24-hour averaged air temperatures of days with ice ablation (Ambach 1963, Tab. 42, p. 268) and on daily ice ablation measurements (Ambach 1963, Tab. 58, p. 301). The ice ablation measurements were taken at 10 stakes within an area of 80 m in diameter. A linear relation results between cumulative ice ablation and cumulative positive-degree-days values with $\beta = 18.6\ \mathrm{kgm^{-2}K^{-1}d^{-1}}$ (Ambach 1988, Fig. 1, p. 60).

ii) *Qamanârssûp Sermia:*

Ablation area SW Greenland, near the ice margin 790 m a.s.l. (Braithwaite and Olesen 1984).

With W_R/W_M = 0,57 (Braithwaite and Olesen 1984, Tab. 3, p. 160) and with the same α-value as in case (i) by Eqs. (8), (15) it results

$$\alpha = 4.22 \quad \mathrm{kgm^{-2}K^{-1}d^{-1}} \tag{20}$$

$$n \equiv 0.84 \tag{21}$$

$$\beta = 7.8 \quad \mathrm{kgm^{-2}K^{-1}d^{-1}} \quad \text{(calculated)} \tag{22}$$

$$\beta \equiv 7.3 \quad \mathrm{kgm^{-2}K^{-1}d^{-1}} \quad \text{(measured,Braithwaite and Olesen 1984, Table 4,p 164)} \tag{23}$$

For the Nordborggletscher similar figures were obtained ($W_R/W_M = 0.58$, $\beta = 7.1\ \mathrm{kgm^{-2}K^{-1}d^{-1}}$; Braithwaite and Olesen 1984, Tables 1 and 3). It is noteworthy that the same α-value was applied for Camp IV-EGIG and Qamanârssûp Sermia. This may be correct when the same roughness parameters, wind velocities, and atmospheric pressures are valid at the two sites.

The correction for nonadiabatic stratification is small and only amounts some percent due to high values of the shear velocity and low values of the gradient of the potential temperature above the surface. Therefore this correction was neglected.

Conclusions

By the present interpretation the following remarks are evident:

i) High shares of sensible heat flux density to the heat of melt are related to low positive-degree-days factors.

ii) Great ablation values are related to low positive-degree-days factors.

These conclusions are explained by the following considerations. In general, great ablation values originate from great sensible heat flux densities. Two cases are compared

$$\text{Case 1:} \quad \beta_1 \equiv [W_{NT}(1) + W_T(1)]/PDD(1) \tag{24}$$

$$\text{Case 2:} \quad \beta_2 = [W_{NT}(2) + W_T(2)]/PDD(2) \tag{25}$$

$$\begin{aligned} \text{with} \quad & W_{NT}(2) \equiv W_{NT}(1) \\ & W_T(2) \equiv 2\, W_T(1) \\ & PDD(2) \equiv 2\, PDD(1) \end{aligned}$$

The sensible heat flux density is doubled in case 2 where W_{NT} remains constant. Finally it yields

$$\beta_2 = [W_{NT}(1) + 2\, W_T(1)]\,/2\, PDD(1) \tag{26}$$

$$\beta_2 \approx \beta_1 - [W_{NT}(1)/2\, PDD] \tag{27}$$

$$\beta_2 < \beta_1 \tag{28}$$

Therefore it is concluded that low values of the positive-degree-days-factor correspond to high values of ablation.

The positive-degree-days factor significantly varies with heat balance characteristics. The dependence on altitude is given by the fact that high values of wind velocity and high values of air temperature occur in the lower ablation area. This results in a great relative share of the sensible heat in the heat balance. Therefore, small positive-degree-days factors are obtained in the lower ablation area, in agreement with actual measurements.

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Notation

b, b_o – Atmospheric pressure (b_o standard pressure)

c_p – Specific heat of the air at constant pressure

n – Factor of proportionality

PDD – Cumulative positive-degree-days value

T_{200} – Air temperature at 200 cm above the surface

v_{200} – Wind velocity at 200 cm above the surface

W_M, W_R, W_C, W_L, W_S – Water equivalent related to heat flux densities. Subscripts: $M \equiv$ Melt, $R \equiv$ Net radiation balance, $C \equiv$ Conduction, $L \equiv$ Latent, $S \equiv$ Sensible

W_T – Water equivalent related to heat flux densities proportional to positive air temperature

W_{NT} – Water equivalent related to heat flux densities independent of air temperature

z – Height above the surface

z_{oT} – Roughness parameter of the temperature profile

z_{oW} – Roughness parameter of the wind profile

α – Heat transfer coefficient

β – Positive-degree-days factor

$\varkappa$ – Karman constant

ϱ, ϱ_o – Air density (ϱ_o standard density)

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Address:

Institut für Medizinische Physik,
Universität Innsbruck,
Müllerstr. 44,
A-6020 Innsbruck,
Austria.